J.A. CULLER

GENERAL PHYSICS

ELECTRICITY,
ELECTROMAGNETIC WAVES,
AND SOUND

J.A. CULLER

GENERAL PHYSICS

ELECTRICITY, ELECTROMAGNETIC WAVES, AND SOUND

PREFACE

SEVERAL aims have been kept in mind in the preparation of the following text: (1) To make the descriptions, proofs, statements, and illustrations clear to the average student. (2) To emphasize the physical side of physics, point out its applications in the commercial world, and give more than an outline in the development of a topic. (3) To incorporate in the body of the discussions and in their proper place the electronic and electromagnetic theories now so well established.

The discussion of electricity begins with the experimental evidence which led to a belief in the electron, and the ordinary phenomena of electricity are explained in accordance with this theory.

Magnetism comes early in the discussion and emphasis is placed on the fact that a magnetic field is the result of an electric charge in motion.

The explanation of electrolysis is given in accordance with modern theories, and an unusual amount of space has been given to the very important subject of electromagnetic induction.

After a study of electricity the next natural and logical step is to electromagnetic waves. These embrace the whole subject of ether disturbance, including those short waves which produce light. Light is therefore treated as a subdivision of this general heading. Here also is introduced the evidence in favor of the belief that light is an electromagnetic disturbance resulting from rapid vibrations of electric charges, such, for example, as the spectroscopic effect when a source of light is in a magnetic field, the phenomena of radio-activity, etc.

No attempt has been made to produce a compendium of physics but rather a logical development of the live topics which, it seems, should be included in a text-book for college students.

We desire to thank the Leeds & Northrup Co. for cuts showing standard resistance, shunts, and inductance; the General Electric Co. for Figs. 101, 102, 106, 127, and 128; the Weston Electrical Instrument Co. for Fig. 69; Wm. Gaertner & Co. for Fig. 97; and the Electric Storage Battery Co. for Fig. 55.

J. A. CULLER.

CONTENTS

CHAPTER I

ELECTRICITY

CHAPTER II

MAGNETISM AND THE ELECTRIC CURRENT

CHAPTER X

ELECTROMAGNETS

CHAPTER XI

ELECTROMAGNETIC INDUCTION

CHAPTER XII

ELECTROMAGNETIC WAVES

CHAPTER XIII

SOUND

GENERAL PHYSICS

ELECTRICITY, ELECTROMAGNETIC WAVES, AND SOUND

CHAPTER I

ELECTRICITY

1. What Electricity Is.—Various theories as to the nature of electricity have been proposed from time to time. All have been of service in the advancement of science, and each in its time was probably the best that could be formulated from the experimental data then at hand.

A one-fluid theory was proposed by Benjamin Franklin in 1750. According to this theory electricity was assumed to be a self-repellant fluid distributed through all matter. All bodies in a normal state were assumed to contain a definite quantity of this fluid. It was then explained that a body deficient in its normal quantity of fluid would attract a normal body but would repel another body which was also deficient. This involved the assumption that bodies devoid of any electric fluid would repel each other.

Thus a body was regarded as positively charged when it contained an excess of electric fluid, and negatively charged when it contained less than what naturally belonged to it.

Another theory prevalent at that time was the two-fluid theory which assumed that there were two weightless and continuous fluids in every body of matter. One, such as that on glass when rubbed with silk, was called positive. The other, such as that on rosin or hard rubber when rubbed with cat fur or woolen cloth, was called negative. In a normal body these two fluids were assumed to be present in equal quantities, thus neutralizing each other. When a conductor was brought into an electric field, the two fluids present in it would be separated, one being repelled and the other attracted. In case of insulators such as mica, glass,

silk, etc., the fluids were not easily separated, and when bodies were electrified by rubbing one on the other only the surface layers were affected, an excess of positive fluid being found on one and an excess of negative on the other.

This theory was probably never intended as an explanation of the nature of electricity, but it has been valuable as a means of describing and investigating electrical phenomena and as such has been extensively used.

A modern theory which is offered in place of the two-fluid theory assumes that atoms of matter are composed of positive and negative particles or corpuscles. These may become detached from their atoms and are then free to move along a conductor under the influence of an outside electrical force. Insulators would then be explained as substances in which corpuscles are not easily separated from their atoms. The chief difference between this and the old two-fluid theory consists in the assumption that instead of two continuous fluids there are two kinds of discrete particles whose presence and movement cause the electrical phenomena which are observed.

A modern theory which seems most plausible because most nearly in accord with experimental results is a modification of the one-fluid theory. Each atom of a substance is here regarded as made up of a number of minute particles called corpuscles or electrons all of which are negative. These are very minute, having a mass of about $\frac{1}{1800}$ that of an atom of hydrogen. There is a great deal of experimental evidence tending to prove the existence of these minute negative particles. It is assumed that there is in each normal atom sufficient positive electricity to neutralize the electrons all of which are negative. Positive particles are always found to have nearly the mass of the atom. Consequently it is believed that in conductors there are many roaming electrons which have been detached from atoms and which may at times connect themselves to other atoms. Under the influence of an electric force these are set in motion, causing what is called the electric current. In solid conductors the positive portion of the atom does not move.

Whenever a body contains an excess of corpuscles it is said to be charged negatively, *i.e.*, there are more electrons than are needed to neutralize the positive. Whenever a body is deficient

in electrons it is said to be charged positively, for there are not enough electrons to neutralize all the positive.

It is unfortunate that the terms positive and negative were not reversed in their first application to the two kinds of electricity.

Preference will be given to this so-called electron theory in the following discussions.

2. Evidence for the Electron Theory.—A theory cannot be regarded as established nor is it accepted by the modern scientific world until experimental tests show that it is not only in accordance with actual results but that the line of laboratory tests which the theory suggests gives results which would be expected. A theory is often crude at first and yet may serve as a working hypothesis. From time to time it may be modified by experimental results until, if false, it is abandoned, or, if true, is established and generally accepted. It must not be understood, however, that a theory is first formulated and an effort then made to confirm it by experimental results. There is rather first a great mass of facts often isolated and not satisfactorily explained. A theory is then proposed which is in accord with these facts and which gives a rational basis for the explanation of all. The important thing in science is not the theory but the experimental facts, and the true scientist works diligently for the latter without being prejudiced by the former.

The electron theory is not the result of a sudden discovery but rather a growth from the accumulated evidence of years of experiment.

Until near the end of the 19th century the atom was the smallest particle of which science had any experimental knowledge. Experiments with electrons had been made before that time but they were not recognized as such. Now it is believed that the atom is composed of many perfectly distinct particles. The atom holds its place in our conception of matter just as before, but it is no longer regarded as an indivisible particle nor as the ultimate particle of which all matter is composed.

In Fig. 1 is shown a glass bulb with terminals A and C sealed in the walls. If the air is now pumped out until the pressure within is about .01 mm. of mercury and a current of high voltage, such as that from an induction coil or electric machine, is passed in at A and out at C, there will issue from C a stream of particles

called *cathode rays*. These were so called because, according to conventional terms, C is the cathode and A the anode when the current flows in the direction indicated. If the direction is reversed, cathode rays will stream from A. These rays move in a straight line normal to the surface from which they come. The glass at S will fluoresce with a greenish-yellow light. Various kinds of crystals will become luminescent when cathode rays are directed upon them. A light wheel will be vigorously turned when these rays are directed against the paddles on one side of the wheel. A metal disc placed in the path of the rays will effectively screen the space beyond.

It was at first thought that cathode rays were some kind of ether waves, but this notion has been entirely abandoned. Sir

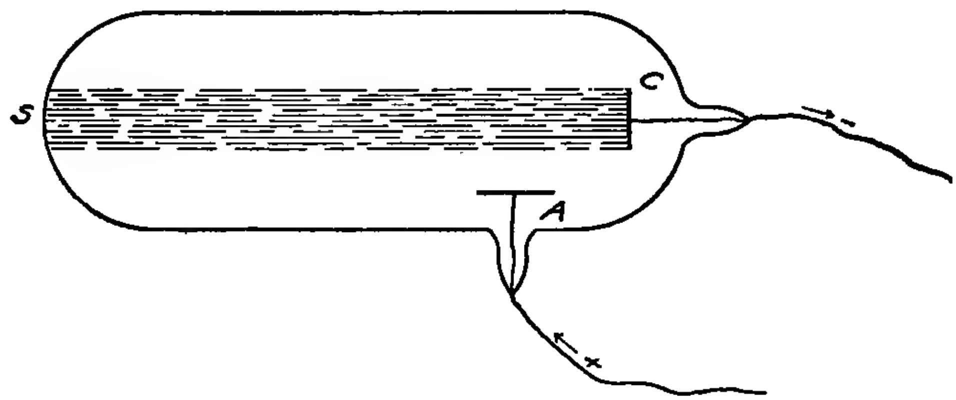

FIG. 1.

William Crookes maintained that cathode rays consisted of numerous material particles projected from the cathode. A strong evidence in favor of his claim is that when a magnet M (Fig. 2) is held near the tube as shown, the rays are drawn down between the poles of the magnet or repelled in the opposite direction, depending on the position of the magnet; *i.e.*, if one position attracts the rays. Then on turning the magnet so the poles are reversed the rays are repelled. This effect is just the same as that of a magnet on a conductor carrying a current of electricity (see Fig. 95). Ether waves are not affected in this manner by a magnetic field.

If the cathode stream is in fact composed of discrete particles charged with electricity it would seem at first thought that they would be about the size of ordinary atoms and so could not pass out through the walls of the tube any more than atoms of air

could pass into the vacuum within. Lenard in 1893 constructed a tube with a small aluminum window and, by directing cathode rays against the aluminum, showed that the particles which compose these rays pass out and move on through several centimetres of air before they are stopped, while at the same time no air could enter the tube. This seemed to be a strong objection to the claim that cathode rays were composed of projected particles of matter. But when it is once assumed that this cathode stream is composed of minute parts of atoms, each bearing about the same relation to the dimensions of the atom as a speck of dust to the dimensions of a room, it is seen that these minute particles could pass through the thin partition of aluminum and on for a distance into the air before they would all be stopped by atoms which might lie in their paths.

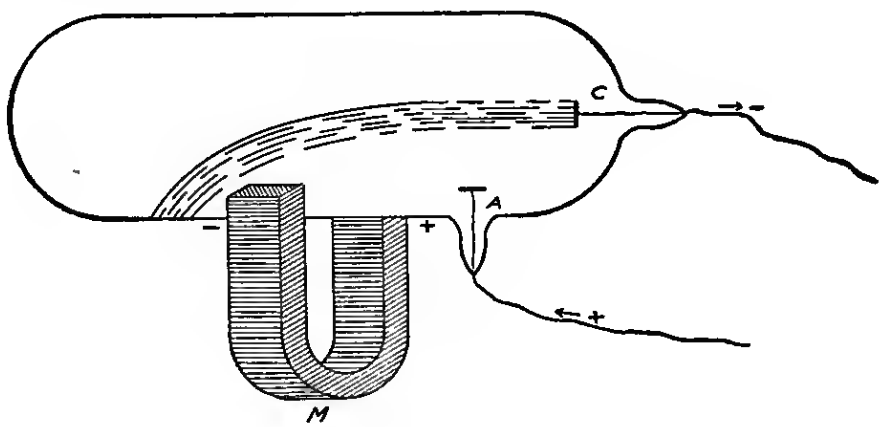

Fig. 2.

The mass of these small particles—electrons—have been calculated many times from different experimental data. The results always show a mass about $\frac{1}{1800}$ that of the lightest atom—that of hydrogen. A method of making this calculation is given in § 133.

An important property of cathode ray particles is that they carry a negative charge of electricity, *i.e.*, all electrons are negative. This may be shown by use of a tube designed by J. J. Thomson. Here *A*, Fig. 3, is the anode and *C* the cathode. A perforation in *A* permits some of the cathode rays from *C* to pass into the chamber *T* and fall upon the wall at some point *P*. Now by use of a magnet as explained above the rays may be deflected from their course, and instead of falling on *P* may be made to

fall on n which is a conductor connected at E to an electroscope. The conductor n is surrounded by a metal tube which is connected to the ground and thus shields n from any electrification except that caused by the cathode rays. The effect on the electroscope shows that cathode rays always carry a negative charge. The direction in which the rays are deflected by a magnet also shows that the charge must be negative.

Further evidence of the electron is found in radio-active substances such as uranium, radium, etc., which give off several kinds of rays as will later be described more fully. Among these are the *beta* rays, which possess properties almost identical with cathode rays and so are not atoms but minute parts of atoms which are being thrown off in natural changes which are taking place in matter.

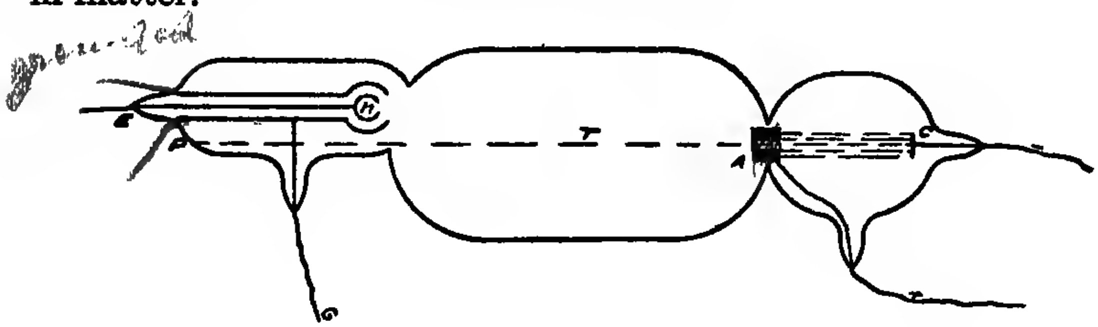

FIG. 3.

These negative particles called electrons—also, by some, called negative corpuscles—are the parts of the atom with which we can deal in experiment, *i.e.*, we get them off by themselves, observe their properties, and calculate their mass and velocity. They appear to be constituents of all kinds of matter. They do not differ when different electrodes and different gases are used in the vacuum tube. But if electrons are constituent parts of the atom, then, according to our present knowledge of electricity, it is necessary to assume that in each atom there is a quantity of electricity in some form which will just neutralize the negative of the electrons and give us the neutral atom with which we commonly deal. Rays which carry a positive charge are observed in experiment, but in all cases the particles which compose these rays have nearly the dimensions and mass of the atom itself. It is probable that these positive particles are only atoms from which one or more electrons have been detached.

Other strong evidences of the electron theory have been dis-

covered and will be described in connection with other topics. Enough has been said at this point to indicate the line of thought and experiment which has led up to this theory.

3. Kinds of Electricity.—Long before the corpuscular nature of electricity was known it had been agreed to call that charge *positive* which appears on glass when it is rubbed with silk. In some way, not well known, the contact and separation of glass and silk cause a deficiency of electrons on the glass and an equal quantity in excess on the silk. Likewise it had been agreed to call that charge *negative* which appeared on ebonite or hard rubber when these were separated after contact with fur or woolen goods. Here electrons are found in excess on the ebonite while the fur and wool have less than the normal number.

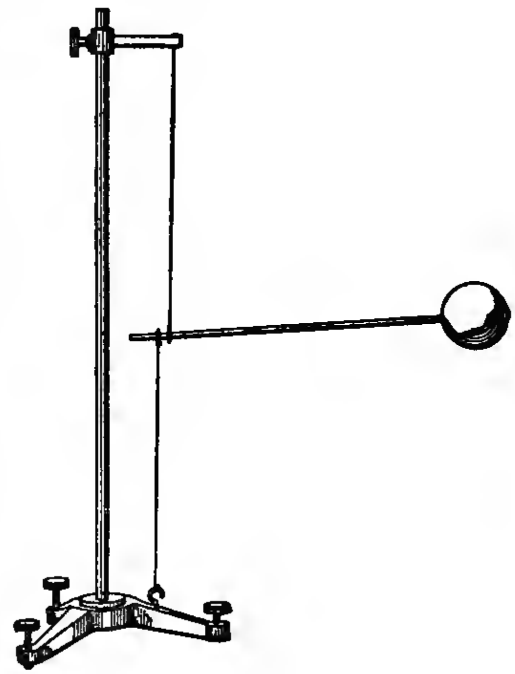

FIG. 4.

If a pith ball is suspended from a silk fibre and touched with a charged rod of ebonite, some electrons will pass from the rod to the ball, thus charging it negatively. The rod will then repel the ball. If a charged glass rod be touched to another pith ball some electrons will pass from the ball to the glass, leaving the ball positively charged. The two pith balls will now attract one another. Hence the law:

Like charges repel and unlike charges attract one another.

A convenient instrument for testing these and other phenomena of this so-called static electricity is shown in Fig. 4. A pith ball is attached to the end of a slender fibre of glass. The glass is supported by silk fibres as shown. The ball will swing horizontally in response to a very small force. If, by use of the ebonite rod, the ball is given more than its normal number of electrons it will be charged negatively and can then be used as an electroscope to detect the kind of charge in other bodies.

4. Quantity of Electricity.—The quantity of static electricity is measured by the force which it would exert upon a known

quantity at a given distance from it. If Q_1 and Q_2 are two quantities at a distance r from one another, then, as experiment shows, the force F between them will vary directly as the product of these quantities and inversely as the square of the distance between them. This is shown in formula by:

$$F \propto \frac{Q_1 Q_2}{r^2} \tag{1}$$

For unit quantity r is 1 cm. and Q_1 is equal to Q_2. Then when, in air, Q_1 is such a quantity that F is 1 dyne, Q_1 or Q_2 is the electrostatic unit of quantity. In words, *the electrostatic unit of quantity is that quantity which when placed at a distance of* 1 *cm. in air from an equal quantity will act upon it with a force of* 1 *dyne.* Equation (1) may then be written:

$$F = \frac{Q_1 Q_2}{r^2} \tag{2}$$

This will give the correct value of F in dynes, provided the experiment is made in air. Strictly, however, the definition should specify " in vacuum " instead of " in air," but the difference in the value of F would be very slight, and it is much more convenient to use air as a medium between the charges. If, however, some other substance such as a plate of glass, mica, or sulphur is interposed between the charges, the value of F is diminished in nearly all cases, only a few gases being exceptions. The substance between the charges is called the *dielectric*, and the ratio of the value of F in air to its value when some other dielectric is used is called the dielectric constant and is usually designated by K. Equation (2) for any dielectric may then be written:

$$F = \frac{Q_1 Q_2}{K r^2} \tag{3}$$

Values of K for various dielectrics are given in Appendix, Table 10.

5. Field of Force.—A *field of force* is any region in which force may exist. The *strength* or *intensity* of an electrical field is the force with which a unit charge, as defined above, would be urged if placed at that point. A *unit field* is one in which a unit charge is acted on by a force of 1 dyne. The *direction* of a field is the direction in which a positive charge would move if placed in that field.

A field is often represented by lines called *lines of force.* These are lines whose direction at any point is the same as the direction of the field. Each square centimetre of surface at right angles to the direction of the field is regarded as including as many lines of force as there are units of field intensity at that point. This is true of magnetic, gravitational, and electrical fields, though in the last the term *tubes of force* is frequently used instead of *lines of force*, and the number of tubes is numerically equal to the intensity of the field divided by 4π. By so doing the number of tubes indicates the number of unit charges on the surface of a conductor and also the electrical density on any portion of the surface. Hence we may say that 1 sq. cm. of surface at right angles to the direction of the electrical field will include as many tubes of force as there are units of intensity or field strength at that point divided by 4π. The reason for this will appear from the following: If a unit of electricity is included within a given area on the surface of a positively charged body and lines are drawn from every point of the boundary of this area in the direction of the field of force, these lines will include a tube of force. These tubes are always regarded as passing out from a positive charge and ending on a negative one. The greater the charge the greater the number of tubes of force per square centimetre at any point in the field.

Let a very small spherical conductor B, Fig. 5, be charged with Q units of positive electricity. Q tubes of force will then radiate from B in all directions. Let A be a portion of an imaginary surface of a large sphere concentric with B and at a distance r cm. The area of the surface of A is $4\pi r^2$ and it intercepts the Q tubes of force from B. Hence the number of tubes per square centimetre on A is $Q/4\pi r^2$. Also, since B is very small, its charge may be assumed to be at the centre of the sphere A. The strength of the field at a distance r from the Q units on B is Q/r^2 as shown by equation (2) and the definition of strength of field. But

$$\frac{Q}{r^2} \div 4\pi = \frac{Q}{4\pi r^2}$$

and it has just been shown that $Q/4\pi r^2$ is the number of tubes per square centimetre. Hence the number of tubes of force per square centimetre at right angles to the direction of the field is

equal to the intensity or strength of the field divided by 4π. Instead of drawing tubes, the field may be represented quite as well by drawing one line along the axis of each unit tube.

If two charges, equal but opposite in kind, are placed near each other, the lines or tubes of force would be as shown in Fig. 6, *A*, but if the charges are the same in kind, the lines would be as in *B*. In a uniform field the tubes are everywhere parallel and each has the same cross section throughout its length.

The method of representing a field by lines of force originated with Faraday. He regarded the lines as being under tension, *i.e.*, as elastic bands stretched between the two charges, and also, though not like elastic bands, as repelling each other laterally. The direction of the lines and the stress between the two charged bodies would point to such a conception. The two oppositely charged bodies in Fig. 6, *A*, would accordingly be drawn together while those at *B* would be pushed apart.

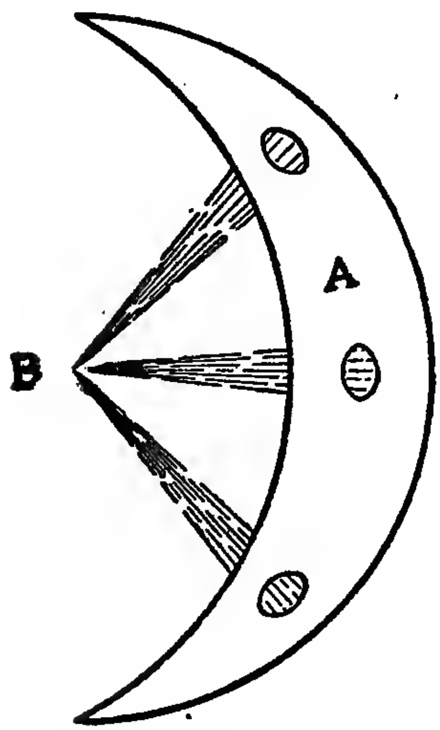

FIG. 5.

The old method of explaining attraction and repulsion of magnets and electric charges as "action at a distance" is simply a statement of a condition which could not be explained. It is not possible for one body to affect another except through some medium of communication between them. This medium is now believed to be the ether and Faraday's lines are not simply a convenient fancy, but represent a strain in the medium—the ether—in the region of the charged bodies. The bodies will then always move in such a direction as to relieve the strain and reduce the amount of potential energy.

6. Potential.—The potential of a point in an electrical field has the same meaning as potential energy in reference to a point in the earth's gravitational field. The potential of any point above the earth's surface may be defined as the amount of energy required to lift a unit mass from the earth to that point. To lift it to a greater height would require more energy or work and so

the potential energy would be greater. When the body is raised to a point at the limit of the earth's gravitational field, *i.e.*, to an infinite distance, its potential in reference to the earth is the greatest possible. It is customary, however, to regard the outmost boundaries of a field as having absolute zero of potential and to define *absolute potential* of any point in a field as the quantity of work required to bring unit mass—if the field is gravitational—from infinity to that point. In case of gravity the potential of a point defined in this manner is evidently negative, for work is not being done by an external agent against a resisting field but the field itself moves the mass nearer to the earth.

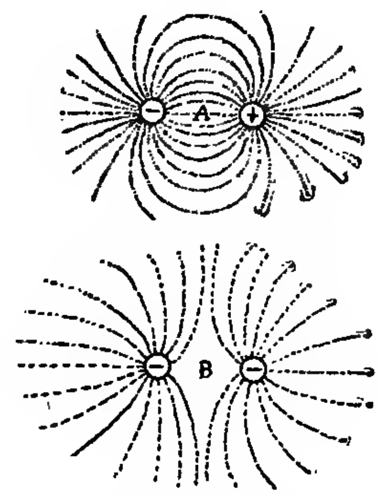

FIG. 6.

In a similar manner, *absolute electrical potential of a point is defined as the work required to bring a unit positive charge from an infinite distance to the point.* If the field is one due to a positive charge, the potential of any point in the field is positive, for work must be done by an external agent against a resisting field and so potential energy of the unit positive charge will be increased when it is moved nearer the charge which is repelling it.

The potential difference (P.D.) between two points is defined as the amount of work required to move unit positive charge from one point to the other.

When an electrical P.D. exists between two points on a conductor, electricity will flow from the higher to the lower potential. Hence if two oppositely charged bodies, or bodies having the same kind of charge but at different potential, are connected by a conductor, a current will flow from positive to negative or from a point of higher to a point of lower potential. This is the conventional direction of current. In fact, if the electron theory is correct, the only thing that moves in a solid conductor is the electron, and its motion is always from a negative to a positive charge or, in general, in such a direction as will relieve any strain caused by a separation of electrons from their atoms. We will see that in case of electrolytes and ionized gases both positive and negative charges move, but in solids the electron is the only

part that appears to move and its motion constitutes what is called an electric current. If, then, + and −, Fig. 6, *A*, are connected by a conducting wire, electrons will pass from − to + and in so doing will carry one end of lines of force with them. Thus the lines are shortened and finally disappear. Ether strain is thus removed as far as these electrons are concerned.

7. The P.D. Due to a Charged Point.—Let *A*, Fig. 7, be a point charged with Q units of electricity. It is desired to find the P.D. between p_1 at a distance r_1 from *A*, and p_2 at a distance r_2 from *A*. The strength of the field at p_1 is $\frac{Q}{r_1^2}$ and at p_2 is $\frac{Q}{r_2^2}$ (see section 5 and equation 2). By definition given above, P.D. is the work required to move unit positive charge from p_2 to p_1, *i.e.*, over a distance r_2-r_1. If the strength of field were uniform between p_2 and p_1, the amount of work could be easily obtained

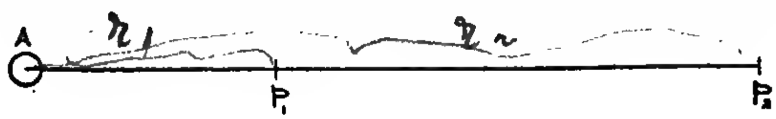

FIG. 7.

by multiplying the force, *i.e.*, the strength of the field, by the distance r_2-r_1. This force, however, varies inversely as the square of the distance from *A*. The average force in such a case is the geometric mean of the forces at p_1 and p_2, *i.e.*, it is the square root of the product of $\frac{Q}{r_1^2}$ and $\frac{Q}{r_2^2}$ or $\frac{Q}{r_1 r_2}$. This average strength of field times the distance gives the work or P.D. sought. Hence

$$\text{P.D.} = \frac{Q}{r_1 r_2}(r_2-r_1) = Q\left(\frac{1}{r_1}-\frac{1}{r_2}\right) \tag{4}$$

If the absolute potential, V_1, of the point p_1 is desired, move p_2 to an infinite distance from *A*. Then the P.D. between the two points will be the work required to bring unit positive charge from infinity to p_1. But if $r_2=\infty$, $1/r_2=1/\infty=0$. Hence, from equation (4),

$$\text{P.D.} = V_1 = Q\left(\frac{1}{r_1}-0\right) = \frac{Q}{r_1} \tag{5}$$

when not in air $\frac{Q}{k r_1}$

Therefore the potential due to a charged point varies directly as the charge and inversely as the distance from the charge. (See also Appendix 1.)

It may be shown that if this charge were placed on the surface of a sphere having A as a centre, the potential at p_1 would not be changed, *i.e.*, if the sphere were enlarged till its radius is r_1, the potential at its surface would be the same as at p_1 when the charge was at A.

8. Equipotential Surfaces.—An equipotential surface is one on which no work is performed in moving an electric charge from one point to another, *i.e.*, there is no P.D. between points on the

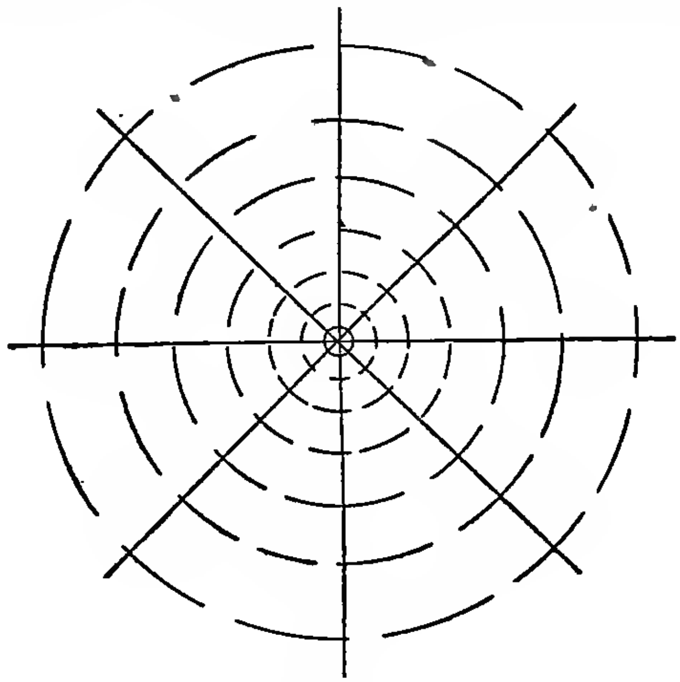

FIG. 8.

surface. Such surfaces are spherical shells concentric about a charged spherical conductor. There is a P.D. between the shells, but all points on the same shell have the same potential.

The tubes of force from an electric charge are always at right angles to the equipotential surfaces, otherwise there would be a component of force along the surface and this times the distance between two points would indicate that work must be done in moving a charge from one point to another on the surface. Thus when the lines of force in any electrical field have been determined, the equipotential surfaces may be drawn everywhere at right angles to these lines. The simplest representation of such a field

is shown in Fig. 8, where the radiating lines are lines of force and the circles are cross sections of equipotential shells around the charge.

9. Electric Induction.—Electric induction is the phenomenon observed when an insulated conductor is brought into an electrical field. Let *AB*, Fig. 9, represent a brass rod supported on a glass stand. According to the electron theory the uncharged brass contains a sufficient number of electrons to neutralize all the atoms there. Many of the electrons, however, may at any time be detached from their atoms and are then free to move under the influence of an electrical force. If a rod of hard rubber *R* is electrified by contact with cat fur, it is surrounded by a field of force. If then the brass rod is moved into this field the free electrons will be driven toward *A*, and consequently negative electricity

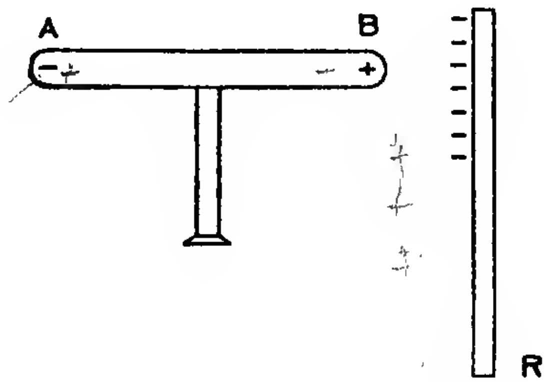

FIG. 9.

will be in excess of positive at that end of the rod while, in consequence of a deficiency of electrons at *B*, there will be an excess of positive at that end.

The fact of opposite charges at the ends of *AB* may be readily shown by use of the proof-plane *P*, which is simply a disc of metal attached to an insulating handle. If *P* is touched to *B*, electrons will pass from *P* to *B* and the disc will then be positively electrified, as may be shown by bringing it near an electroscope, such as the charged pith ball of Fig. 4. Now neutralize *P* by touching it to the hand or some body connected to the earth. Then touch it to *A*. The disc will receive an excess of electrons and will be negative, as the electroscope will show. On removal of the rod *R* from the region of *AB*, electrons in excess at *A* will be distributed throughout the brass rod and the atoms will again be neutral.

10. Charging by Induction.—If a conductor, as *AB* in Fig. 9, while in a negative electrical field, is touched at any point by the finger or by any conductor connected to earth, electrons will escape from it. If then the finger and afterwards the rod *R* be removed, the whole body *AB* will show a deficiency of electrons, *i.e.*, will be positively charged. This will be the case no matter at what point *AB* is touched, for contact of *AB* with another body increases the region into which electrons may be driven.

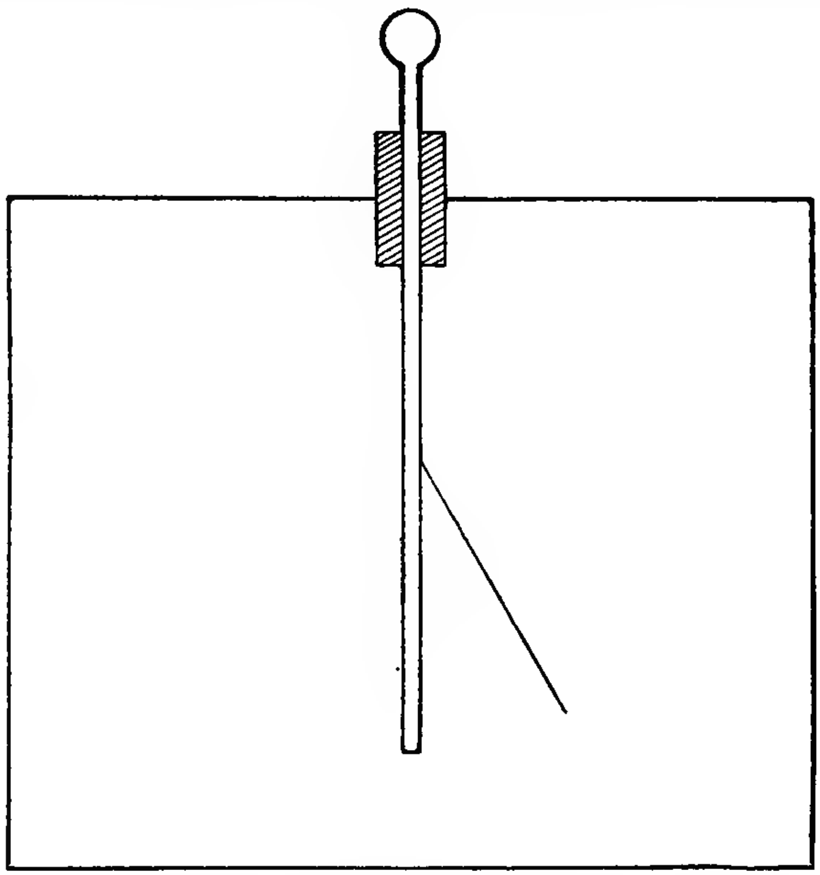

FIG. 10.

Then after contact is broken and the field removed, those electrons which were driven to the end *A* are not sufficient in number to neutralize the positive of the whole body.

If the field had been positive electrons would have passed from the finger into *AB*, which would then have been found to be negatively charged.

Such is the method of charging any body by induction. The gold leaf electroscope is usually charged in this manner. Gold is used because leaves of it can be made very thin and light. The leaves are attached to a conducting rod which is carefully insulated

from other bodies. Fused sulphur is a good insulator for this purpose. When this insulated rod with gold leaves attached is charged by induction either positively or negatively, the leaves will be driven apart, because like charges repel one another. In the form shown in Fig. 10 a single leaf is used which will swing away from a brass strip that is similarly charged. The electroscope can therefore be used to detect the presence and character of other charges. Suppose the leaves are charged negatively, then if a positively charged body is brought near the top of the electroscope, the leaves will fall toward one another because they have become neutral by the withdrawal of electrons from them. If the positive charge is brought still nearer or is made stronger, the leaves will fall completely together and will then diverge again, for so many electrons have been withdrawn that the leaves have become positively charged. In a similar manner the action of the leaves may be explained when they are first charged positively.

11. Capacity of a Conductor.—If an isolated conductor is given a charge Q, its potential is raised a certain amount, *i.e.*, a certain amount of work must be done to bring unit positive charge to the conductor. If the charge is doubled, the strength of the field is also doubled and so likewise the potential. If the conductor is made larger, the same quantity Q will not, however, cause as great a change in potential (see § 6). Capacity, C, may therefore be defined as the charge per unit change of potential, or as the ratio of the charge Q to the potential V. Expressed in formula,

$$C=\frac{\overset{\text{charge}}{Q}}{\underset{\text{potential}}{V}} \qquad V=\frac{Q}{C} \qquad Q=V\cdot C \tag{6}$$

Capacity of a conductor varies not only with size and shape but is also directly dependent on the dielectric constant K. Equation (3) shows that the strength of the field varies inversely as K. Hence when K increases, less work will be done in moving unit charge through the field, *i.e.*, the potential at the surface of the conductor is less. Hence when the dielectric is other than air, equation (5) should be written:

$$V=\frac{Q}{Kr} \tag{7}$$

Substituting this value in (6),

$$C=Kr \tag{8}$$

The capacity of a spherical conductor in air is therefore numerically

equal to its radius r, but, in another medium, capacity equals radius times dielectric constant.

12. Location of the Charge.—If a metal vessel such as a spherical shell with an opening to the inside, or a cylinder made of wire gauze, is connected to an electroscope and a charged metal ball suspended from a silk string is lowered into the vessel, a charge like that on the ball appears on the outside of the vessel and an unlike charge on the inside. This is simply a case of electric induction. If the ball is charged negatively—*i.e.*, has an excess of electrons—a number of electrons equal to this excess will be driven to the outside of the vessel and also into the electroscope. Hence there will be a deficiency on the interior walls which the excess of electrons on the ball will just be able to supply. If the ball is now touched to the inner wall and then removed, it is found to be neutral. The leaves of the electroscope still stand apart and are not affected by again touching the ball to the inner walls or by its removal. The excess of electrons driven to the outside remain there. Since electrons repel each other, they will occupy the surface of largest area, which is the outer surface of the vessel. This is Faraday's "ice pail experiment," so called because the vessel he first used for this purpose was a metal ice pail. His explanations, however, were not given in terms of electrons as here. Faraday also noted that there was a divergence of the gold leaves whether the vessel was filled with air, oil, or other nonconducting medium. Hence he gave the name dielectric to media of this character.

The fact of a charge appearing only on the outside of a vessel may be shown by charging any hollow metal body and then by use of the proof-plane and electroscope, testing the inner and outer surfaces.

13. The Field within a Hollow Charged Conductor.—When a closed hollow conductor is charged with electricity, the interior at all points is devoid of any resultant field intensity. The reason for this in the case of a spherical shell is apparent from a consideration of Fig. 11. When this sphere is charged, the electric surface density will be the same at all points. By density is meant the quantity of electricity on each square centimetre of surface. Let P be any point within the sphere. Let two lines be drawn through P, making very small angles with one another. These lines

enclose areas which are sections of cones with vertices at P, their bases s and s_1 being portions of the shell. Then if electric density is d, the quantity of electricity on s and s_1 is respectively sd and s_1d. The strength of field at P due to the charge on s is, by equation (2), sd/r^2, and that due to the charge on s_1 is s_1d/r_1^2. These forces are in opposite directions. In similar cones the areas of their bases are directly as the squares of the altitudes, *i.e.*, $s/s_1 = r^2/r_1^2$. Hence $ds/ds_1 = r^2/r_1^2$, *i.e.*, the quantities of electricity here considered vary directly as the squares of their distances from P.

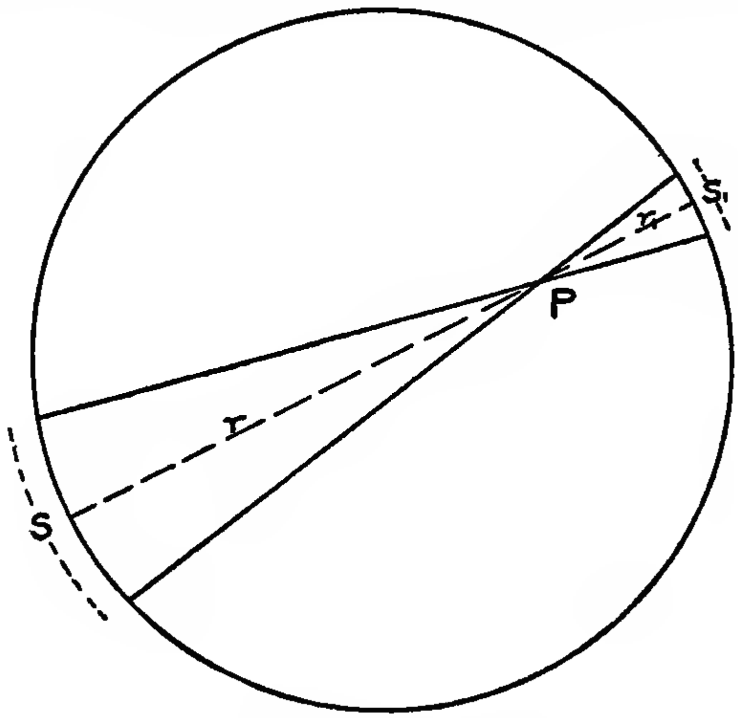

FIG. 11.

But the forces at P due to these charges vary inversely as the square of the distances. Consequently the forces at P are equal and opposite and so there is no resultant force at that point. Since the whole interior may be regarded as made up of similar sets of cones, and since P is any point within the sphere, what is true of P is true of any point within the sphere.

The same is true of an enclosed cylinder or other body. Faraday made a large cubical enclosure covered with tin-foil. When he entered this with a delicate electroscope, he could not detect any field of force while the outside was highly charged. A similar condition may be shown by inverting a wire gauze vessel over an electroscope and then electrifying the cylinder by contact with the charged ebonite stick.

14. Distribution of the Charge on a Conductor.—The charge on an insulated conductor is most dense at points where the curvature is greatest. On a sphere electrical density is uniform over the surface, but on other surfaces, such as that of an ellipsoid, density is found to be greatest at points of greatest curvature. An explanation of this may be found in the fact that adjacent portions of any given charge repel one another. Consequently the components a, a, Fig. 12, of forces driving electricity toward P are greater than a_1, a_1, which tend to drive the charge to P_1.

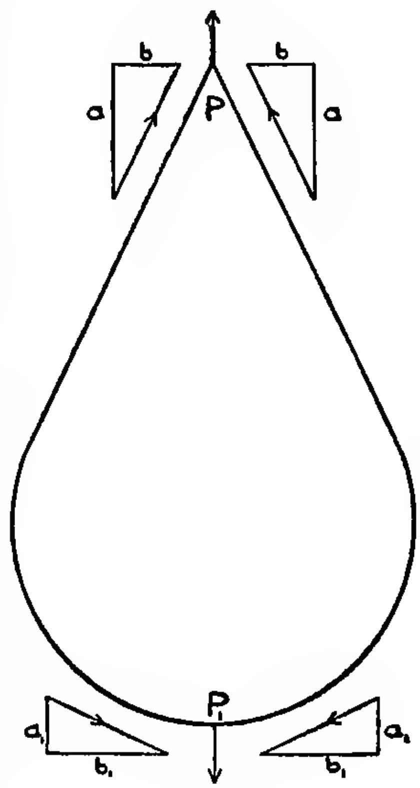

FIG. 12.

Another method of explaining the increased density at points is that since density is the quantity of electricity per square centimetre, that on the surface of a sphere of radius r_1, when charged with Q_1 units, is $Q_1/4\pi r_1^2$. Let another sphere of radius r_2, less than r_1, be in contact with the first sphere. It will contain a charge Q_2 and the density on its surface is $Q_2/4\pi r_2^2$. Since the spheres are in contact the potential, V, is the same on both. Then from equation (5), $Q_1 = Vr_1$ and $Q_2 = Vr_2$. Substituting these values of Q_1 and Q_2 in the above expressions for density,

$$\frac{d_1}{d_2} = \frac{\dfrac{\cancel{V}r_1}{\cancel{4\pi} r_1^2}}{\dfrac{\cancel{V}r_2}{\cancel{4\pi} r_2^2}} = \frac{r_2}{r_1} \tag{9}$$

This shows that the densities on the spheres are inversely as their radii. The greater the curvature the greater the density. Now when one portion of a surface has a greater curvature, *i.e.*, shorter radius, than other portions, the former may be considered part of the surface of a small sphere attached to a larger one. Hence the increased density on the portion of greatest curvature.

When the curvature becomes very great, as in case of fine

points, the strain may be such as to break down the dielectric. A discharge into the air follows, causing the so-called "electric wind," which may be detected by holding a candle flame in front of the point. The force which sets the air in motion would tend to move the point in an opposite direction, as is shown by the "electric whirl." A point discharge in the dark has the appearance of a brush and so is called a brush discharge. This is a common phenomenon in nature. On the masts of ships it is known as St. Elmo's fire.

15. The Electric Spark.—Gases are ordinarily poor conductors of electricity. Two bodies oppositely charged are almost perfectly insulated from one another by dry air between them as long as the P.D. is not too great. If two knobs are mounted as shown in Fig. 13 so that the space between them may be adjusted, then when one is charged positively and the other negatively a spark will pass between them whenever the P.D. is sufficiently great

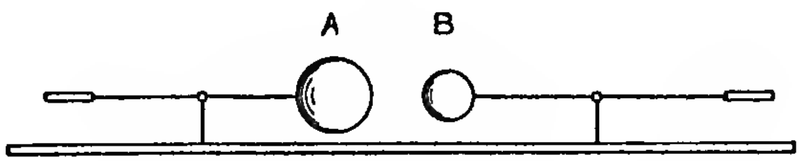

FIG. 13.

for that distance. If the knobs are pushed closer together, a smaller P.D. will cause a spark. The length of the spark gap depends on several conditions such as atmospheric pressure, the size and character of surface of the knobs, and the presence of ions in the air as explained below. It may be stated roughly that under ordinary atmospheric conditions, with knobs more than 2 cm. in diameter and more than 2 mm. apart, 30,000 volts are required for every centimetre of the air gap. This is the same as 100 electrostatic units of potential.

If, however, cathode rays or X-rays are projected into the region between the knobs, the air there becomes a good conductor and the knobs are soon discharged. When air is thus made a conductor it is said to be ionized. An atom from which an electron has been separated exhibits a positive charge, and one which carries an electron in excess of the normal number is negatively charged. Cathode rays, X-rays, a flame, and many other agencies will ionize a gas in this manner. As long as the ionizing agent

continues, the gas will conduct electricity in proportion to the number of ions present. After the ionizing agent ceases, ions will continue for some time in the gas but those of opposite charge will be gradually drawn together and neutralized; or, in an electrostatic field such as that between the knobs of Fig. 13, a positive ion will move with the tubes of force, *i.e.*, toward the negative knob, while the negative ion will move in the opposite direction. Thus the gas is soon cleared of most of its ions and again becomes a nonconductor. Now when the P.D. between the two knobs is gradually increased, the air at first acts as an almost perfect nonconductor, for there are very few ions present in normal air. With increase of P.D. a point is reached where the electric force will of itself ionize the gas and thus cause it to become a good conductor. A discharge then takes place in the form of an electric spark. Succeeding sparks readily follow because of ions produced by former sparks. A gas flame held between the knobs facilitates the passage of a spark because a flame ionizes the air.

16. The Energy of a Charge.—The energy of an electric charge is equal to the amount of work that must be done to produce the charge. Suppose a body A is devoid of any charge. Let unit charge be brought to it from an infinite distance or from some body such as the earth which, for purpose of reference, may be assumed to have zero potential. Both the charge and the potential of A will thus be raised. Let other unit charges be brought in succession to A until the total charge is Q and the potential V. Each increase in charge increased the potential and also the work required to bring up the next quantity of electricity. The potential at first was zero and increased uniformly to V. Hence the average potential is $V/2$. The total work, therefore, is the sum Q of all the unit charges times $V/2$. If work or energy is represented by W, then

$$W = \tfrac{1}{2} QV \qquad (10)$$

Substituting in (10) the value of V from (6),

$$W = \frac{Q^2}{2C} \qquad (11)$$

or by substituting the value of Q from (6),

$$W = \tfrac{1}{2} CV^2 \qquad W = \frac{CV^2}{2} \qquad (12)$$

17. Lightning.—Lightning is an electric spark of great magnitude which occurs between different portions of a rain cloud, between two clouds, or between a cloud and the earth. The origin of the electric charge in the atmosphere or in a cloud is not yet a settled question. Clouds are formed by condensation of moisture in air, but this cannot occur unless there are nuclei of some kind on which a droplet may begin to be formed. It has been shown by experiment that it is difficult to cause condensation of moisture in air which has been freed from dust particles, but if this air be ionized in the manner explained in § 15, the ions serve as nuclei and condensation readily occurs, first on negative ions and later on positive ones. If at any time there is a sufficient number of ions in air which is saturated with moisture, they may serve as centres of condensation and charges of opposite kind may thus be formed.

Each droplet is charged to a certain potential but when several of these unite into a larger drop the potential rapidly rises. Suppose n small drops unite into a large one, then since volumes of spheres vary as the cubes of their radii, the radius of the large drop will be $n^{\frac{1}{3}}$ times that of one of the small ones. Since the capacity of a sphere is proportional to the radius (§ 11), the capacity is increased $n^{\frac{1}{3}}$ times. Also, since potential is equal to charge divided by capacity and there are n small charges united on the large drop,

$$V=\frac{n}{n^{\frac{1}{3}}}=n^{\frac{2}{3}}$$

Hence the potential on the large drop is $n^{\frac{2}{3}}$ times that on one of the small ones, *e.g.*, if 27 small charged drops unite, the potential will be raised nine times. Thus, after the small drops are once charged, the union of these may produce a high potential. It does not seem possible, however, to secure by this means a potential sufficiently great to cause a lightning stroke one mile in length. Their lengths vary from a short stroke up to even two miles in length. A potential great enough to cause a spark one mile long would certainly produce an enormous brush discharge—a thing which is not observed. Most lightning strokes pass from one portion of a cloud to another portion. Comparatively few strokes reach the earth. It is possible that the potential in a cloud may not be very great and that a disruptive discharge at one point is

Spark

followed by a succession of discharges throughout the length of an electrified region. These follow so rapidly that they appear as one stroke. When a large body of electrified raindrops fall toward the earth, a stroke occurs between it and the earth.

A stroke of lightning may do much damage, particularly where numerous electrical lines lead to electrical plants, telephone centres, etc. To provide against this, various forms of "lightning arresters" are provided and by them lightning is diverted to the ground before it can pass into and destroy expensive machinery. A simple form of such arrester is a conductor from the line wire to the ground with a short air gap at some point in the conductor. The current on the line is therefore not grounded unless, as in case of lightning, the potential should rise to such a point that the current would cross the air gap and pass to the ground. A similar device consists in breaking the ground wire with a plate of mica. This prevents grounding of the ordinary current, but at a certain potential, depending on the thickness of the mica, the dielectric breaks down and the current is grounded. Another device with many advantages is the aluminum cell. This consists of aluminum plates covered with a film of aluminum hydroxide and placed in a suitable electrolyte. One terminal may be connected directly to the line and the other to the ground. The cell prevents the passage of a current until the potential rises to a certain critical point which can be fixed in the structure and operation of the cell. For any higher potential, as in case of lightning, the cell permits a free discharge to the ground.

In the protection of buildings, lightning rods are serviceable if good conductors of sufficient size and without breaks extend from moist earth, five or six feet below the surface, to sharp points which rise well above the roof and chimneys. These protect in two ways. First, the sharp points slowly discharge electricity to the ground and, by thus relieving the dielectric strain, minimize or prevent a stroke. Second, if a disruptive discharge occurs, the rod may serve to conduct it to the ground. A few terminals over a building, however, even when well grounded, are not a guarantee of protection from lightning. The interior of a metal building would not be injured by lightning. A building with a metal roof connected to ground at three or four points by rods or heavy copper wires will be fairly well protected.

18. Condensers.—A condenser is an arrangement by which the capacity of an insulated conductor is increased by the presence of another conductor, the two being separated by a dielectric. The second conductor is usually connected to the earth or to the opposite pole of an electrical generator. To understand how this arrangement increases the capacity of a conductor, let A, Fig. 14, represent a portion of a spherical conductor charged to a certain potential. Its field of force would extend out indefinitely in all directions, as shown in Fig. 8, and its potential is the amount of work required to bring unit positive charge through this field to the conductor. Now let a concentric spherical shell B be placed in the field near A and connected by a wire to earth. The potential of B will then be the same as that of the earth, *i.e.*, zero. Hence no work will be required to bring unit charge from the earth to B. The potential of A is now only the P.D. between A and B, for the only work done in moving unit charge to A is that caused by the P.D. between B and A. The P.D. between A and any point on B is the same as before B was placed in the field, hence the fall of potential at A is the same as that at B. To restore A to its original potential, so that as much work would be done in moving unit charge from B to A as was originally required to move it from infinity or earth to A, the sphere A must receive a greatly increased charge.

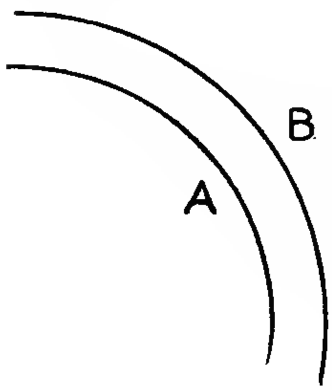

FIG. 14.

19. Capacity of a Spherical Condenser.—Let A, Fig. 15, be an insulated spherical conductor surrounded by a spherical shell B which is connected to earth. Let r and r_1 be the respective radii of these spheres and let A have a charge of Q units. The potential on the surface of A is Q/r (see equation 5). At a distance r_1 from the centre the potential is Q/r_1. When B was connected to earth its potential fell to zero, *i.e.*, it lost Q/r_1 units of potential. Since Q/r included the quantity Q/r_1 before B was reduced to zero, the potential V on A must now be

$$V = \frac{Q}{r} - \frac{Q}{r_1} = Q\left(\frac{1}{r} - \frac{1}{r_1}\right) = Q\left(\frac{r_1 - r}{rr_1}\right) \qquad (13)$$

Since capacity C is defined as the ratio of charge Q to potential V (see equation 6),

r^2

$$C = \frac{Q}{Q\left(\frac{r_1 - r}{rr_1}\right)} = \frac{rr_1}{r_1 - r} \tag{14}$$

The two conductors A and B are very close to one another and so no serious error will be introduced by using r^2 in place of rr_1. Then if $r_1 - r$ is represented by d,

$$C = \frac{r^2}{d} = \frac{4\pi^2}{4\pi d} = \frac{S}{4\pi d} \tag{15}$$

where S is the total surface of the sphere A. The values of C in (14) and (15) are correct when air is the medium between the

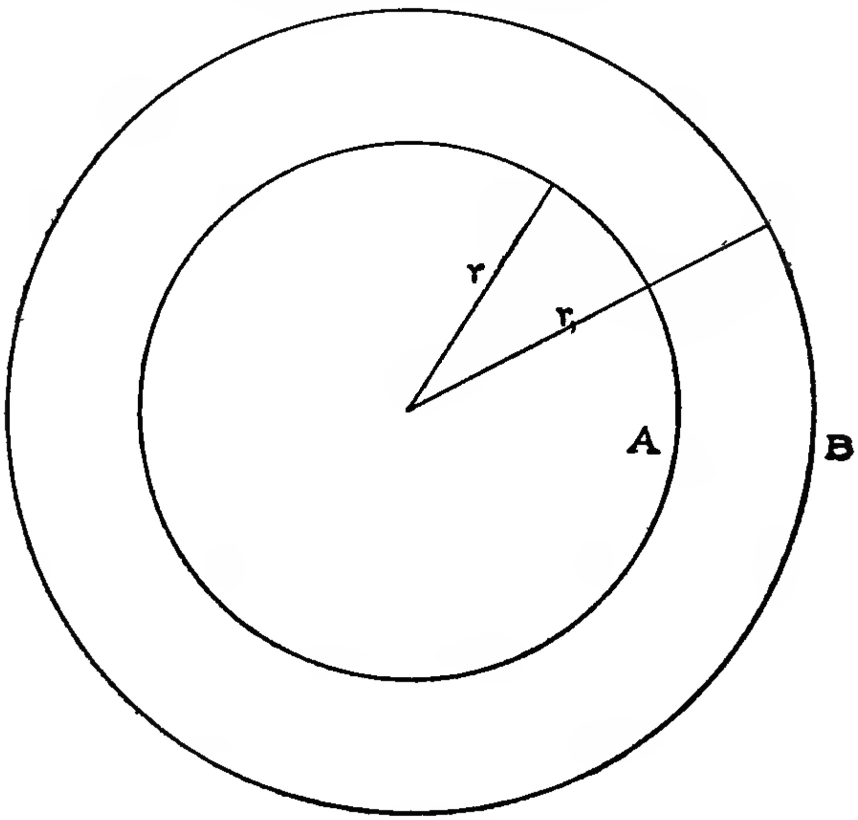

Fig. 15.

plates. For other media the values must be multiplied by the dielectric constant K. Thus

$$C = \frac{KS}{4\pi d} \tag{16}$$

In case of parallel plate condensers the area of the plates may be regarded as portions of the areas of very large spheres. The ratio of the area of one of the plates to the area of surface of this large sphere is the same as the ratio of the capacity of the plate condenser to that of the large spherical condenser. Hence if C_p stands for the capacity of the plate condenser and S_p for the area of one of the plates,

$$C_p = \frac{KS_p}{4\pi d} \tag{17}$$

where d is the thickness of the dielectric.

Plate condensers, instead of being composed of two large plates separated by a dielectric, are built up in the manner shown in Fig. 16, where sheets of tin-foil or other thin metal are separated by thin sheets of mica, paraffined paper, or other dielectric. The alternate sheets of foil are connected to A and the others to B. In this way a condenser of large capacity can be made up in compact form. In calculating capacity by use of equation (17), S_p is the area of all the plates on one side, *i.e.*, the area of all the foil connected to A in Fig. 16, or, in other words, S_p is the area of that portion of the sheets of mica or other dielectric which is covered by tin-foil on both sides. The reason for this will appear on a reconsideration of § 18 and Fig. 14.

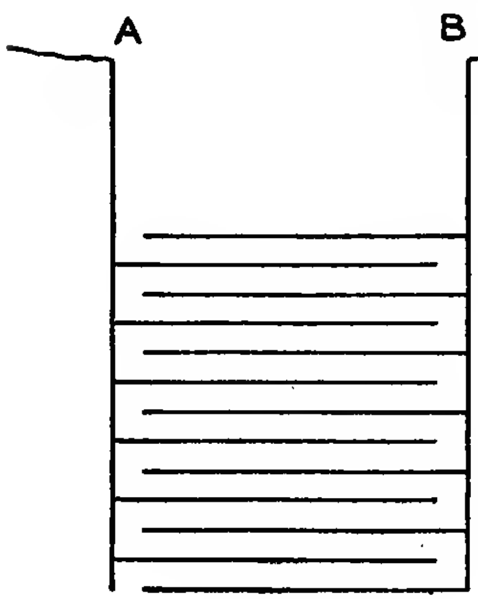

FIG. 16.

To charge a condenser, let A of Fig. 16 be connected to one terminal of an electric machine or battery and B to the earth or, better, to the other terminal of the machine.

Another common form of condenser is the Leyden jar, which is simply a glass jar covered inside and outside to about three-fourths of its height with tin-foil. The jar with its coverings may then be regarded as a plate condenser in form of a cylinder. Its capacity may be roughly calculated by use of equation (17).

In calculating the capacity of a condenser by use of equation (16) or (17), the quantity obtained is, according to equation (6), the number of electrostatic units of quantity that must be placed on one plate, the other being connected to earth, to cause a change of one erg in the amount of work that must be done in moving unit charge from one plate to the other, *i.e.*, to cause unit change in the P.D. between the plates.

Further consideration of condensers is given in later chapters.

20. Electrometers.—To measure the P.D. between two plates which are oppositely charged, as in a condenser, various instruments have been devised. In one invented by Lord Kelvin, the plates A and B, Fig. 17, are connected respectively to any two points whose P.D. is sought. A disc cut from the central portion of the upper plate is suspended from one arm of a balance and may move freely up or down. Sights are provided to determine when the disc is exactly in the plane of A. The P.D. between the plates will be the same as that between the points to which they are connected.

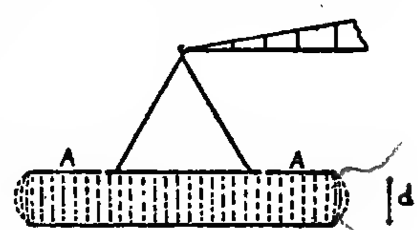

FIG. 17.

The disc will then be attracted toward the lower plate with a force F, which may be determined by placing weights in the pan at the other end of the balance beam until the disc is returned to its position in the plane of A. The part of the plate A surrounding the disc is called the guard ring. It insures a uniform field between the disc and an equal portion of B directly beneath. At the edge of the plates tubes of force do not run straight across but curve outward from edge to edge, hence the field there is not uniform. Now let S be the area of the disc and d the distance between the plates. This forms an air condenser whose capacity is

$$C = \frac{S}{4\pi d} \qquad \text{(See equation 15)}$$

From equation (12) the energy is

$$W = \tfrac{1}{2} CV^2$$

Eliminating C from these two equations,

$$\frac{2W}{V^2} = \frac{S}{4\pi d}$$

Since the field is uniform the work which must be done in moving the disc from B to A may be expressed by Fd. This may then be put in place of W above. Hence

$$\frac{2Fd}{V^2}=\frac{S}{4\pi d}$$

$$\therefore V=\sqrt{\frac{8\pi d^2 F}{S}} \qquad (18)$$

Knowing S, d, and F it is possible to calculate the number of ergs of work required to carry unit electrostatic charge from one point to the other, *i.e.*, to find the P.D. between the points. Since P.D. is found in this manner and not by comparison with some standard, this instrument is called an *absolute electrometer*.

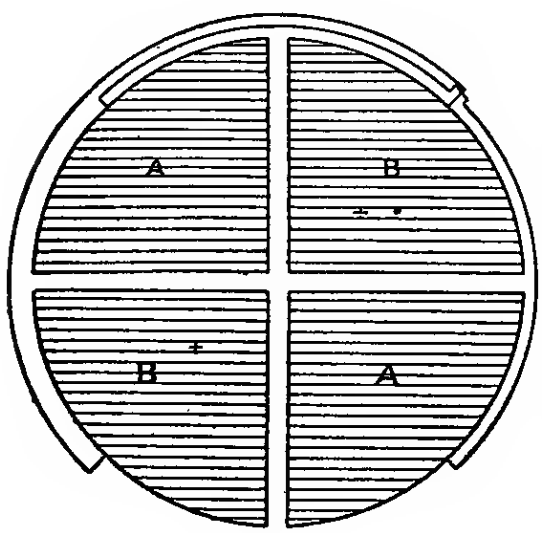

FIG. 18.

Another instrument used for a similar purpose but much more sensitive than the one just described, is called the *quadrant electrometer*, also an invention of Lord Kelvin. A simplified form now in common use is the Dalezalek electrometer. The instrument consists essentially of a metal drum, Fig. 18, which is divided into quadrants or quarters, each quadrant being supported on an insulating pillar and connected by a wire with the one opposite, forming two pairs. Within this drum is suspended a "needle" of light silver paper attached to a stem which carries a small mirror. The suspending fibre is quartz or a fine bronze wire. Now let all the quadrants be momentarily grounded to bring them to the same potential. The needle is then turned so that its position will be symmetrical in reference to the two pairs of quadrants, *i.e.*, will stand in the line separating the A and B quadrants. The needle is then charged to a high potential. It will not turn toward the A or B quadrants for they are equal in potential. If now the A quadrants are connected to earth and the B quadrants to a body which is charged, say, positively, then the needle, if positive,

will be repelled by the positive quadrants and will turn until its turning moment equals the torsion of the suspending fibre. The deflection of the needle is determined by the movement of a beam of light reflected from the mirror onto a scale or by means of a telescope and scale. If this observation is made for a known P.D. in the pairs of sectors, then, assuming that the deflection of the needle is proportional to P.D., an unknown P.D. may be determined.

This instrument is also valuable in other ways, *e.g.*, in observing the rate of electric discharge, the activity of an ionizing agent, or the strength of radium and other like substances.

21. Specific Inductive Capacity.—The capacity of a condenser differs when different substances are interposed between the plates. The value of a substance in this respect is usually indicated by K and is called the dielectric constant. It was named by Faraday the *specific inductive capacity* and is defined as the ratio of the capacity of any condenser to the capacity of the same condenser when air is used as the dielectric. An approximate value of this constant for various dielectrics is given in the appendix.

22. Residual Charge.—When a condenser, such as a Leyden jar, is highly charged, it may be discharged by means of a wire one end of which touches the outer coating and the other is brought near the knob. A bright spark passes and the jar appears to be completely discharged. A little later, however, another discharge, much weaker than the first, may be obtained. This is called a residual charge. Since the foil or metal plates of a condenser are good conductors, it appears that a residual charge must in some manner depend on the slow recovery from a strained condition of the dielectric. The importance of the dielectric in this respect may be strikingly shown by use of a dissectible Leyden jar. Let such a jar be charged, then remove the two coatings. These may be handled or touched together, but no evidence of a charge will be found in them. When the coatings are then replaced, the jar is found to be charged as strongly as before it was dissected.

If the glass while separated from the coatings is touched with the hand over its entire surface, the charge is removed. It appears from this that the metal plates simply serve as distributors of the charge to all parts of a dielectric, the latter being always a

nonconductor. The inner and outer walls of the glass jar possess charges of the same sign as that of their respective coatings before removal. Apparently electrons are not easily detached from atoms of a dielectric nor can they make their way through substances of that kind. Hence opposite charges on the coatings cause a strained condition in the glass and, like most mechanical strains, a complete recovery does not at once follow the removal of the stress. Therefore several residual charges, each smaller than the preceding one, may be obtained from a condenser.

23. The Electric Machine.—The fundamental principle of the electric machine may be shown by a simple device called the electrophorus. This, as shown in Fig. 19, consists of a plate R of hard rubber, rosin, or any nonconductor that may be electrified.

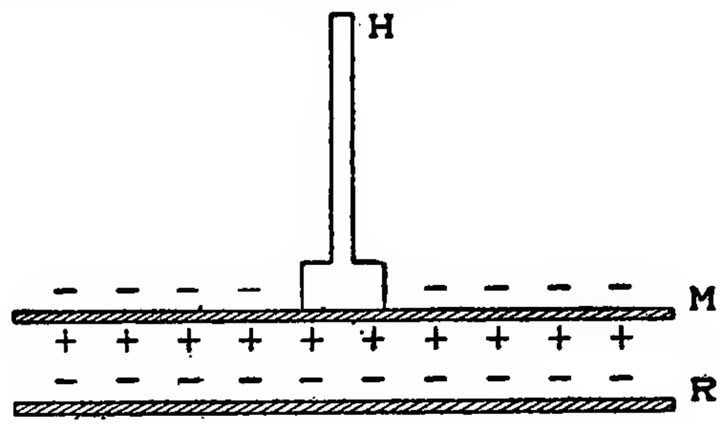

Fig. 19.

Upon this is placed a metal disc M. By use of the insulating handle H, M may be lifted from R. Let R be electrified negatively, *i.e.*, have a greater number of electrons than is needed to neutralize the positive there. This may be done by striking the rubber with cat-fur or woolen cloth. When M is now placed on R, the negative charge there acts inductively on the metal plate, driving a number of electrons to the upper side and thus leaving the lower side positive. Let plate M, while still on R, be touched with the finger or any conductor that leads to the earth. Electrons will escape from the plate. If M is then lifted and removed from the influence of R, it is found to be positively charged, and if brought near to some conducting body, electrons will flow to the plate and restore its neutral state.

This operation may be repeated indefinitely, and the charges thus obtained may be stored in an insulated conductor or a con-

denser such as a Leyden jar. These repeated operations do not impoverish the charge in *R*. The electricity produced represents a certain amount of energy. This, however, comes, not from the charge on *R*, but as a result of work done in separating the two opposite charges when *M* is removed. The action of *R* on *M* is only inductive and there is no transference of electric charges from one to the other.

An electric machine constructed in accordance with the principle just described may be made continuous and automatic in its operation. This will be apparent from Fig. 20. A glass plate

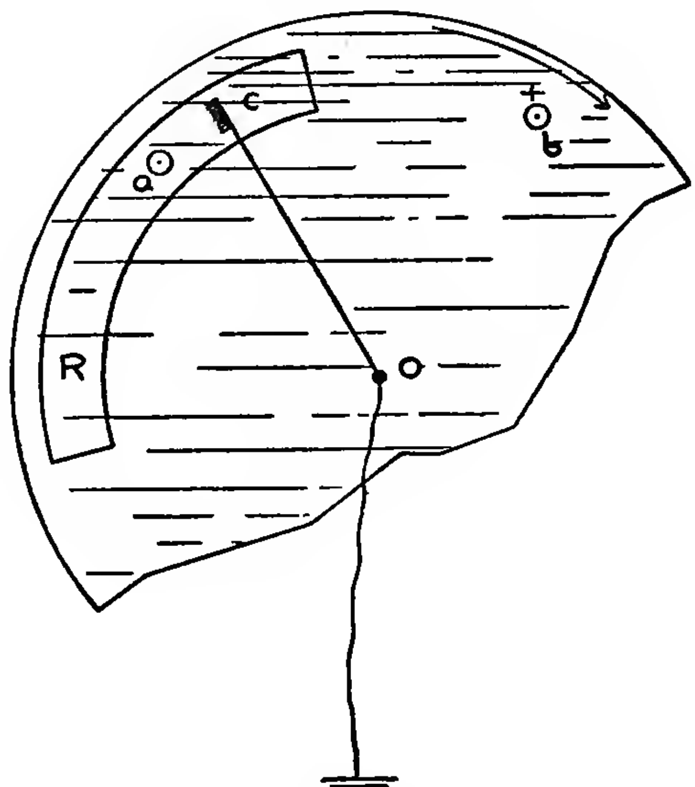

FIG. 20.

which may be rotated on the axis *o* has a number of small metal discs fastened on its front surface, as *a* and *b*. These correspond to the upper plate of the electrophorus. Behind the glass plate is a stationary nonconducting body *R*, which corresponds to the lower plate of the electrophorus. If *R* is now charged, say negatively, it will act inductively on *a*, and by turning the glass plate in the direction indicated by the arrow, *a* is made to touch the metal brush *c*. This is the same as touching the upper plate of the electrophorus. Under the influence of *R* a number of electrons

are driven from a through the metal rod co to the ground. After a passes c it also moves from the influence of R. This is the same as lifting the upper plate of the electrophorus. When a, therefore, has moved on to a position b it is deficient in electrons and hence shows a positive charge. If R had been positively charged, electrons would have passed from c to a at the moment of contact, and so a negative charge would have appeared at b. Thus after R is once charged it is only necessary to rotate the glass plate. Each metal disc will, then, when it reaches b, carry a charge which may be taken off by a conductor and carried to a condenser or other body which is to be charged.

A complete machine which operates in accordance with this principle is shown in Fig. 21. This one is known as the Toepler-Holtz machine. There are two glass plates, one of which, S, is stationary while the other may be rotated. Two pieces of paper, as shown by the dotted lines, are pasted on the stationary plate. These are called inductors. Conducting arms o and o' extend from the inductors over to the front of the revolving plate and end in a brush of tinsel which touches the buttons a, b, c, etc., as they pass. The operation is the same as that described above, except that here the electrophorus is double and the electric charge on the buttons is given up chiefly to the inductors through the conductors o and o'. If the inductor R is negative, the button b will carry a positive charge to R'. Then R' acts on d just as R acted on a, but with an opposite effect, so that e carries a negative charge to R. Thus as the glass plate is turned the inductors are more and more highly charged, up to a limit depending on several conditions such as size of glass plates, atmospheric pressure, ionization of air, etc. The arms Q and Q' are held in position by insulating supports, one branch terminating in fine points which lie close to the front surface of the revolving plate, while the other branch terminates in the knobs E and E'.

Under the influence of the inductors now highly and oppositely charged, electrons are driven out to one knob, E, and withdrawn from the other, E'. This may continue until a spark passes between the knobs. The P.D. between the knobs falls when the spark occurs, but is soon restored by the operation of the machine, so that a rapid succession of sparks may be produced. The conducting rod k carries a stream of electrons from the end over the

negative inductor to the end over the positive inductor, both inductors acting to produce this effect.

Leyden jars may be attached to Q and Q' to accumulate the charge. Sparks will then pass less frequently between E and E' but a greater quantity of electricity will pass when the spark does occur.

The Wimshurst machine is, in principle, the same as that just described, but in construction and operation it is different. There

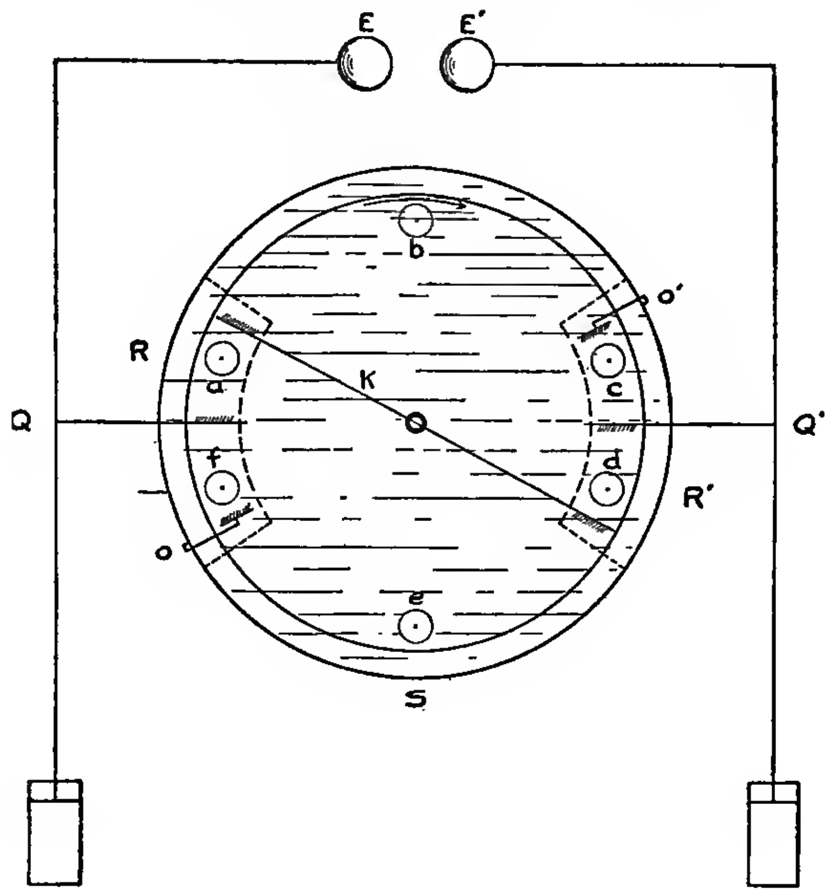

Fig. 21.

are two plates which revolve in opposite directions, each carrying a number of metallic sectors on its outer face. These sectors act alternately as carriers (*i.e.*, like the buttons on the Toepler-Holtz machine) and as inductors.

24. Dimension of Electrostatic Units.—There are three systems of electrical units, *electrostatic*, *electromagnetic*, and *practical.* Only the first of these will be considered here.

Electrostatic units are founded on the definition of unit quantity of static electricity. Q is the c.g.s. electrostatic unit when

it repels an equal charge Q' with a force of 1 dyne, the distance between the charges being 1 cm. in air. From equation (3),

$$QQ' = Fr^2 K$$

and since F is a force, r a distance, and Q is equal to Q', the dimensional equation for unit quantity is

$$Q^2 = [MLT^{-2}]\,[L^2]\,[K]$$

$$\text{or} \qquad Q = [M^{\frac{1}{2}} L^{\frac{3}{2}} T^{-1} K^{\frac{1}{2}}] \tag{19}$$

The strength of a field has been defined as the force per unit charge, *i.e.*, the force with which unit charge would be urged if placed in the field. Hence

$$\text{Strength of field} = [MLT^{-2}] \div [M^{\frac{1}{2}} L^{\frac{3}{2}} T^{-1} K^{\frac{1}{2}}]$$

$$= [M^{\frac{1}{2}} L^{-\frac{1}{2}} T^{-1} K^{-\frac{1}{2}}] \tag{20}$$

Unit strength of field, or field intensity, is 1 dyne.

Unit P.D. exists between two points when one erg of work must be done in moving unit charge from one point to the other, *i.e.*, potential difference is the work per unit charge. Hence, letting V represent this unit,

$$V = [ML^2T^{-2}] \div [M^{\frac{1}{2}} L^{\frac{3}{2}} T^{-1} K^{\frac{1}{2}}]$$

$$= [M^{\frac{1}{2}} L^{\frac{1}{2}} T^{-1} K^{-\frac{1}{2}}] \tag{21}$$

Capacity is defined as the ratio of the charge to the difference of potential, hence

$$C = [M^{\frac{1}{2}} L^{\frac{3}{2}} T^{-1} K^{\frac{1}{2}}] \div [M^{\frac{1}{2}} L^{\frac{1}{2}} T^{-1} K^{-\frac{1}{2}}] = LK \tag{22}$$

The strength of current in this system is the number of electrostatic units of quantity that pass a cross section of a conductor in unit time. Hence, letting i represent strength of current,

$$i = [M^{\frac{1}{2}} L^{\frac{3}{2}} T^{-1} K^{\frac{1}{2}}] \div [T]$$

$$= [M^{\frac{1}{2}} L^{\frac{3}{2}} T^{-2} K^{\frac{1}{2}}] \tag{23}$$

Problems

:16 dyne

1. Two points, 5 cm. apart, are each charged with 20 units of negative electricity. With what force will the charges repel one another?

2. What is the intensity of an electrical field at a point 10 cm. from a charge of 500 electrostatic units?

3. If a charge of 25 units is placed at one corner of an equilateral triangle and 70 units at another corner, the sides being 30 cm. long, what will be the intensity at the third corner?

4. A spherical conductor, whose radius is 3 cm. and its potential 130 units, is touched to an uncharged conductor. The potential fell to 20 units. What is the capacity of the second conductor?

5. A sphere whose diameter is 22 cm. is charged with 136 units of static electricity and then connected by a fine wire, whose capacity may be neglected, to an uncharged sphere 10 cm. in diameter. What will then be the potential of each sphere?

6. How much electrical energy is stored in a sphere whose capacity is 25 cm., when the charge is 1000 electrostatic units?

7. What is the P.D. between two charged discs 8 cm. in diameter when a force of 10 g. is needed to hold one 5 cm. from the other?

8. A spherical conductor of radius r is surrounded by a spherical shell of radius r'. If r' is connected to the earth and air is the dielectric, how much is the capacity of the conductor increased?

The capacity of the conductor is r while that of the condenser is $rr'/r'-r$. Subtract and then multiply both numerator and denominator by 4π. Take $r'-r$ as equal to d and S equal area of surface.

Ans. 1. 16 dynes.
2. 5 dynes.
3. .0946 dyne.
4. 16.5 cm.
5. 8.5 units.
6. $2(10)^4$ ergs.
7. 350 units.
8. $\frac{S}{4\pi d}$.

CHAPTER II

MAGNETISM AND THE ELECTRIC CURRENT

25. What Magnetism Is.—It has already been stated that, according to the electron theory, an electric current is the movement of electrons within a conductor. Each electron possesses a negative charge of electricity and, while it is at rest, is surrounded only by an electrostatic field in which like charges are repelled and unlike charges are attracted. But when an electron is set in motion, it carries its tubes of force with it and sets up another field whose lines of force are at right angles to the electrostatic lines and also to the direction of motion of the electrons. This second field is magnetic. Whenever electrostatic lines of force are moved through the ether, a magnetic field is produced. Magnetism, then, is a condition of the ether and is caused by the motion of an electrostatic charge.

The existence of such a field may be shown by passing a strong current, *i.e.*, a stream of electrons, along a conductor, as in Fig. 22. Some iron filings sprinkled on the card will become magnets and will arrange themselves in concentric circles about the conductor. Such a magnetic field exists at all points along the conductor while the electrons are moving through it.

It may be shown by experiment that an electric charge will, when moving through the ether, set up a magnetic field. Rowland in 1876 made an experimental investigation of this subject. He mounted a gilded ebonite disc in such a manner that it could be rapidly rotated and also charged to a high potential. A delicate magnetic needle suspended above the disc and shielded from all influences except that of a magnetic field was observed to turn to the right or left according to the sign of the electric charge, *i.e.*, the needle turned just as it would if placed near a wire on which a current is passing in one direction or the other. If the conducting wire is bent around in form of a circle, as shown in Fig. 23 where the effect is increased by use of several parallel strands, then the current flowing around the circle causes a strong magnetic field within, as shown by the effect on iron filings. If a magnetic needle is placed within this circle it will promptly take

a position parallel to the lines of filings. If the direction of the current is reversed, the needle will turn through 180° and so will point in an opposite direction. In Fig. 24 is a conductor wound in form of a slender spiral coil the ends of which are attached, one to a strip of zinc and the other to a strip of copper. These

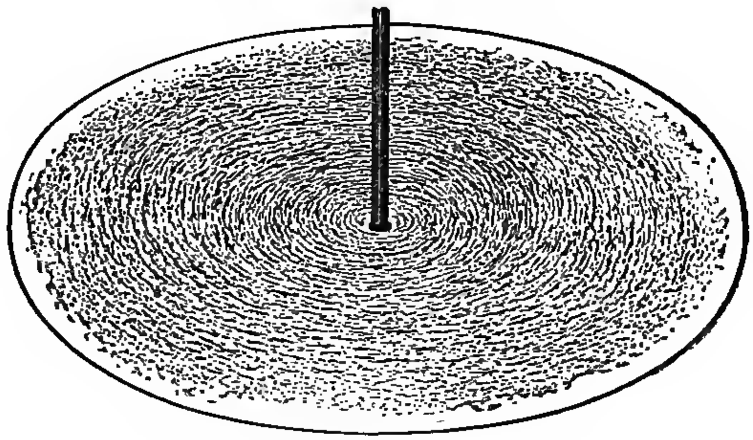

FIG. 22.

project into a battery solution, the whole being floated on water as shown. When this floating battery is placed in the field caused by a current flowing around the large coil, the small coil will take a position at right angles to the plane of the large coil, or, if the current is reversed, will turn just as did the magnetic needle in Fig. 23.

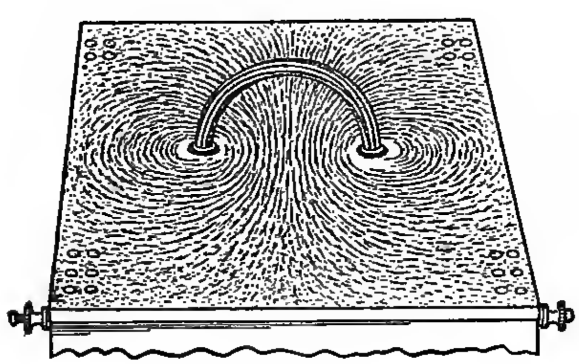

FIG. 23.

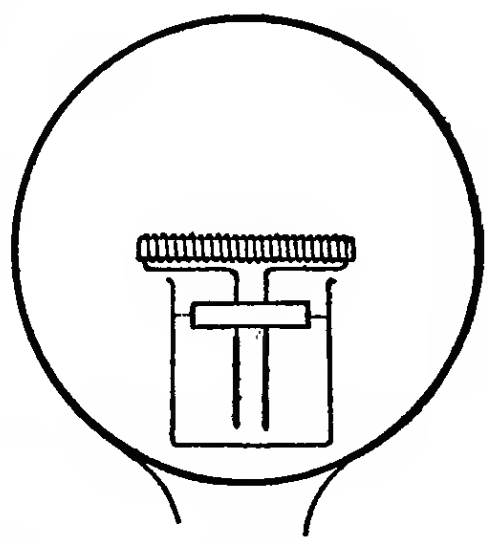

FIG. 24.

It appears from experiments such as these that the fundamental thing regarding magnetism is electricity in motion, *i.e.*, the movement of electric charges which here are the electrons. A theory which attempts to explain magnets, such as a magnetic

needle or a bar magnet, should therefore be based on the motion of electric charges as a cause. Such a theory has been worked out by P. Langevin. Before considering it, attention should be given to the matter presented in the next two paragraphs.

26. Permeability.—The lines of filings shown in Fig. 23 indicate a magnetic field set up in air. If a body of iron is placed in such a field, magnetic lines will crowd toward it and there will be a greater magnetic flux through the iron than through the same region in air. (See Fig. 25.) That property which a body possesses in its relation to the flux of magnetic lines through it is called *permeability*. A technical definition will be given later. The permeability of air is taken as unity. A body which is more permeable than air is said to be *paramagnetic* or simply *magnetic*. Such a body will be attracted by a magnet. A needle formed from such a substance will, when suspended in a magnetic field,

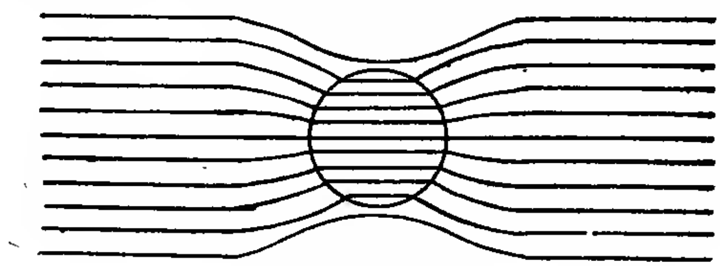

FIG. 25.

set itself parallel to the lines of force. The most strongly paramagnetic substances known are iron, nickel and cobalt. To these should also be added certain alloys discovered in 1903 by Heusler and known by his name. These are composed of manganese, aluminum, and copper. Two parts by weight of Mn to 1 part of Al. The Mn is dissolved in molten copper and the Al then added. The metal is then moulded in desired form and is found to be about as permeable as cast iron. Several other magnetic alloys have since been found, all of which contain either manganese or chromium.

A substance which is less permeable than air is said to be *diamagnetic*. It will, when placed in a magnetic field, move to the weaker parts of the field, and a needle made of a diamagnetic substance will set itself at right angles to lines of force in a magnetic field. Magnetic lines will bend away from a diamagnetic body, and the lines within it are less crowded than they would be in air. Most substances are diamagnetic. The one most strongly so is bismuth.

The above classification refers to the action of bodies in air. In another medium the action may be different. A body which is paramagnetic in air will, when immersed in a liquid more paramagnetic than itself, appear diamagnetic. This may be illustrated by filling a slender glass tube with a weak solution (about 50 per cent.) of ferric chloride and suspending it in a magnetic field in air. The tube will turn so as to stand parallel to the magnetic lines. Now place in the field a beaker filled with a concentrated solution of the ferric chloride, and the tube will, when immersed, turn to a position at right angles to the magnetic lines.

Iron, nickel, cobalt, and certain alloys such as the Heusler alloys, are intensely affected by the magnetic field and so are usually classed by themselves as ferromagnetic.

Other paramagnetic substances such as chromium, cerium, and oxygen, and also all the diamagnetic substances, exhibit only feeble effects.

27. Magnets.—The term magnets is usually applied to a body of iron in such a state that it sets up a magnetic field and attracts other magnetic substances. A valuable iron ore called magnetite, Fe_3O_4, is found in various localities.* It is magnetic by nature and was first observed on the Ægean coast in Magnesia. It was called the Magnesian stone, whence our word magnet. If a long, slender piece of this ore is suspended at its middle point, it will turn under the influence of the earth's magnetic field and take a stand with its long axis pointing north and south. Thus it was used as a guide to mariners and was called lodestone. The Chinese appear to have used magnets in this manner as early as 1100 B.C.

The most powerful and most valuable magnet now in use is the electro-magnet. This consists of a soft-iron core around which insulated wire is wrapped. A magnetic field is produced while the electric current flows around the core, but ceases when the circuit is broken. A magnetic field of any strength, within certain limits to be described later, may therefore be produced by regulating the strength of the current. This may vary in degree from the feeble field which operates a delicate relay to powerful electric cranes where masses of iron weighing several tons are lifted by

* Magnetite is found chiefly in Norway, Sweden, Finland, the Urals, New York State, Pennsylvania, New Jersey, and Michigan.

simple contact with the magnet and may be dropped by breaking the circuit. The electromagnet occupies a very important place in the industrial life of the world.

A permanent magnet is one which retains its magnetic property independent of any magnetizing agency. It is made of steel, and when placed in a strong magnetic field, such as that produced by a strong electromagnet, will continue to be a magnet after the field is removed. Such magnets are useful in many ways but cannot be compared with electromagnets in strength and adaptability.

28. Theory of Magnets.—A knowledge of the fact that the electron is a constituent part of an atom, that the electrons are in rapid orbital motion within the atom, that each carries a charge or is a charge of electricity, and that the movement of an electrical charge causes a magnetic field, furnishes a basis upon which a fundamental theory of the magnet may be constructed. If all material substances may be regarded as composed of molecules, the molecules in turn being composed of atoms, and the atoms made up of positive and negative electric charges, the negative at least being known and believed to have an orbital motion, then it should be possible to explain the behavior of different substances when placed in a magnetic field. Suppose in the first place that, in case of certain substances, the structure of the atom is perfectly symmetrical, *i.e.*, that for every electron revolving in one direction there is one revolving in an opposite direction. The sum of their magnetic effects would then be zero. Now when such a substance is brought into a magnetic field, the effect is to accelerate the motion and increase the orbits of electrons moving in one direction and to decrease the orbits of those moving in an opposite direction (see Fig. 61). The effect is to destroy the symmetry and produce a resultant field, which is the same in kind as that of the external magnet. Since likes in magnetism repel one another, a substance of the kind described would be repelled by a magnet. Such substances are diamagnetic. The effect of a magnetic field on a diamagnetic substance is, however, always feeble.

If a substance contains electrons which revolve in different orbits, some larger and some smaller, and symmetry is wanting, then the molecule taken as a whole would most probably exhibit resulting magnetism, *i.e.*, the magnetism produced by electrons

revolving in one direction would not be neutralized by others, in an opposite direction. Such a molecule may therefore be regarded as a small magnet and in a magnetic field will be affected accordingly. (See Fig. 24.) Substances of this kind are called paramagnetic or simply magnetic.

If the resultant magnetism of the group of atoms within a molecule is large and the molecules influence each other across the space which separates them, the substance is said to be ferromagnetic, *i.e.*, it is very strongly paramagnetic.

In all these cases the first effect of a magnetic field is that described for diamagnetic substances, and so all substances may be said to be diamagnetic, and paramagnetic effects are the result of stronger forces in an opposite direction.

Such, briefly, is a statement of the most probable theory of magnetism in the light of our present knowledge of electricity and the constitution of matter.

29. Poles of a Magnet.—According to the theory just explained, a magnet, as the term is ordinarily used, is a rod of soft iron, hard steel, or other magnetic substance, in which the planes of the orbits of electrons are parallel and facing in the same direction. Not all the orbits are thus turned but only those which produce a predominating magnetic field. The molecules as a whole are turned to such a position that their resultant fields are parallel and in the same direction. All paramagnetic and ferromagnetic phenomena are therefore molecular in character. In soft iron this arrangement of molecules is maintained only while the iron is under the influence of an outside field, while in hard steel the arrangement persists after the field is removed.

If a bar of steel is uniformly magnetized and then dipped in iron filings, the filings become magnets and will cling to the ends of the bar, diminishing in quantity along the sides toward the middle where no filings are found. It was, therefore, wrongly assumed that there was a greater quantity of magnetism at the ends of a magnet than at other points. The ends were called *poles*, and magnetism was once regarded as a kind of fluid which could flow from one point to another or could exist in a free state. It is evident from the electron theory, however, that there is as much magnetism at the centre of the bar as at any other point. The greater magnetic effect on filings or other magnetic bodies

near the ends of the bar is a resultant effect of the magnetism distributed all along the bar. In Fig. 26 let SN represent a permanent magnet with the so-called poles at S and N. Let an electron orbit, e, with its magnetic field be placed midway between N and S in the field of the bar magnet. Then e will face about so that its lines are parallel to and in the same direction as the lines outside the bar. But since it is impossible to have one pole of a magnet

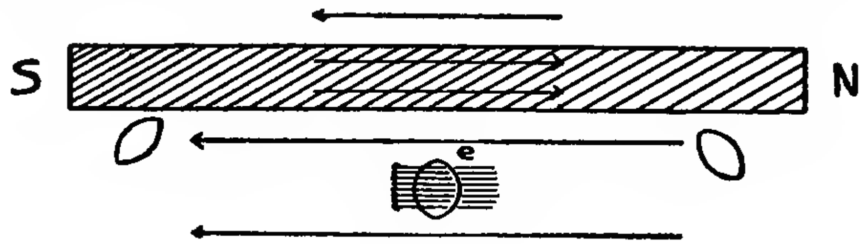

FIG. 26.

without having another of opposite magnetic effect and exactly equal in strength, then N is equal to S and each is at the same distance from e. Hence e is under the stress of balanced forces of attraction. If e is placed to the right or left of the centre of the bar, it will still keep its lines parallel to those of the field in which it moves and will be more and more strongly attracted by that end of the bar to which it moves.

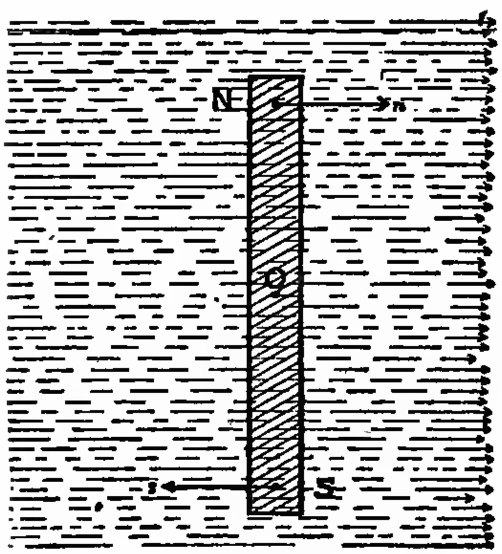

FIG. 27.

A pole of a magnet is technically defined as the point of application of the resultant of all the forces acting on one-half of the magnet and tending to produce rotation about the magnetic centre when the magnet is placed at right angles to a uniform magnetic field. Thus, in Fig. 27, n and s represent the resultants of a number of parallel lines which increase in length from o to each end where they are longest. If the sum of all the moments at each end is divided by the sum of all the forces, the quotient will be the distance from o, where, if the sum of all the forces is applied, the moment will be the same as before. The positions of poles thus

defined, in case of a permanent bar magnet, are distant from one another about five-sixths, or less, of the length of the magnet.

The pole which points northward when a magnet can freely rotate in the earth's magnetic field is called the north-seeking pole and the other the south-seeking. Often they are called simply the north and south poles, but since like poles repel one another while unlike poles attract, it seems inconsistent to call that pole north which points to the earth's north magnetic pole. This is avoided by using the terms north-seeking and south-seeking.*

Any number of poles, called consequent poles, may exist between the two at the ends. If a straightened piece of clock spring, for example, is touched at several points with a magnet, iron filings will show magnetism at these points. (Fig. 28.) This is the principle of an instrument called the magnetophone by which sound may be recorded on a steel wire or disc and later reproduced. The steel wire may be wrapped spirally on a cylinder and when the cylinder is rotated successive portions of the wire are moved under an electromagnet which varies in intensity in response to waves of sound as in the transmitter of a telephone. The different portions of the wire are therefore magnetized differently and, when the cylinder is turned in the same direction under a suitable diaphragm, the same sounds will be produced.

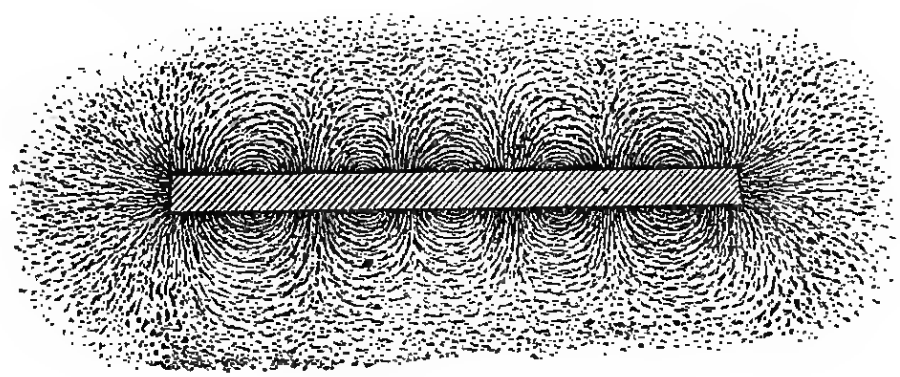

FIG. 28.

It is possible to have a magnet without any poles. A magnetised iron ring will have all its lines within the iron. If, however, a portion of the ring is cut away, poles will be apparent at the ends of the gap.

* In France the end of a magnetic needle which points northward is called the south pole and the other end the north pole.

30. Law of Magnets.—According to a law announced by Coulomb in 1785, the force of attraction or repulsion between two magnetic poles is directly proportional to the product of the pole strengths and inversely proportional to the square of the distance between them. This may be written

$$F \propto \frac{m_1 m_2}{r^2} \tag{24}$$

where F is the force, m_1 and m_2 the strengths of the poles, and r the distance.

If, for any system of units that may be adopted, the value of F is always determined in vacuum, then equation (24) may be written

$$F = \frac{m_1 m_2}{r^2} \tag{25}$$

but for different media between the poles the attraction or repulsion is different. This depends on the permeability of the medium. The more permeable a medium is the less will be the force. Hence equation (25) should be written

$$F = \frac{m_1 m_2}{\mu r^2} \tag{26}$$

where μ stands for permeability.

The value of μ for vacuum is 1, and for air is 1.000005. Hence the permeability of air may also be taken as 1.

31. Unit Magnetic Pole.—As stated in a previous section there are three systems of electrical units (§ 24). One of them is the electromagnetic system which depends on the definition of the unit magnetic pole. It should be remembered, however, that magnetism of one kind cannot, like an electric charge, be set off by itself. It is not possible to have an N pole without an S pole. It is possible, however, in case of a long, thin magnet to have the poles so far apart that the action of one may be considered without reference to the influence of the other. The conception of a single pole is often an advantage in this discussion.

From equation (26) we see that it is possible to make m_1 equal to m_2 and of such strength that F is one dyne when r is 1 cm. and the medium is air. Such a strength of pole may then be taken

as the unit. Hence, *unit pole is a pole of such strength that, when placed in air,* 1 *cm. from an equal and similar pole will repel it with a force of one dyne.*

32. Magnetic Lines of Force.—The region about a magnetic pole possesses properties which are not found when the magnet is removed. The ether in this region is assumed to be under strain. A ferromagnetic body placed in such a field will become a magnet and will move or tend to move in such a direction as to reduce the strain. This is somewhat analogous to mechanical strains which cause a stress tending to restore a body to an unstrained condition. A magnetic field is any region in which

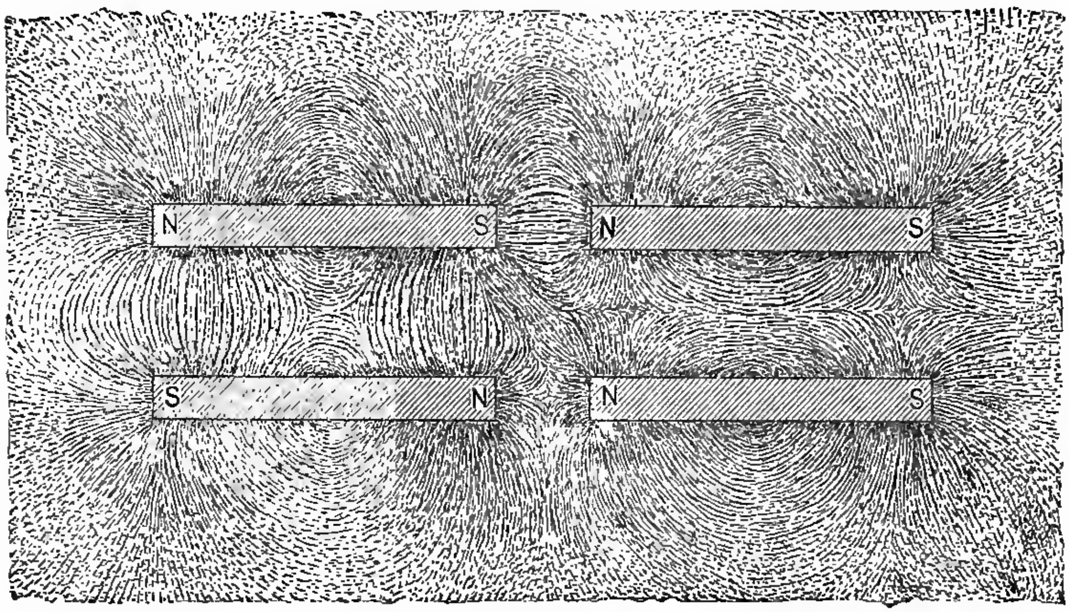

FIG. 29.

magnetic effects are produced. Faraday represented such a field by lines or tubes of force. These lines are assumed to pass from the south-seeking (S) to the north-seeking (N) pole within the magnet but from N to S in the field outside the magnet. Outside the magnet lines always start at the N pole and end on the S pole. The direction of the lines at any point in a field is the same as that in which a small N pole would be urged if placed at that point. Some idea of these lines in one plane of the field may be obtained by sprinkling iron filings over a paper beneath which a magnet or magnets are placed. Thus in Fig. 29, where four magnets are arranged as shown, the filings become magnets and, clinging together, arrange themselves along the lines of force.

Faraday thought of these lines as being under tension. He also assumed a repulsion between lines. Hence there would be attraction between the two upper magnets in the figure and repulsion between the two lower ones.

33. Intensity of a Magnetic Field.—The intensity or strength of any point in a magnetic field is the force with which unit pole would be acted on if placed at that point. If the strength of field is represented by H and the force by F, then $F=H$ as long as the force is only that on the unit pole. If the unit pole is replaced by one of strength m, then

$$F=Hm \tag{27}$$

A unit field is one in which the force on unit pole is 1 dyne. Such a field is usually represented by one line per square centimetre of surface at right angles to the field. Unit strength of field is called a *gauss*. The number of lines per square centimetre then represents the strength in gausses.

34. Magnetic Induction.—Magnetic induction is denoted by B and is defined as the total number of lines per square centimetre appearing in a body under the influence of a magnetic field. This number includes both the lines which are induced in the body and those which would be in the same space if the body were removed. Let a bar of iron, Fig. 30, be placed in a field of intensity H. The molecular magnets of the iron will be turned so that their lines coincide with those of the field. The bar therefore becomes a magnet, and a much greater number of lines now pass through the iron from S to N. This additional magnetism has not been created. It was already in the iron and has only been made by the field to have a common direction. If the total induction per square centimetre is B and the number of lines in the same region of the field is H, then $B-H$ is the number of lines due to the molecular magnets of the iron.

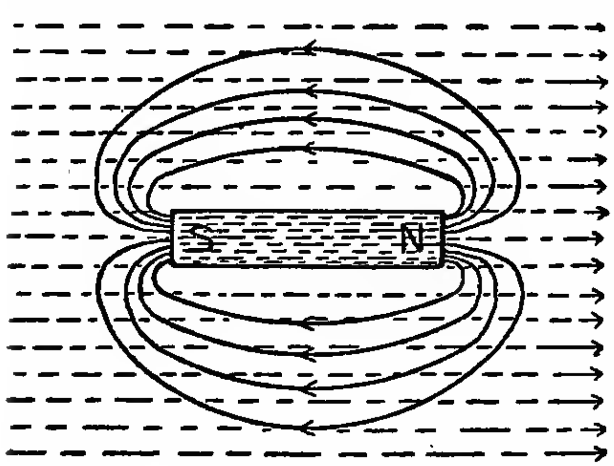

Fig. 30.

If a unit pole is placed at the centre of a sphere 1 cm. in radius, then the strength of the field at any point on the surface of the sphere will be unity. Since there are 4π square centimetres of surface there must be 4π lines coming from the unit pole. If there are m unit poles at the centre of the sphere, $4\pi m$ lines would arise from it. Hence when the pole strength per square centimetre of the face of the iron bar is m, there will be $4\pi m$ lines coming out from that area at the north-seeking end, passing through the air to the south-seeking end, and thence through the iron from S to N. Consequently the total induction B is expressed by

$$B=H+4\pi m \tag{28}$$

35. Intensity of Magnetization.—Intensity of magnetization is denoted by I and may be defined as *the number of unit poles per square centimetre induced in a bar of iron or other magnetic substance when placed in a magnetic field.* This means the number of unit poles per square centimetre of the face of the magnet. From what has been said in the previous section we may, according to this definition, write

$$I=\frac{B-H}{4\pi} \tag{29}$$

Comparing this with equation (28) we see that $I=m$. Hence we may write

$$B=H+4\pi I \tag{30}$$

36. Susceptibility.—Magnetic susceptibility is usually denoted by k and is defined as *the ratio of the intensity of magnetization to the strength of the field, H, which produces it.* This may then be expressed by the equation

$$k=\frac{I}{H} \tag{31}$$

37. Permeability.—Permeability has already been discussed in § 26. We are now ready to define it in terms of other quantities. It is usually denoted by μ and may be defined as the ratio of the total induction, B, to the strength of field H. Hence

$$\mu=\frac{B}{H} \tag{32}$$

Combining equations (28), (29), and (30),

$$\mu=1+4\pi k \tag{33}$$

38. The B-H Diagram.—When a bar of soft iron or other magnetic substance is placed in a magnetic field that may be varied at will, then as the field, H, is increased step by step, the induction, B, will also be changed. If the values of H and B are determined for each change and are plotted as abscissæ and ordinates respectively, we obtain a curve like that in Fig. 31. The curve is different for different substances, but, as a rule, three different stages of magnetization may be observed. From o to a the induction increases slowly. From a to b a small increase in H will produce a large increase in B. Then the ratio of B to H begins to grow less and the curve finally becomes a straight line which, if B and H are plotted on the same scale, will be inclined 45° to the axes, *i.e.*, the increase of lines of force in the iron is the same as it would be in air at that place if the iron were removed. Iron in this final state is said to be *saturated.*

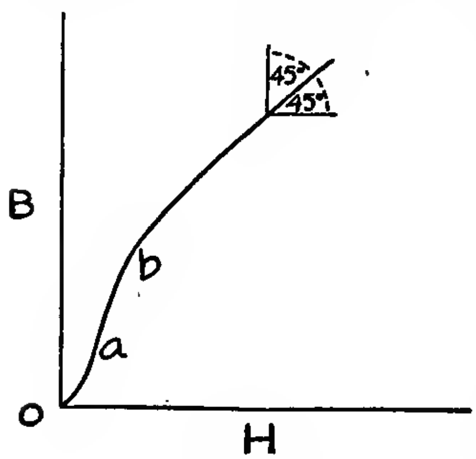

FIG. 31.

A physical interpretation of the B-H curve would probably be that ferromagnetic molecules of an unmagnetized bar of iron are influenced by the magnetic fields of their neighbors and bound together loosely in irregular groups which can be broken up only by the application of force. This would explain the curve from o to a. After the orientation of the molecules begins, a slight force would suffice to turn them to a position where their fields would more nearly be in the same direction, and when all were turned, a further increase in the field H could have no effect in increasing the magnetization of the iron.

An I-H diagram would be similar to that of Fig. 31, but, since I, from equation (29), is proportional to $B-H$, then I includes only the lines which are induced as a result of H and not the lines of H. Hence this curve at the point of saturation would be parallel to the abscissa.

39. Hysteresis.—Hysteresis is a term which means lagging behind. It is observed when iron is subjected to the influence of an alternating magnetic field, for then the induction B lags behind the magnetizing force H. This will be understood by a

consideration of the curves in Fig. 32 where the abscissa represents the magnetizing force and the ordinate the resulting induction. When an alternating current flows in a coil of wire, the field in the coil is alternately directed one way and then the other. A core of soft iron within the coil will then be subjected to a field which changes its direction each time the current is reversed. Suppose that the iron is completely demagnetized and therefore in a state represented by *o* in the figure. Now let *H*, which is zero at *o*, increase in a positive direction until the iron is saturated. (+ and − here signify only opposite directions of the magnetizing field.) The induction *B* will increase from *o* to *B* and the *B-H* curve will be *oaC*.

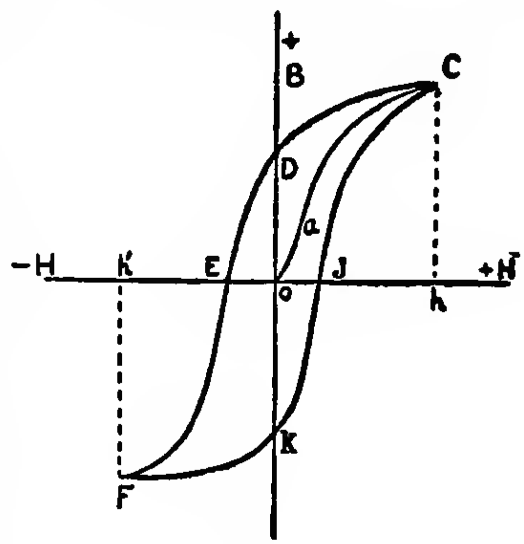

FIG. 32.

If now the strength of the current in the coil is decreased, the field *H* is decreased and may be restored to the state *o* where it is zero. But the curve, instead of returning along the path *Cao*, follows a path *CD*, and although there is now no magnetizing force, yet the induction is still the large quantity *oD*, called ***residual magnetism*** or the ***remanence***. The ratio of *oD* to the value of *Ch* at saturation is called ***retentivity***. It is observed that the induction lags behind the magnetizing force and hence the application of the term ***hysteresis***. In order to remove this residual magnetism, *i.e.*, to extend the curve *CD* to *E*, it is necessary to reverse the current and thus apply the force of a reversed magnetic field *oE*. This quantity *oE* is therefore called the ***coercive force***. In case of hard steel this force is very large and hence the possibility of permanent magnets made of that material. In soft iron and mild steel coercive force is small.

If the reversed magnetic field be increased in strength to *h'*, the curve *CDE* will be extended to *F*, where the induction will be the same as at *C*, but with the lines of force in an opposite direction, *i.e.*, the molecular magnets in the iron have been made to turn through 180° while the field changed from *h* to *h'*.

Now let the negative field decrease in strength from *h'* to *o*

and the induction will then be oK. Here again B is lagging behind H and a certain coercive force oJ is needed to remove the residual magnetism. A further increase of the positive field to h restores the state C. Thus a cycle has been completed. Any subsequent cycle of changes in the field from h to h' and back to h will result in a cycle of changes in the induction which, when traced on a B-H diagram, will describe a *hysteresis loop* $CDEFJC$.

At all points of the loop induction lags behind the magnetizing force, which shows that it requires a certain force to cause the molecules of iron to face about. If no such force were required, the lines oE and oJ would, as far as they represent coercive force, become zero, *i.e.*, E and J would coincide with o and instead of a loop there would be a single curved line which would be retraced each time H changed from h to h' or from h' to h. Under this condition the area of the loop would be zero, but since a force has been applied in producing a change, the area of the loop represents the amount of work done in one cycle of operation. This work is lost as far as this cycle of operations is concerned and appears as heat in the iron.

It is evident from this that the quality of the iron is an important consideration in the construction of electric machines where the induced magnetism must rapidly change direction under the influence of a rapidly alternating field as in case of transformers, armatures of dynamos, etc.

40. Effect of Temperature on Magnetism.—Since both heat and magnetism are treated as molecular phenomena, we would expect that a change in temperature would result in a change of the magnetic properties of a magnetic substance. When a bar of soft iron is placed in a weak magnetic field and heated, no sensible increase in permeability is noticed till the temperature rises above 600° C. Then there is a rapid rise in permeability which later falls to zero at about 800° C.—*i.e.*, at 800° C. the iron both loses its magnetism and ceases to be magnetic. As the iron cools it regains its magnetic property but at a lower temperature.

In stronger magnetic fields the changes are not so abrupt, but in all cases there is a so-called critical temperature for magnets at which they cease to be magnetic. Nickel and cobalt possess this same property, but their critical temperature is lower.

It appears that when the molecules are greatly agitated by heat the magnetic field is not able to keep the molecular magnetic fields of the iron in line.

41. Magnetic Moment.—Magnetic moment is defined as the product of the pole strength and the distance between the poles. If M is the magnetic moment and m the pole strength, then

$$M = ml \tag{34}$$

This is the same as the turning moment which a magnet would experience when placed at right angles to a unit magnetic field. If in Fig. 33 the value of the uniform field, H, is unity—*i.e.*, such that unit pole placed at any point would be acted on by a force of one dyne—then m unit poles will be acted on by a force of m dynes in one direction, while an equal force will act on the other pole of the magnet in an opposite direction. This gives a mechanical couple of which the moment is ml. The magnetic moment, therefore, varies only with the strength of the poles of a magnet and the distance between the poles.

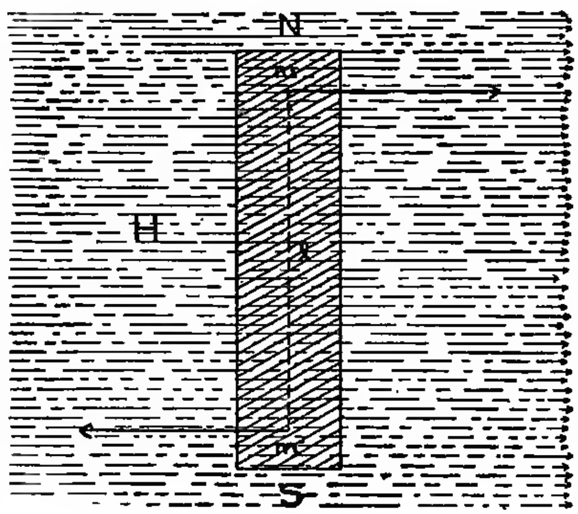

FIG. 33.

If the magnetic moment is known, it is plain that the moment of force tending to turn a magnet placed as in Fig. 33 in a field of strength H is Hml or HM.

If we use the expression Fh to represent moment of force we may write

$$Fh = HM \quad \text{or } Hml \tag{35}$$

If, however, the magnet is inclined at angle θ to the direction of the field as in Fig. 34, then the component of Hm which is effective in turning the magnet is $Hm \sin \theta$ and the moment of the couple is $Hml \sin \theta$. Hence a general equation for any position of the magnet is

$$Fh = Hml \sin \theta = HM \sin \theta \tag{36}$$

If the magnet is free to turn it will take a stand parallel to the field just as a compass needle turns to a position where it is parallel to the earth's magnetic field. If then a magnet is suspended as

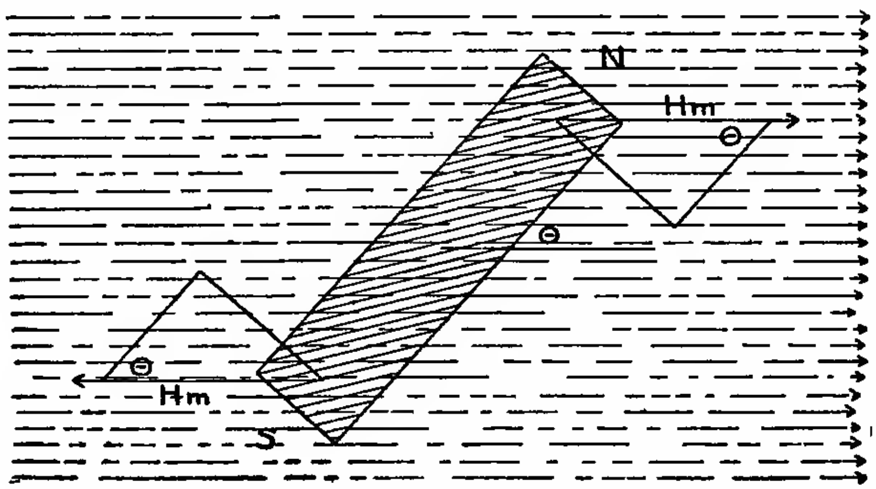

FIG. 34.

shown in Fig. 35 and is made to swing as a torsion pendulum, it will execute simple harmonic motion provided θ is so small that sin θ may be regarded as equal to θ measured in radians. With this understanding we may write equation (36) as follows:

$$\frac{Fh}{\theta} = HM \qquad (37)$$

Since H and M are constant quantities, θ must vary directly as Fh. This is a condition of S.H.M. Consequently, from equation 127 of "Mechanics and Heat" we may write

ask

$$HM = \frac{4\pi^2 I}{P^2} \qquad (38)$$

or

$$P = 2\pi\sqrt{\frac{I}{HM}} \qquad (39)$$

Hence, if we determine the moment of inertia, I, of a magnet and its period of oscillation P, we can readily calculate the value of HM from equation (39).

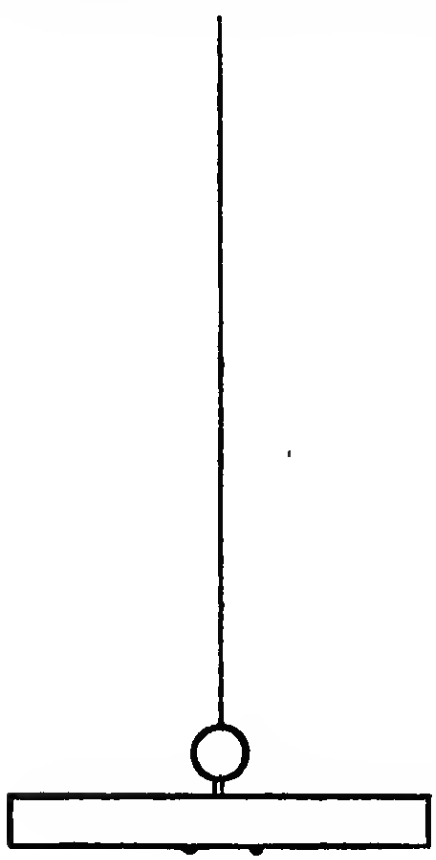

FIG. 35.

42. The Field Produced by a Magnet.—Since it is impossible to have an isolated magnetic pole, the strength of field at any

point near a magnet will be a resultant effect of both poles. For example, let SN, Fig. 36, be a magnet 10 cm. long, and suppose it is desired to know the strength of field at G, a point 10 cm. from the north-seeking pole N. Let the pole strength of the magnet be 50 units, *i.e.*, 50 unit poles. Then since strength of field at G is the force with which unit pole at that point is urged, we have from equation (25), for the field due to the N pole, $50/10^2$, and for that due to the S pole, $50/20^2$. But since the poles are opposite in kind, their resultant effect at G would be their difference or .375 gauss. [illegible] to strength of field

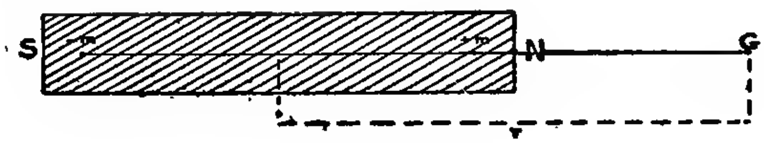

FIG. 36.

It is desirable to know the field strength, F, due to a magnet, in terms of the magnetic moment of that magnet and the distance from the centre of the magnet to the point of the field being considered. Let this distance be r, Fig. 36, the length of the magnet l, and the poles $+m$ and $-m$. Then the strength of field at G would be, as shown above, $\frac{+m}{(r-l/2)^2}$ due to the N pole, and $\frac{-m}{(r+l/2)^2}$ due to the S pole. The resultant field at G is therefore the algebraic sum of these quantities. Hence

$$F=\frac{2mlr}{\left(r^2-\frac{l^2}{4}\right)^2} \tag{40}$$

But ml is the magnetic moment, hence we may write

$$F=\frac{2Mr}{\left(r^2-\frac{l^2}{4}\right)^2} \tag{41}$$

If the distance r is many times greater (20 times or more) than the length of the magnet, no appreciable error will result from the omission of $l^2/4$. Hence we may express the strength of field by

$$F=\frac{2M}{r^3} \tag{42}$$

43. Action of a Magnetic Needle in Two Fields at Right Angles.—It has already been shown how it is possible to find the strength of the field due to a magnet. Let *NS*, Fig. 37, be the magnet and let a short magnetic needle, suspended so that it may freely turn, be placed in this field at a distance r from the centre of the magnet. The effect of the magnet alone would be to turn the needle to a position parallel to its lines of force F. Let another field, such as the horizontal component of the earth's magnetism, act with an intensity H at the same point but in a direction at right angles to F. The effect of H alone would be to turn the needle parallel to H. The needle will therefore occupy

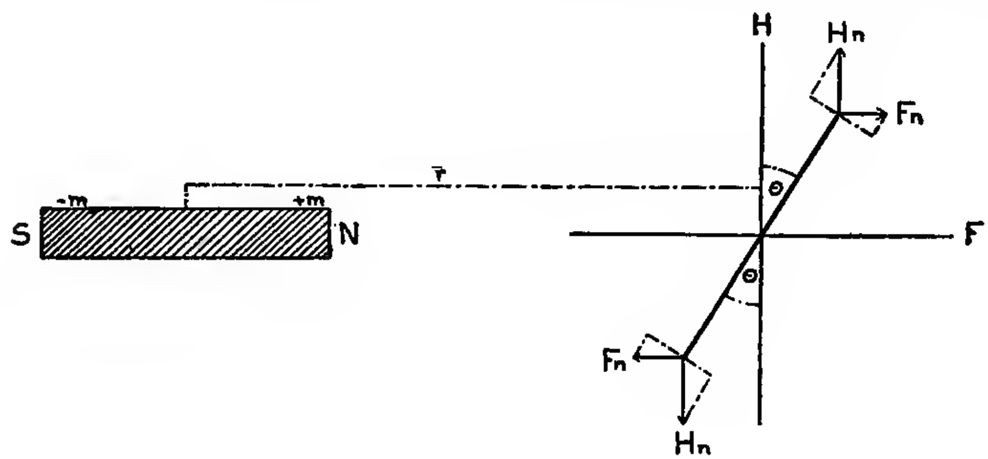

FIG. 37.

a position of equilibrium between the two forces. Let this position be such that the needle makes an angle θ with the field H, and let n be the pole strength of the needle whose length is l'. Then the turning moment of the H-field is $Hnl' \sin\theta$, while that of the F-field is $Fnl' \cos\theta$. Since the needle is in equilibrium under these two moments, we have the equation

$$Hnl' \sin\theta = Fnl' \cos\theta$$

or $$F = H \tan\theta \tag{43}$$

Substituting the value of F from equation (42),

$$\frac{2M}{r^3} = H \tan\theta$$

$$\frac{M}{H} = \frac{r^3 \tan\theta}{2} \tag{44}$$

44. Determination of M and H.—The value of HM may be found from equation (38) by the use of the vibration magnetometer shown in Fig. 35. The value of the moment of inertia I can be calculated from the dimensions of the magnet and the period of vibration, P, can be observed. Also the ratio of M to H can be found from equation (44) by use of a deflection magnetometer shown in Fig. 38. Here a small magnet fastened to the back of the mirror serves as the magnetic needle, and the magnet used in Fig. 35 is placed so that its axis is at right angles to the length of the needle and at a distance r, as shown in diagram Fig. 37. The mirror turns with the needle so that by use of a telescope and scale the value of θ can be observed.

If equations (44) and (38) be multiplied together we get the equation

$$M^2 = \frac{2\pi^2 I r^3 \tan\theta}{P^2} \tag{45}$$

in which all the terms on the right can be determined. Hence the magnetic moment, M, can be found.

The division of (38) by (44) gives

$$H^2 = \frac{8\pi^2 I}{P^2 r^3 \tan\theta} \tag{46}$$

from which the intensity of a field, as H in Fig. 37, may be determined.

It is also observed from equation (38) that M and I are constant quantities for the same magnet, consequently different fields may be compared by the relation

$$\frac{H_1}{H_2} = \frac{P_2^2}{P_1^2}$$

FIG. 38.

i.e., the intensities of two fields vary inversely as the squares of the periods of vibration of the magnet in those fields. This and equation (44) will give the total intensity only when the lines of force of the field are parallel to the plane of vibration of the magnet. Otherwise the magnet will be influenced only by that component of the field to which it is parallel. The lines of the earth's magnetic field, for example, are not horizontal except at

points near the equator. Hence, since the plane in which the magnet vibrates is horizontal, the methods described above will give, in this case, only the horizontal component of the earth's magnetic intensity.

45. Terrestrial Magnetism.—The earth is surrounded by a magnetic field of moderate intensity. The lines of force in a general way run north and south and lead in one direction to a north magnetic pole at a point northwest of Hudson Bay, about 97° W. longitude and 70° N. latitude. In the other direction the lines lead to a south magnetic pole south of Australia, about 148° E. longitude and 74° S. latitude. These poles, however, are not fixed but are slowly shifting.

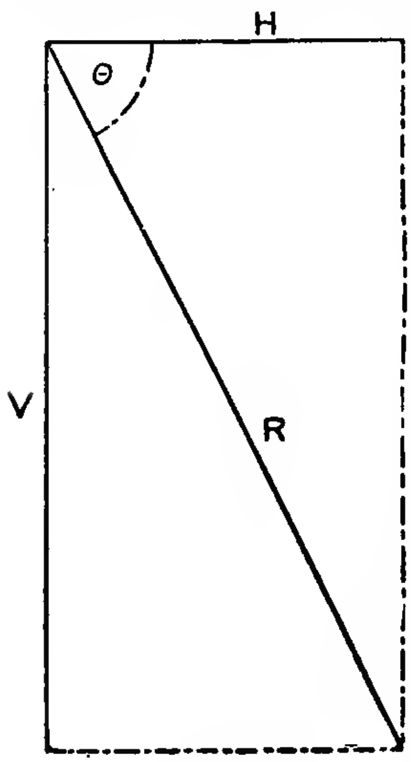

FIG. 39.

If a magnetic needle is mounted so that it is free to swing in all directions it will take a position parallel to the earth's lines of force. At the equator, or near it, the needle will be horizontal. As it is moved northward the north-seeking end will point more and more downward, until at the north magnetic pole it will stand in a vertical position. At the south pole the needle will again be vertical but with south-seeking end downward.

The angle which the needle makes with a horizontal plane at any point on the earth's surface is called the *inclination* or *dip*. This angle has been determined for many points on the earth. Lines drawn through points of equal dip are called *isoclinic* lines. These, though irregular, correspond in a general way to parallels of latitude.

When a magnetic needle is mounted so that it can swing only in a horizontal plane, as in the case of compasses used in navigation and surveying, it is evident that, except where there is no dip, only a component of the field intensity is effective in directing the needle. In Fig. 39 let R be the direction of the magnetic field. It is also the direction of a dipping needle. Then θ is the dip or inclination. Let the line R represent the total intensity

of the field, then V and H are its vertical and horizontal components. The value of H is of most importance and one method of finding it has been described in previous paragraphs, where it is shown that by use of the vibration and deflection magnetometers all the values for equation (46) may be found. When H has been found the total intensity R may be calculated if the dip, θ, is measured, for by Fig. 39,

$$R = \frac{H}{\cos \theta} \tag{47}$$

For example, if H is .2 gauss—*i.e.*, the horizontal intensity of the field is such as will exert a force of .2 dyne on unit pole—and if the dip at that point is 70°, then the total intensity R is about .58 gauss. It is also observed that V may be found from

$$V = H \tan \theta \tag{48}$$

If now we confine our attention to the position of a needle in a horizontal plane we find that it does not in general point in the

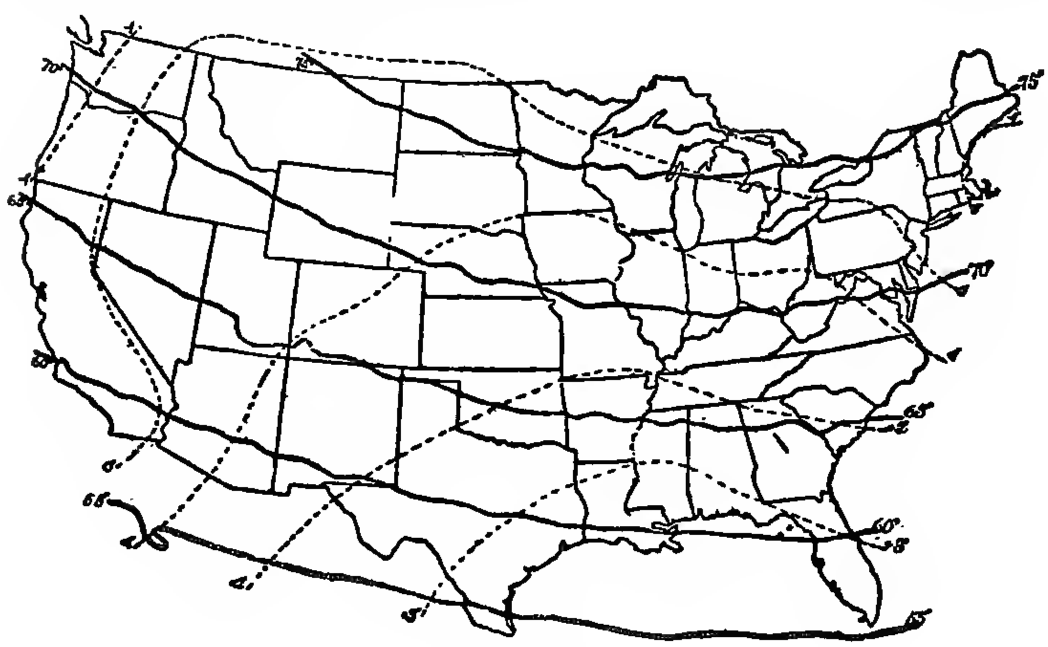

FIG. 40.

direction of a geographical meridian but makes an angle with it. This angle is called the *declination* or *variation* and lines which pass through points of equal declination are called isogonic lines. These correspond in a general way to geographical meridians but, as shown in Fig. 41, are very irregular lines.

A line drawn through all points of zero declination is called an agonic line. The agonic line in the United States passes through Michigan, Ohio, Kentucky, Tennessee, North Carolina, and South Carolina.

At points east of the agonic line a magnetic needle points west of north, and, west of the line, east of north. On the map, Fig. 40, the heavy chain lines indicate the dip at points through which they pass. The broken lines show the number of minutes of change in dip each year. One of these is marked 0′, *i.e.*, there is no change in dip at points on this line. Below this line the dip is increasing

Fig. 41.

a certain number of minutes each year as marked at the end of the line. In New England and on the Pacific coast dip is decreasing as the map shows. This map is taken from one prepared by the Coast and Geodetic Survey and includes all observations up to 1907. The map shown in Fig. 41 is taken from the same source and includes observations on declination up to the year 1910. The heavy chain lines are the isogonics. The one marked 0° is the *agonic*. The broken lines indicate the annual rate of change in declination. One of these marked 0′ is a line on which there is no annual change. All broken lines west of this are moving eastward and those on the east side are moving westward at a rate per year indicated by the number of minutes at the end of the line.

Declination, inclination, and magnetic intensity are called the *magnetic elements*. These are now being determined at many points in all civilized countries. Accurate observations have been made in England since the year 1540. It is expected that when sufficient data covering a long period of time are collected some law in regard to magnetic changes in the earth's field will be established and also that some plausible theory of the origin of the earth's magnetism may be evolved.

One thing that is clearly shown is that the magnetic poles and all the magnetic elements are changing. The most important of these is the so-called *secular* change which it is presumed will complete a cycle in about 470 years. If a magnetic needle were pivoted at the centre in such a manner that it is free to indicate both inclination and declination at the same time, each pole will very slowly describe an irregular curved line which, it is thought, will form a closed curve. Such a curve may be plotted, as far as the data go, from observations of inclination and declination that have been made from time to time. From observations such as these it appears that the secular change in England should complete a cycle in about the year 2020 A.D.

In addition to the secular change there are also cycles of change each day and each year, though these are slight. The magnetic field is also modified by the influence of the moon, by conditions which give rise to sun spots, and by magnetic storms.

Problems

1. If the strength of a magnetic pole, regarded as isolated from its opposite pole, is 147 units, at what distance in air will the strength of the field be 3 gausses?

2. The pole strength of a magnet is 350 units. The distance between poles is 10 cm. What is the strength of field at a point 10 cm. distant from the N pole and in line with the axis of the magnet?

3. What is the force exerted on a pole whose strength is 3 units when placed at a point 3 cm. directly above the S pole of a horizontal magnet 4 cm. long, the pole strength of the latter being 50 units?

4. A rod of soft iron placed in a magnetic field acquires a pole strength of 200 units. How many lines of force are thus made to pass through the rod?

5. If the distance between the poles of a magnet is 4.2 cm. and the magnetic moment is 50 c.g.s. units, what is the pole strength?

6. The magnetic moment is 300 c.g.s. units and the magnet is placed in a uniform field whose strength is 12 gausses. What will be the moment of the couple?

7. If the strength of the earth's magnetic field is .2 gauss and a needle whose magnetic moment is 300 c.g.s. units is placed so that it is inclined 30° to the field, what is the moment of the couple tending to turn the needle?

8. What is the permeability of a sample of wrought iron when a field strength of 2 gausses causes an induction of 8000 gausses?

9. If an induction of 8000 lines per square centimetre results from placing an iron bar in a field of 12 gausses, what is the intensity of magnetization?

10. If the period of vibration in a magnetometer at a place where the earth's field is .19 gauss is 15.12 seconds, what is the strength of field at another place where the period of the same instrument is 16 seconds?

11. What is the total strength of the earth's field where the horizontal component is .2 gauss and the dip is 70°?

Ans. 1. 7 cm.
2. 2.725 gausses.
3. 12.39 dynes.
4. 2513.
5. 11.9 units.
6. 3600 dyne centimetres.
7. 3 dyne centimetres.
8. 4000.
9. 635.6 unit poles.
10. .17 gauss.
11. .585 gauss.

CHAPTER III

ELECTROMAGNETIC AND PRACTICAL UNITS

46. Unit Strength of Current.—In 1820 Oersted of Copenhagen made the discovery that when a current of electricity flows in a conductor, a magnetic field is set up at right angles to the direction of the current. He showed that when a magnetic needle is placed in this field it tends to take a position at right angles to the conductor, the N pole of the needle being driven in one direction along the lines of force and the S pole in the opposite direction. If now this conductor is bent in form of a circular loop, Fig. 42, the magnetic fields which surround the conductor at all points will create a magnetic field at the centre of the circle, the intensity of which will depend on the strength of the current.

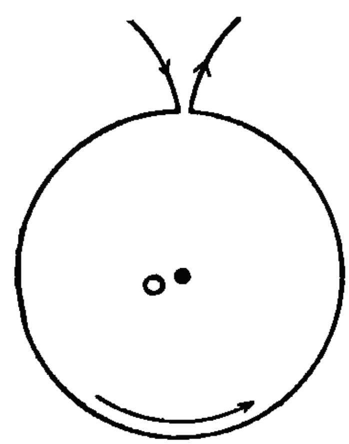

FIG. 42.

This principle has been used in determining a unit electromagnetic strength of current, the definition of which is as follows: *The electromagnetic unit of current is that current which when flowing through an arc* 1 *cm. long, of a circle* 1 *cm. in radius, will act on unit pole at the centre with a force of* 1 *dyne.*

The loop shown in Fig. 42 is filled with lines of force, as has been shown in Figs. 22 and 23 for one plane in the field. A convention for finding the direction of lines of force is as follows: *Grasp the conductor with the right hand so that the thumb will point in the direction the current is flowing. The fingers will then point in the direction of the lines of force of the magnetic field.* If then a positive pole is placed at o in Fig. 42 it will be moved along the lines of force in a direction at right angles to the plane of the coil. A south-seeking pole would be urged in an opposite direction.

According to the definition for unit electromagnetic (*e.m.*) strength of current the strength of field at the centre of unit circle would be 2π dynes, for there are 2π cm. in the complete circle. For n loops of wire in unit circle the strength of field at o would be $2\pi n$. If in addition the strength of current be made i instead

of one unit as defined, the strength of magnetic field becomes $2\pi ni$. If, finally, the radius of the circle is made r cm. instead of 1 cm., then, since the length of a conductor looped in form of a circle varies directly as r and the magnetic force from any point varies inversely as r^2, we have, letting F represent the magnetic intensity at o,

[illegible]

$$F = \frac{2\pi rni}{r^2} = \frac{2\pi ni}{r} \tag{49}$$

This gives the strength of field, *i.e.*, the number of gausses, or dynes of force on unit pole, at o when i is taken in *e.m.* units.

47. The Ampere and Coulomb.—For commercial purposes the *e.m.* unit as defined above is inconveniently large. So a practical unit called the *ampere* has been adopted. It is one-tenth (10^{-1}) of the *e.m.* unit.

The *e.m.* unit of *quantity* of electricity is the quantity per second which passes any cross section of a conductor in which the *strength* of the current is one *e.m.* unit. When the strength of current is one ampere, the quantity which passes per second is called one *coulomb*. The coulomb is then the practical unit of quantity and is one-tenth (10^{-1}) of the *e.m.* unit of quantity.

By *strength of current* is meant the quantity rate at which electricity is transferred on a conductor, while *quantity* of electricity, measured in coulombs, is independent of the time in which it may be made to flow from one point to another. By analogy the strength of a current as measured in amperes may be compared to the number of gallons of water per second flowing through a pipe. We may speak of the strength of the stream in terms of the number of gallons per second. But the quantity of electricity as measured in coulombs would correspond to the number of gallons of water in a vessel, no matter how or at what rate it had been placed there.

The strength of current is one ampere when one coulomb per second passes any cross section of a conductor.

48. Comparison of e.m. and e.s. Units.—In the electrostatic system, as has been shown, the unit *quantity* of electricity was first defined and then unit strength of current, i, would be the number, Q, of *e.s.* units of quantity per second which pass a cross section of the conductor. This is expressed by

$$i = \frac{Q}{t} \tag{50}$$

This equation also expresses the proper relation of the quantities in the *e.m.* system, but there it was i that was first defined, while in the *e.s.* system Q is defined as the fundamental unit.

Probably a more fundamental unit of current would be the number of electrons which pass any point per second or a strength which would cause a repulsion or attraction of 1 dyne between two conductors at a distance of 1 cm. But the systems which we have were established at a time when it was not so well known as now that all magnetism is electromagnetic, and that when a charge of static electricity is set in motion it will create a magnetic field in the same manner as does the current from a battery.

If a given quantity of electricity is measured first in *e.s.* units, and then in *e.m.* units, in a manner which will be indicated later, it will be found that the former is, in round numbers, $3(10)^{10}$ times larger than the latter. This means that the *e.m.* unit of quantity is $3(10)^{10}$ times as large as the *e.s.* unit.

49. Unit Difference of Potential.—The P.D. between two points has already been defined (§ 6) as the number of ergs of work that must be performed in moving a unit quantity of electricity from one point to the other. This is the definition no matter what system of units is used, but since the *e.m.* unit of quantity is $3(10)^{10}$ times as large as the *e.s.* unit, $3(10)^{10}$ times as many ergs of work must be done on the *e.m.* unit quantity as on the *e.s.* unit for the same actual difference of potential. Hence the P.D. indicated by 1 erg of work in the *e.m.* system is a very small quantity. *In the e.m. system two points have unit potential difference when* 1 *erg of work is done in moving unit e.m. quantity from one point to the other.*

The unit P.D. in the *e.m.* system is therefore $\frac{1}{3}(10)^{-10}$ part of the corresponding unit in the *e.s.* system.

50. The Volt. — Since the *e.m.* unit P.D. is inconveniently small, $(10)^8$ of them have been taken as a practical unit called the *volt. The volt then may be defined as the P.D. between two points when* $(10)^8$ *ergs of work must be done to transfer unit e.m. quantity of electricity from one point to the other.*

The volt is therefore $\frac{1}{300}$ of the *e.s.* unit P.D., for $\frac{1}{3}(10)^{-10}$ times $(10)^8 = \frac{1}{3}(10)^{-2}$.

51. Electromotive Force.—When potential difference is regarded as a cause of the flow of electricity it is usually called

electromotive force. Thus in Fig. 43, if a is charged positively and b negatively, there is a P.D. between a and b which is the E.M.F. causing a flow of current when the points are connected by a conductor. If we follow a convention and say that a current flows from a to b, there will be a drop of potential at each successive point from a to b, the sum of which is equal to the P.D. between a and b. If two points, as c and d, are joined by a conductor, a current will flow on *dec* for the potential at d is higher than at c. This P.D. may be called the E.M.F. of the current *dec*.

The greatest P.D. which a battery is capable of creating is called the E.M.F. of that battery; for example, the P.D. between the terminals of a cell on open circuit. The same may be said of other electric generators. How a battery produces a P.D. that acts as an E.M.F. will be described in a later section.

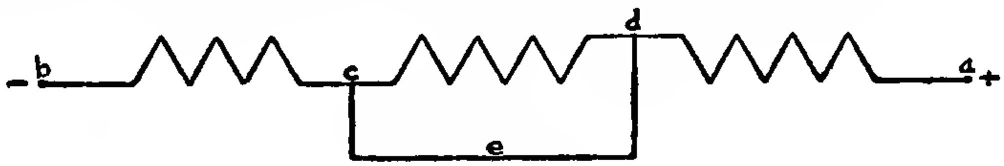

FIG. 43.

Electromotive force, like potential difference, is measured in volts.

52. Unit of Resistance. Ohm's Law.—The German physicist, G. S. Ohm, in 1827 announced a law, known by his name, that *for any given conductor the strength of the electric current is directly proportional to the electromotive force.* Thus if i is strength of current and E is E.M.F., then Ohm's law may be stated by

$$\frac{E}{i} = R \tag{51}$$

where R is a constant quantity which has been called the *resistance.* If, however, the conductor is changed in dimensions, state, or kind of material, the resistance will change and the E which was formerly used will give a different value for i. Equation (49) is therefore better written

$$i = \frac{E}{R} \tag{52}$$

The absolute e.m. unit of resistance is the resistance of a conductor when a P.D. of one e.m. unit maintained at its terminals causes a current of one e.m. unit.

The practical unit of resistance is called the *ohm*. *An ohm is the resistance of a conductor in which a P.D. of one volt will cause a current of one ampere.*

Since a volt equals $(10)^8$ absolute *e.m.* units of potential and the ampere equals $(10)^{-1}$ absolute units of current, then, from equation (51), the ohm equals $(10)^8 \div (10)^{-1} = (10)^9$ absolute units of resistance.

53. Concrete Standards.—The c.g.s. electromagnetic units as defined above are the fundamental units of the *e.m.* system. But although these units are real they are not practical in actual application for comparison, measurement, or legal action. Hence certain concrete units, made to conform as nearly as possible to the fundamental ones, have been prepared. This is much like the fundamental definition of the metre as one-ten-millionth of the earth's quadrant, but the practical metre is a platinum-iridium bar which was made to agree as nearly as possible with the fundamental definition.

The *ohm*, *ampere*, and *volt* have been defined in terms of concrete standards and are called *international units.*

The international ohm is the resistance offered to an unvarying current by a column of mercury 106.3 *cm. long, of constant cross-sectional area, and at the temperature of melting ice, the mass of the mercury being* 14.4521 *grams.* The area of cross section of such a column is practically one square millimetre.

The international ampere is the unvarying electric current which when passed through a solution of nitrate of silver in water, in accordance with standard specifications, deposits silver at the rate of .001118 *g. per second.*

The international volt is that electromotive force which will produce a current of one international ampere in a conductor whose resistance is one international ohm.

International units as defined by these concrete standards agree so closely with the fundamental ones that for all practical measurements no distinction need be made.

54. Energy Relations.—Since a volt is such a P.D. between two points that $(10)^8$ ergs of work must be done to transfer 1 *e.m.* unit of quantity of electricity from one point to the other, then to transfer one coulomb (10^{-1} *e.m.* unit) would require 10^8 times $10^{-1} = 10^7$ ergs. This is one joule of energy. If the number of

coulombs is represented by Q and the number of volts by V, then

$$\text{Energy in joules} = VQ \tag{53}$$

Now the strength of a current in amperes is the number of coulombs which are transferred per second. Hence if amperes are represented by i and t is the time in seconds, $Q = it$ as in equation (50). Hence equation (53) may be written

$$\text{Energy in joules} = Vit \tag{54}$$

Also from equation (52), where E is the same as V, $V = Ri$. Substituting this value of V in (54),

$$\text{Energy in joules} = i^2Rt \tag{55}$$

A current of electricity consists of electrons moving from the negative to the positive terminal of a conductor, *i.e.*, the current is a negative one, according to the electron theory, and the direction in which electrons move is opposite to that in which an electric current has been assumed to move. While electrons thread their way between molecules of a conductor, they are impeded in their movement, particularly by poor conductors, and their energy appears as heat. At least this is one conception we may have of how an electric current heats a conductor. The speed of an electron may not be great, possibly about 1 cm. per second when the P.D. is one volt, but the speed with which an impulse travels along a line of electrons, setting them all in motion, is enormously great—about $3(10)^{10}$ cm. or 185,000 miles per second.

When work is thus done in moving electricity against the resistance of a conductor and the only resistance is that due to the nature of the conductor itself, then all the energy expended appears in the conductor as heat. If the conductor is immersed in water in a calorimeter, the number of calories of heat may be measured by simply multiplying the mass of water plus the water equivalent of the calorimeter by the rise of temperature. But the mechanical equivalent of one gram-calorie is 4.187 joules, hence, from equations (54) and (55), V or i or R may be found when the other quantities are known. Also the number of calories of heat may be calculated when the strength of current in amperes, the resistance of the conductor in ohms, and the time are known.

Thus if both sides of equation (55) are divided by J, the mechanical equivalent of heat (see p. 246, " Mechanics and Heat "),

$$\frac{\text{Energy in joules}}{J}=\frac{i^2Rt}{J}$$

or

$$\text{number of calories}=\frac{i^2Rt}{4.187}=.24i^2Rt \tag{56}$$

A similar equation could be deduced from (52′), and it is plain that these relations furnish an excellent means of determining the mechanical equivalent of heat if the electric quantities involved can be accurately measured.

Now Vit is the total energy of a current where V is volts, *i.e.*, 10^8 ergs; i is amperes, *i.e.*, 10^{-1} *e.m.* unit; and t is time in seconds. If t is 1 second, then Vit is 10^7 ergs or one joule. One joule per second is taken as the unit rate of electrical work and is called the *watt.* The watt is the unit of electrical power and may be found by multiplying volts by amperes. One kilowatt is 1000 watts. One kilowatt-hour is the quantity of work done in one hour when the power is one kilowatt.

There are 746 watts or .746 kilowatts in one horse-power.

Problems

1. What power is required to maintain a current of .5 ampere at a pressure of 110 volts?

2. What will it cost to maintain 50 lamps, the resistance of each being 220 ohms, the current required being .5 ampere, when the rate is 10 cents per K.W. hour? 220 × 50 = 1100 : 2 = 5500 × 0.5 = 2750

3. If $2.7(10)^9$ electrostatic units of quantity of electricity pass through a conductor per second, what is the strength of current as measured in amperes?

4. How much work is done in moving 25 coulombs of electricity through the space between two points whose P.D. is 2 *e.s.* units?

5. What will be the strength of field at the centre of a coil of wire, the diameter being 30 cm., the number of turns 20, and the strength of current 2 *e.m.* units?

6. How much heat will be developed by the passage of 500 coulombs of electricity through a wire whose resistance is 25 ohms?

Ans. 1. 55 watts.
2. 27.5c. per hr.
3. 0.9 ampere.
4. 15,000 joules.
5. 16.75 gausses.
6. 2985.43 calories.

CHAPTER IV

CONDUCTION OF ELECTRICITY THROUGH SOLUTIONS

55. An Electrolyte.—An electrolyte is a solution which serves as a conductor of electricity. Electrolytes of greatest importance are aqueous solutions of strong acids, bases, and salts. Pure water is a nonconductor of electricity and so likewise is pure sulphuric acid, but a solution of sulphuric acid in water is a good conductor. We shall see, however, that the method by which electricity is conveyed through an electrolyte is very different from that in a metallic conductor. In case of electrolytes, as we have seen, osmotic pressure, lowering of freezing point, and elevation of boiling point are all abnormal. (See p. 220, "Mechanics and Heat.")

56. The Electrolytic Cell.—Faraday in 1832 made a close study of the effects of passing a current through solutions—a process which he called electrolysis. The solution *E*, Fig. 44, is the electrolyte which we will say for our present purpose is copper sulphate ($CuSO_4$). Plates *A* and *C*, extending down into the electrolyte, are called electrodes, *A* being the anode * (the way up) and *C* the cathode (the way down) when the current flows in the conventional direction, as indicated by the arrow heads. A battery *B* creates a potential difference between *A* and *C*. The effects observed in this particular case are a deposit of pure metallic copper on the cathode and a passing of copper from the anode into the solution.

57. Action within an Electrolytic Cell. Dissociation.—Various attempts have been made to construct a theory which would consistently explain experimental facts of electrolysis. In 1805 Grotthus advanced the theory that molecules of a dissolved substance are composed of positive and negative atoms and when these are placed in an electrostatic field, as between the electrodes in Fig. 44, they are turned so that their positive sides are toward

---

* The terms anode and cathode originated with Faraday. He placed the electrolytic cell so that the current passing through it would be parallel and in the same direction as the earth currents which were assumed to flow from east to west or in the direction of the sun's apparent motion. Hence the plate of the cell on the side where the sun comes up was called the anode, and on the side where the sun goes down, the cathode.

the cathode and the negative sides toward the anode. Then when the P.D. becomes sufficiently great, the positive and negative atoms were supposed to be forced apart and caused to move by steps—*i.e.*, from molecule to molecule—toward that electrode by which they are attracted. Thus it was supposed that molecules continued as such in a solution and were broken up only by the action of an electric force.

Faraday likewise regarded the electric current as the immediate cause of the breaking up of molecules in a solution. The parts into which molecules are separated possess a charge of electricity, either positive or negative. Faraday called these *ions* (from the Greek word *to go*). The ions which move toward the cathode are called the *cations*, and those moving in the opposite direction, *anions*. Thus the ions of hydrochloric acid are $\overset{+}{H}$ and $\overset{-}{Cl}$; sodium chloride, $\overset{+}{Na}$ and $\overset{-}{Cl}$; sulphuric acid, $\overset{+}{H_2}$ and $\overset{-}{SO_4}$; copper sulphate, $\overset{+}{Cu}$ and $\overset{-}{SO_4}$.

An ion is an atom or a group of atoms possessing a charge of electricity.

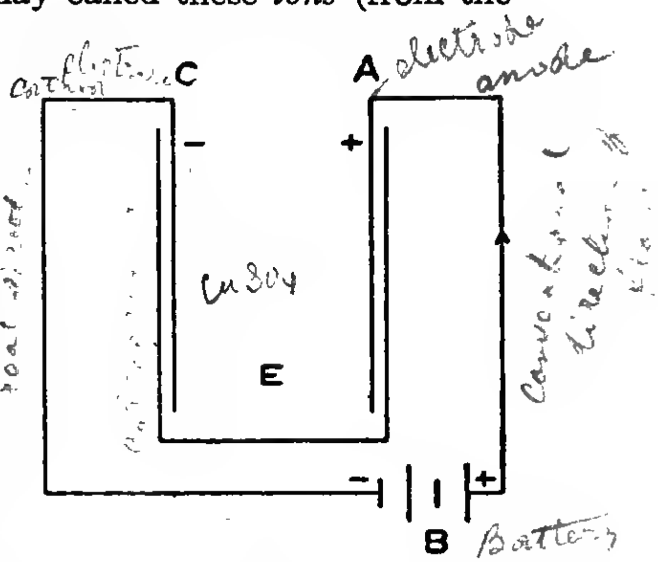

FIG. 44.

An objection to these theories was brought forward by Clausius in 1857. His objection was based on the fact that very little or none of the electric energy of the current is expended in the separation of molecules into ions as the Grotthus theory would require. In a cell like that shown in Fig. 44 where the electrodes are copper and the electrolyte $CuSO_4$, the current flows as soon as there is a P.D. between the electrodes and is proportional to the P.D. as Ohm's law requires for metallic conductors. It is true that when the substance deposited by the cation is different from that of the cathode there is a counter E.M.F. due to the tendency of the substance to go back into solution and therefore a certain critical P.D. of the electrodes, different for different substances, is then necessary before electrolysis will begin, but when once started the current is proportional to E.M.F. whereas, according to the

older theory, the current should at once greatly increase as soon as the molecules have been broken up into ions.

Clausius, therefore, proposed a modification known as the *dissociation theory*. This has already been alluded to under Osmotic Pressure. The theory assumes that in an electrolyte the ions of molecules are dissociated whenever a solution is formed. No current is needed to break up the molecules. In fact the presence of ions with positive and negative charges is a necessary condition of an electrolyte, and without these a solution is not a conductor of electricity. A solution of sugar in water is a nonconductor of electricity because the molecules of sugar are not separated into positive and negative ions.

It is not necessary to assume that all the molecules of an electrolyte are ionized. In a strong solution, as will be shown later, the ionization is only partial, though in a very dilute solution all the molecules may be separated into ions. Free ions may not remain so, but in strong solutions are constantly uniting with others of opposite sign and then separating. At any given time, however, there are a number of free ions and a number of neutral molecules where ions have united, except in very dilute solutions where all ions may be assumed to continue free. The effect, then, of a difference of potential between electrodes immersed in an electrolyte is not to *cause* ionization but to *direct the motion* of ions already in the solution.

Attempts have been made to explain the cause of dissociation. Clausius was of the opinion that it resulted from violent impacts against water molecules. Another possible cause may also be given. It has been shown (equation 3) that the force between two electric charges depends on the medium in which the charges are placed—*i.e.*, the force varies inversely as the dielectric constant of the medium. Now for water, which is by far the most important solvent, the value of K is 80 as compared to air. It is possible that this weakening of the bonds which hold together the ions of a molecule may, in certain substances, render effective the impacts of water molecules and result in dissociation.

This theory did not find a ready acceptance until the publication in 1887 of two remarkable papers,* one by Van't Hoff on the

* See Modern Theory of Solutions: Harper's Scientific Memoirs. Pub. Harper & Bros.

laws of osmotic pressure of dilute solutions and the other by Arrhenius giving an explanation of abnormal osmotic pressure as due to dissociation. The same number of molecules of any substance in a given volume of solution should give the same osmotic pressure. To obtain the same number of molecules a normal solution is made up, *i.e.*, one containing a gram-molecule of the substance to a litre of solution.* A gram-molecule is a number of grams equal to the molecular weight. For sodium chloride, one gram-molecule is $Na(23.05) + Cl(35.45) = 58.5$ g. In like manner one gram-molecule of grape sugar is 342.2 g. If such a quantity of these substances be dissolved, each in say two litres of water, there would be an equal number of molecules per cubic centimetre in each solution. It would be expected, then, that the osmotic pressure would be the same in each case, but experiment shows that in dilute solutions the $NaCl$ solution exerts a pressure twice as great as that of the sugar solution. The natural explanation would be that when $NaCl$ is dissolved in water the molecules are dissociated into ions, each of which acts as a physical molecule and affects the osmotic pressure as much as would one undissociated chemical molecule of sugar. A solution of $NaCl$ is an electrolyte. All electrolytes produce an osmotic pressure abnormally great, but never more than twice as great in case of substances which cannot break up into more than two ions.

In addition to this evidence in favor of dissociation, Raoult has shown by an extensive series of experiments that the lowering of the freezing point and elevation of boiling point of electrolytes are abnormally great. This, as above, would be explained as a result of dissociation, each ion then acting as a physical molecule and virtually increasing the concentration of the solution.

Again, if the theory of dissociation as explained above is correct, the electric conductivity of a dilute electrolyte should be greater than one containing the same member of molecules in a concentrated solution, for in the former case the ions are so far separated that most of them (in very dilute solutions all of them) remain separated from their neighbors and act as carriers of electric charges; while in more concentrated solutions a part of

* In case of a substance like $CuSO_4$, a gram-molecule per liter is usually called a molar solution. One-half of this amount per litre of solution is then a normal solution, for Cu carries two positive charges, *i.e.*, its valence is 2.

them, though not always the same ones, would be united with others of opposite sign, thus forming neutral molecules. Many experiments have been performed to test this point. The best way to do this is to determine what is called the *molecular conductivity* of a solution, *i.e.*, the conductivity with different dilutions but with the same number of molecules of the substance in solution. This may be done by use of an electrolytic cell like that shown in Fig. 45, where two platinum plates, *a* and *b*, are set at a fixed distance apart. Then some normal solution is introduced—enough to cover the plates. A current can then pass from *A* to *C* only by passing through the electrolyte from *a* to *b*. The resistance of that part of the circuit between *a* and *b* is then determined by use of a Wheatstone bridge as described in § 85. The reciprocal of this resistance is the conductivity; *e.g.*, if the resistance of any conductor is three times as great as some standard of resistance, its conductivity would be one-third as great. Now remove one-half of the electrolyte and put in its place an equal quantity of pure water. This is dilution 2. Find the conductivity as above and multiply by 2, since one-half the molecules have been removed and we desire the conductivity when the number of molecules is the same for all dilutions. Hence, multiplying by 2 gives the same result for this dilution as would have been obtained if none of the electrolyte had been removed and a volume of water equal

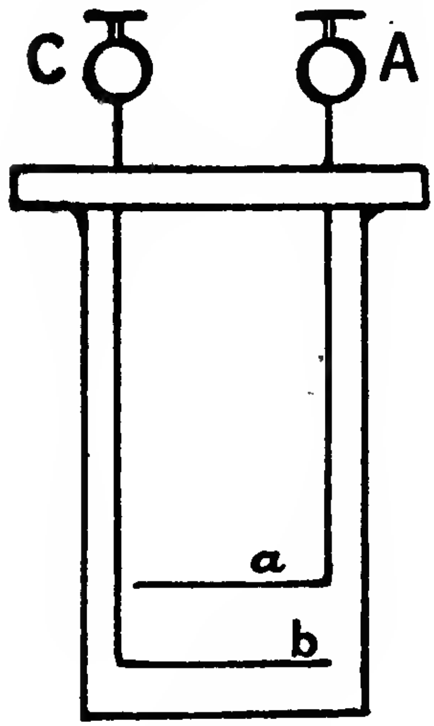

FIG. 45.

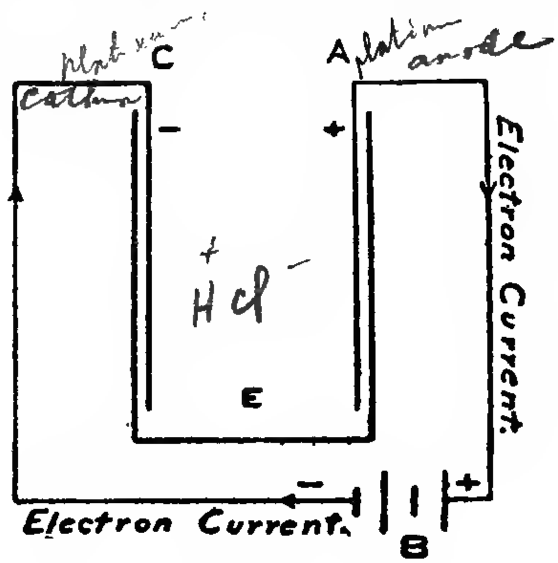

FIG. 46.

to the total volume of the electrolyte has been added. Again remove one-half of the electrolyte and add an equal volume of water. This is dilution 4. Find the conductivity and multiply by 4. The next operation would give dilution 8. Thus the process may be continued until the electrolyte is very dilute. Details of this experiment may be found in laboratory manuals.

An experiment of this character shows that, as the dilution increases, molecular conductivity also increases and approaches a certain fixed value. The physical interpretation of this would be that as the dilution increased a greater and greater number of molecules were permanently ionized. At very great dilutions the percentage of ionization may be regarded as 100.

58. Transfer of Electricity in Electrolytes.—The laws of electrolysis, as far as they apply to deposition of metals, the evolution of gases, or the principle of dissociation, are definite and as well established as are most of the laws of physics. But when we attempt to get a distinct physical concept of just what takes place between the electrodes and the electrolyte there is a great deal of uncertainty and lack of agreement among scientists. It may be profitable, however, to consider one possible way of looking at the matter.

For a first case let the electrolyte *E*, Fig. 46, be a solution of hydrochloric acid in water, the electrodes being strips of platinum. An electron current of electricity flows from the negative pole of the battery *B* to the electrode *C*, and from *A* to the positive pole. These electrons, however, do not pass through the electrolyte. *C* is still called the cathode and *A* the anode according to established conventions. The ions in this electrolyte are $\overset{+}{\mathrm{H}}$ and $\overset{-}{\mathrm{Cl}}$. The $\overset{+}{\mathrm{H}}$ ions will steadily move toward the cathode *C* which is negatively charged, *i.e.*, has an excess of electrons. When $\overset{+}{\mathrm{H}}$ reaches *C*, electrons will pass to H, making it negative, when it will at once combine with a hydrogen ion which has not yet reached *C* and will form a neutral hydrogen molecule H_2 which, with others, rises as a bubble through the electrolyte. On the other side the Cl ions move to *A*, where electrons pass from $\overset{-}{\mathrm{Cl}}$ to *A*, leaving the chlorine ion positive, when it at once combines with one of the Cl ions which has not yet parted with its excess

of electrons, thus forming chlorine gas which is absorbed in the water of the electrolyte. According to this explanation, each of one-half of the $\overset{+}{H}$ ions took two electrons from *C*, and the same number of $\overset{-}{Cl}$ ions gave two electrons to *A*.

Now let the electrolyte be a solution of copper sulphate, $CuSO_4$, and the electrodes, strips of copper. The ions are then $\overset{++}{Cu}$ and $\overset{--}{SO_4}$. The $\overset{++}{Cu}$ ions move up to the cathode and four electrons pass from *C*, making Cu negative, when it combines with a positive Cu, forming a molecule of copper which is deposited on the cathode. On the other side the $\overset{--}{SO_4}$ unites with an atom of the copper anode and brings it into solution as fresh $CuSO_4$. This permits four electrons to escape from *A* to the positive pole of the battery. This is the principle used in electroplating. The solution maintains its strength, but the mass of the anode grows less.

If platinum electrodes are substituted for copper ones, copper will be deposited on the cathode as before, but at the anode the $\overset{--}{SO_4}$ displaces $\overset{--}{O}$ of a water molecule and combines with $\overset{++}{H_2}$, forming H_2SO_4. The $\overset{--}{O}$ then gives up four electrons to the anode, thus becoming positive, and then unites with a negative O, forming a neutral molecule of oxygen gas. These collect and escape as bubbles of oxygen. In this case the solution soon becomes weak in copper ions, for copper is deposited on the cathode and is not replenished at the anode.

In the electrolysis of water a cell like that shown in Fig. 47, called a water voltameter, may be used. Since pure water contains very few, if any, ions, it cannot be electrolyzed directly. By the addition of a little H_2SO_4 the water will swarm with $\overset{++}{H_2}$ and $\overset{--}{SO_4}$ ions and, by use of them, molecules of water, H_2O, may be separated into their constituent parts. In this cell the electrodes are platinum. The cell is filled with water acidulated with H_2SO_4. When a battery having sufficient E.M.F., at least 1.5 volts, is connected as shown, *A* becomes the anode and *C* the cathode. The $\overset{++}{H_2}$

ions then move up to *C*, take four electrons, and combine with $\overset{++}{H_2}$ near by, thus forming two neutral molecules of hydrogen gas, H_2. At the anode the $\overset{--}{SO_4}$ ion displaces the oxygen of a water molecule, then combines with the $\overset{++}{H_2}$ thus set free, forming a molecule of H_2SO_4. The $\overset{--}{O}$ which has been displaced then parts with four electrons to the anode, becoming $\overset{++}{O}$ when it unites with an $\overset{--}{O}$, thus forming a molecule of oxygen, O_2. But since for each $\overset{++}{H_2}$ ion two molecules of hydrogen are formed, then for the same transference of electrons the volume of hydrogen gas evolved should be twice as great as that of oxygen. Experiment shows such a result for the volume of water displaced at *H* is twice as great as that at *O*.

In electrolytic actions such as those described above, the results which are obtained and their quantitative relation to the current employed are simple matters of experimental verification, but a description of just what occurs within an electrolytic cell can be supported only by claim of consistency with known laws of electricity and chemistry.

It will be observed that the word *electrolysis*, meaning *to set loose by means of electricity*, no longer retains its etymological meaning, and, likewise, the terms *cathode* and *anode* should be defined not in reference to the direction in which a current flows but rather in reference to the direction in which positive or negative ions move.

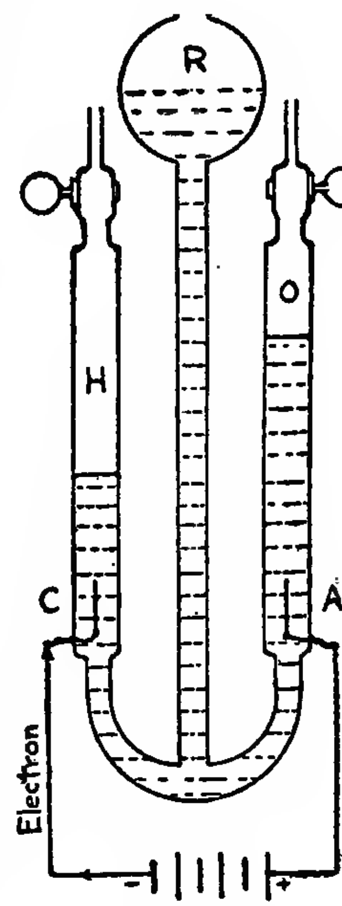

FIG. 47.

59. Faraday's Laws of Electrolysis.—Faraday in 1832 announced two laws which embody the main results of his investigation of the relation between electric currents and the products of electrolysis. It has never been necessary to change his statement of these laws. They are:

1. *The mass of any given substance produced by electrolysis is proportional to the quantity of electricity which flows in the electric circuit.*

2. *The masses of different substances produced by equal quantities of electricity are proportional to the chemical equivalents of those substances.*

The first law states a result which would be expected from the nature of action in an electrolytic cell as described in the preceding paragraph, for a current flows through the circuit, outside the cell, only as a result of ionic charges which are taken from the current at the cathode and an equal number of others which are added to the current at the anode. In other words the battery circuit is open at the electrolytic cell, and a current is kept moving by taking electrons from the negative terminal and adding an equal number of other electrons to the positive terminal. Then, since all the ions of any given substance are alike in their ability to give or take electrons, the quantity of substance deposited or evolved at either the anode or cathode must be proportional to the current.

In reference to the second law a few terms may need explanation. The chemical equivalent of any substance is its atomic weight divided by its valence. The valence of H is 1; that of O is 2. This means that it requires two atoms of hydrogen to combine with one of oxygen in forming a molecule of water, H_2O. Each atom of matter always carries a definite quantity of electricity. The quantity on a hydrogen atom was called by Helmholtz an atom of electricity. The number of atoms of electricity on an atom of matter determines the valence, *i.e.*, an atom of matter may have an excess or a deficiency of electrons, but the number either way is always an exact multiple of the atom of electricity on hydrogen. Hence the valence of an atom or radical is sometimes defined as the number of hydrogen atoms which would need to combine with it to form a neutral molecule. For example, in H_2SO_4 we say that the valence of the radical SO_4 is 2 because it takes two atoms of H to unite with SO_4 and form a molecule of sulphuric acid. Then in $CuSO_4$ the valence of Cu must be 2, for it takes the place of H_2.

A given quantity of electricity will therefore produce in electrolysis twice as many atoms of H as of O because the valence of O is 2, *i.e.*, there need be only half as many atoms of O to carry

the same quantity of electricity. Now the atomic weight of H is 1, and of O, 16. Also the masses of equal volumes of different gases under the same conditions are to each other as their atomic weights. But since the volume of oxygen is one-half that of hydrogen, then 1 g. of hydrogen is chemically equivalent to 8 g. of oxygen. If quantities of hydrogen and oxygen in the proportion of 1 to 8 by weight be united by combustion, none of either gas would be left over, for they are chemically equivalent. Likewise if we are considering the deposit of copper from $CuSO_4$, since the valence of Cu is 2 and its atomic weight is 63.6, the chemical equivalent is 31.8. In case of silver from $AgNO_3$, the valence of Ag is 1 and its atomic weight 107.9, hence its chemical equivalent is 107.9. Therefore the same quantity of electricity that will cause a deposit of 31.8 g. of Cu from $CuSO_4$ will cause a deposit of 107.9 g. of Ag from $AgNO_3$. The mass deposited in any case by a given quantity of electricity varies directly as the atomic weight and inversely as the electric charge (the valence) which the ion carries.

60. Electrochemical Equivalent.—*The mass of any substance resulting from the electrolytic action of one coulomb of electricity is called the electrochemical equivalent of that substance.* The advantage of a knowledge of this quantity will appear from the following:

Faraday's laws may be expressed in form of an equation by

$$m \propto q\frac{a}{v} \tag{57}$$

where m is the mass, q the quantity of electricity, a the atomic weight, and v the valence. By this it is possible to compare the masses of different substances deposited when q, a, and v are known, but we could not find m for any one given substance. If, however, we can find the value of m for any substance on the assumption that q, a, and v are unity, then this value, say k, when multiplied by qa/v would give m for any substance. Hence we may write

$$m = qk\frac{a}{v} \tag{58}$$

The value of k may be found once for all by experiment with any desired electrolyte. The most exact results are obtained from a deposit of silver by use of a cell like that in Fig. 48, where the

anode—a plate of silver—is suspended in a solution of silver nitrate in a platinum cup which is the cathode. By noting the number of amperes, the time, and the increased weight of the cup, the number of grams of silver per ampere-second, or coulomb, is readily calculated. This is found to be .001118 g. per coulomb.

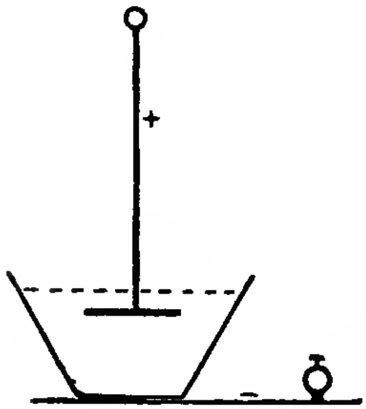

FIG. 48.

Since the atomic weight of silver is 107.9, its valence 1, and q for this value of m is 1 coulomb, equation (58) becomes

$$.001118 = 107.9k$$

or

$$k = 1.036(10)^{-5}$$

The quantity $k\frac{a}{v}$ is by definition the *electrochemical equivalent*, for it is the quantity which when multiplied by the number of coulombs gives the mass deposited.

For hydrogen the atomic weight and valence are both unity, hence, for one coulomb, equation (58) becomes, for this gas,

$$m = k = 1.036(10)^{-5}$$

The value of k is therefore the electrochemical equivalent of hydrogen.

If one coulomb will cause the evolution of $1.036(10)^{-5}$ g. of hydrogen (see Fig. 47), for 1 g. of hydrogen it would require

$$\frac{1}{1.036(10)^{-5}} = 96525 \text{ coulombs}$$

From the second law the mass of substance resulting from any given quantity of electricity is proportional to the chemical equivalent. Hence the 96,525 coulombs which produced 1 g. of hydrogen would, in a proper electrolyte, produce 107.9 g. of Ag, 31.8 g. of Cu, 8 g. of O, or a mass equal to the chemical equivalent of any substance capable of electrolysis.

61. Charge on an Ion.—If the total quantity of electricity carried by ions were known, and also the number of ions concerned, it would be a simple matter to calculate the charge on each ion. In case of hydrogen whose density under standard conditions is $8.95(10)^{-5}$ g. per c.c., the volume of one 1 g. is 11,173 c.c. Some of the best determinations of the number of molecules in 1 c.c.

of hydrogen under these conditions are $2.7(10)^{-19}$. The number of atoms would be $5.4(10)^{19}$. Hence the total number of atoms in 1 g. is

$$11173 \times 5.4(10)^{19} = 60334(10)^{19}$$

Each of these atoms was an ion in the electrolytic cell and the sum of their charges was 96,525 coulombs per gram of the gas. Hence the charge on each ion was

$$\frac{96525}{60334(10)^{19}} = 1.6(10)^{-19} \text{ coulombs}$$

This, then, is what has been called one atom of electricity—the electron. In *e.m.* units it would be $1.6(10)^{-20}$ since 1 coulomb equals $(10)^{-1}$ *e.m.* unit of quantity. In *e.s.* units it would be $4.8(10)^{-10}$ since the *e.m.* unit is $3(10)^{10}$ times as large as the *e.s.* unit.

According to this the fundamental unit quantity of electricity is $4.8(10)^{-10}$ of the electrostatic unit, and this is the charge on all atoms of matter whose valence is 1. If the valence is 2, the atom carries twice this charge.

Assuming this result to be correct, it is possible to calculate the number of molecules per cubic centimetre of any substance deposited by electrolysis. An ion of Cu carries two unit charges or $3.2(10)^{-19}$ coulombs. It would then require, to carry 96,525 coulombs,

$$\frac{96525}{3.2(10)^{-19}} = 30164(10)^{19} \text{ ions}$$

These would form $15{,}082(10)^{19}$ molecules. The mass of copper deposited by 96,525 coulombs is 31.8 g. Hence the number of molecules per gram is

$$\frac{15082(10)^{19}}{31.8} = 4.74(10)^{21} \text{ molecules per gram}$$
$$= 4.03(10)^{22} \text{ molecules per c.c.}$$

62. The Voltameter.—A voltameter, more properly called a coulometer, is an instrument used to measure the quantity of electricity by the electrolytic effects produced. Any of the electrolytic cells which have been described may be used as coulometers.

The silver cell, Fig. 48, is the standard cell. The copper coulometer was used in former times to determine the quantity of direct current used in electric lighting. The change in weight of the cathode and the electrochemical equivalent are the only two factors needed to calculate the quantity of current used.

63. Uses of Electrolysis.—The deposit of a thin layer of one metal over the surface of another is a very common practice in an art called *electroplating*. Gold, silver, copper, nickel, zinc, tin, and platinum are some of the most common metals used for this purpose. It is necessary in all cases that the electrolyte be a solution of some salt of the metal to be deposited, *e.g.*, sulphate or cyanide of copper, chloride of gold, etc., and the anode of the plating vat should be the substance which is to be transferred to the cathode.

Extensive use is also made of this principle in a process called *electrotyping*, by which an exact reproduction of a page of type is made in copper and preserved for future printing.

Ordinary commercial copper contains many impurities. If this is placed as the anode in a copper sulphate solution, an electric current will, when proper precautions are observed, transfer only pure copper to the cathode. This operation is carried out on a large scale and the product is called *electrolytic copper*. About 1 kg. of copper may thus be refined by 250 watt-hours of electrical energy, though the amount differs widely for different conditions and different kinds of copper.

Electrolysis often has an injurious effect on metal pipes laid in the ground. Where the current of an electric circuit is grounded, electricity will flow back to the dynamo, or other point of different potential, on the metal pipes rather than through the ground. The pipe is often surrounded by moist earth containing a solution of electrolytic salts. The pipe then acts as an anode at the point where the current leaves it, and this is where the metal of the pipe wastes away.

NOTE.—In this and the following chapter chemical symbols are, for convenience, kept in the same form after ionization as in the neutral molecule, the number of plus and minus signs indicating the number of atoms of electricity to be taken into account whatever the valence of the substance may be. Some authors write 2H with a plus sign over the H to indicate two ions of H. Others write two plus marks even when they use the form 2H.

Problems

1. A current flowing for one hour increases the weight of the cathode of a copper voltameter 2.9646 g. What was the strength of current?

2. How many coulombs of electricity will produce 50 c.c. of oxygen at 22° C. and under a barometric pressure of 73 cm.?

3. If a certain current produces 3 g. of copper in a copper voltameter, how much nickel would be deposited by the same current? Make a proportion according to the second law.

4. If 46 is the atomic weight of a certain substance and its valence is 2, how much electricity will be required to deposit 11.5 g. of it?

5. How many molecules in 1 c.c. of gold?

6. How long a time must a current of .5 ampere flow to cause a deposit of .4 g. on the cathode of a silver voltameter?

7. What would be the increased cost per kg. of electrolytic copper where a current of 20 amperes at a pressure of 2 volts is maintained at a cost of 10 cents per kilowatt hour?

8. A surface of 10 sq. cm. is plated with gold. A current of 1.3 amperes flows for 30 minutes. How thick will the gold plate be?

Ans. 1. 2.5 amperes.
2. 769.6 coulombs.
3. 2.77 g.
4. 48,262.5 coulombs.
5. $2.95(10)^{22}$.
6. 11 min. 55.6 sec.
7. 16.44c. per kg. or 7.5c. per lb.
8. .08257 mm.

CHAPTER V

BATTERY CELLS

64. The Voltaic Cell.—From the time of the earliest experiments in electricity to the beginning of the 19th century, static electricity, so called, was the only kind known.

Galvani (1737–1798), professor of physiology at Bologna, investigated the relation of electricity to animal life. When a current from his electric machine was passed through the body of a dead frog the muscles would contract. He also noted that when the frogs were hung on brass hooks attached to an iron railing, their legs would be drawn up by muscular contraction whenever they were caused to swing against the iron.

Galvani's explanation was that the electricity was in the body of the animal and, when a conductor connected the outside of the frog's leg to a nerve which led to the inside, a discharge much like that of a Leyden jar took place, thus producing a union of the negative charge on the outer surface with the positive charge within. The metal, as he claimed, only acted as a conductor of these charges.

Volta (1745–1827), professor of natural philosophy at Padua, claimed that Galvani's contention was not correct and that the electricity resulted from the contact of dissimilar metals, the nerve and muscles of the frog acting only as conductors. To prove his claim Volta devised a number of experiments and showed that when dissimilar metals—copper and zinc for example—were placed in contact, the one with the other, one became positive and the other negative, as was shown by a delicate electroscope. This claim is discussed in the next paragraph. He also invented the voltaic pile, which may consist of alternate strips of copper and zinc, paper moistened with acidulated water being placed between each successive pair of plates—*i.e.*, there would be copper, paper, zinc, copper, paper, zinc, etc., piled up to any height. Such a pile may be made to give a very vigorous current of electricity. From this it was an easy step to the voltaic cell as we now know it, consisting of dilute sulphuric acid in which are placed the elements zinc and copper. A wire connecting the zinc and copper plates conducts the electrons

from zinc to copper. This is the only current that actually flows, though according to a custom we speak of a positive current as flowing in the wire from copper to zinc. The former is a negative current and the latter, which is only assumed, is called a positive current.

65. Contact Difference of Potential.—Volta claimed that it was an inherent property of metals that when different kinds were placed in contact there would result a difference of potential between them, and he named a series of metals any one of which would be positive in reference to any other one lower in the series. Experiment seems to confirm the truth of this claim and a differ-

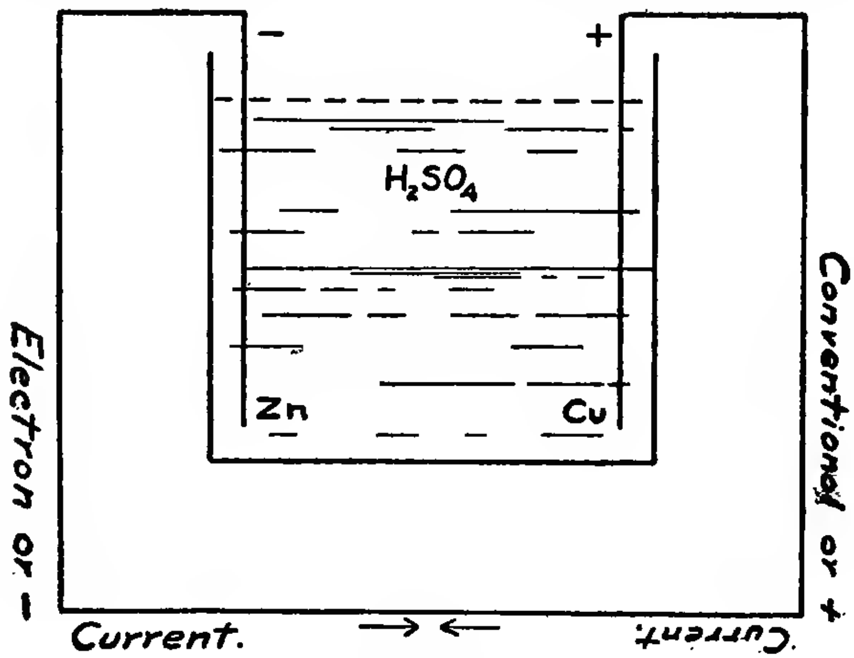

Fig. 49.

ence of potential does exist, but later experiments have shown that when the metals are surrounded by gases other than air, a series entirely different from that given by Volta and others is obtained. This seems to show that the effects observed are a result of the relation of the metals to the surrounding gases rather than an inherent property of the metals themselves. This matter, however, is not yet fully settled.

Between certain metals and an electrolyte there is a definite contact difference of potential which can be measured in several different ways. In the voltaic cell, Fig. 49, it is found that the contact difference of potential between zinc and the electrolyte

is .62 volt and that between copper and the electrolyte is .46 volt, the former being lower and the latter higher than the electrolyte. Hence the total difference between the zinc and copper in the cell is 1.08 volts. This furnishes an E.M.F. which causes a current to flow on the conducting wire.

One method of measuring the contact difference of potential is by use of Lippmann's capillary electrometer. The principle of this instrument is that when a current of electricity is passed through a point of contact between mercury and dilute sulphuric acid there will be a change of surface tension at that point which will cause a movement of mercury one way or the other, depending

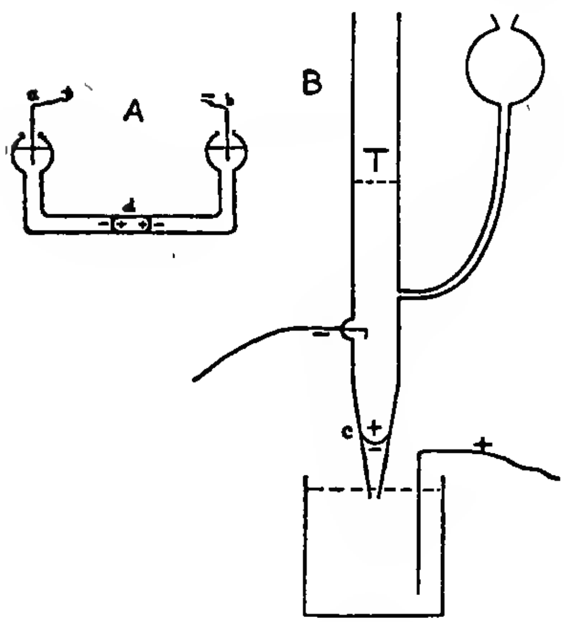

Fig. 50.

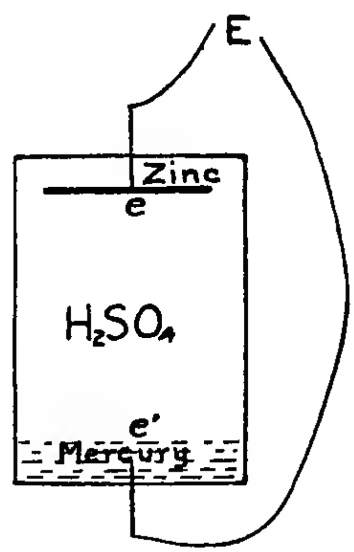

Fig. 51.

on the direction of the current. If, for example, a drop of mercury, *d* (Fig. 50,*A*), is introduced into a glass tube, the remainder of the tube being filled with dilute sulphuric acid, each end of the mercury drop will be positive in reference to the electrolyte with which it is in contact, and as a result of this the surface tension will be weakened at both ends. If now a positive current be passed from the platinum electrode *a* through the liquids to *b*, the effect will be to decrease or neutralize the contact difference of potential on the left end of the mercury and to still further increase the difference at the right-hand end. Thus the surface tension on the left may increase to its natural strength while that on the right will decrease. The mercury will therefore move along the

tube in the direction of the positive current. Lippmann's apparatus consists of a glass tube T drawn to a fine point (Fig. 50, B). It is partly filled with mercury and the lower end is inserted in an electrolyte. There will be a contact of mercury and electrolyte at c and the point c will be lower on account of the contact, for the surface tension of the mercury has thereby been weakened. Now let the meniscus of the mercury at c be observed by a fixed microscope and let a positive current pass from the electrolyte up through c. This will tend to neutralize the contact difference of potential at c, the surface tension will increase, and the mercury meniscus will rise to a wider portion of the tube where the pressure due to surface tension upward is balanced by the pressure of the column of mercury above. By raising the tube S the height of mercury in T may be increased till the meniscus is again brought to its position as first seen through the microscope. As the potential of the current is increased a point will be reached where the height of mercury in T necessary to bring the meniscus back to c will be a maximum. This occurs when the applied difference of potential is just equal and opposite to the contact difference, for any further increase in the potential of the current would cause a difference at c opposite in sign. This would again weaken surface tension and the meniscus would fall to a narrower part of the capillary. Hence the known E.M.F. which must be applied to produce maximum surface tension, as measured by the height of mercury in T necessary to bring the meniscus back to c, is also the contact difference of potential between the mercury and the electrolyte.

Suppose, now, it is desired to know the contact difference of potential between zinc and dilute sulphuric acid. The P.D. at the terminals E, Fig. 51, is the potential difference between the mercury and zinc. This can readily be measured by use of a potentiometer and is evidently the difference between e' and e when these are both considered in reference to the same electrolyte. For example, E is found to be 1.48 volts and e' is found by the capillary electrometer to be .86 volt. Then

$$E = e' - e$$

$$1.48 = .86 - e$$

$$\therefore e = -.62$$

Thus the potential of the mercury is .86 volt higher than the electrolyte, and the electrolyte is .62 volt higher than the zinc. In the voltaic cell, Fig. 49, copper is .46 volt above the electrolyte and the electrolyte .62 volt above the zinc. Hence the P.D. between the copper and zinc is 1.08 volts.

66. Polarization.—The passage of electricity through a voltaic cell or any electric cell is accomplished in the same manner as in an electrolytic cell. The difference between the two cells is chiefly the fact that an electric cell produces its own electromotive force. Polarization is the condition of a cell when a counter electromotive force is set up whereby the flow of current is checked or stopped. As already suggested in the previous chapter, when a small difference of potential is maintained between platinum electrodes in a solution of sulphuric acid, a current will begin to flow but will soon cease. A counter electromotive force has been set up equal and opposite to that between the electrodes. If the P.D. of the electrodes be increased—in this particular case it must be more than about 1.7 volts—the current will again flow notwithstanding the counter E.M.F. which still opposes it. The existence of the counter E.M.F. may be shown experimentally by the arrangement shown in Fig. 52 where, when the switch s is thrown to a, the battery B will cause a positive current to flow as indicated by arrow heads on the battery circuit. The electrolytic cell E, having platinum electrodes and an H_2SO_4 solution, will be polarized. If now the switch s is thrown to b, the battery will be cut out and the cell E will for a short time send out a current in an opposite direction, as will be shown by the galvanometer G.

Thus, as is usually the case in electrolytic action, the passage of a current may cause a change in the character of the surface of one of the electrodes or a change in the electrolyte adjacent to the electrodes, either of which will require the expenditure of external energy in the process of electrolytic action. This occurs when the electrodes are of a different material from that which is produced by electrolytic action.

In the same way most electric cells will soon polarize if the current which they produce is allowed to flow through a conductor from one electrode to the other. The voltaic cell, Fig. 49, is one of this kind, and its current soon becomes weak.

When, on the other hand, the substance deposited by electrolytic action is the same as that of the electrode, there is no polarization. This is the case when the electrodes are bathed in a solution of their own salts, *e.g.*, copper electrodes in a solution of copper sulphate. Here the passage of electricity consists simply of charges on atoms of copper going into solution at the anode and other charged atoms being deposited from the solution upon the cathode. Metals going into solution always carry a positive charge, *i.e.*, are always deficient in electrons. For every positive charge which goes into solution in this case, an equal positive charge goes out and no counter electromotive force is developed.

This suggests the possibility of a nonpolarizing cell and such is realized in the Daniell cell where, in one form, a zinc plate, the anode, is bathed in a solution of zinc sulphate, $ZnSO_4$, and a copper plate, the cathode, in a solution of copper sulphate, $CuSO_4$, the two solutions being kept apart

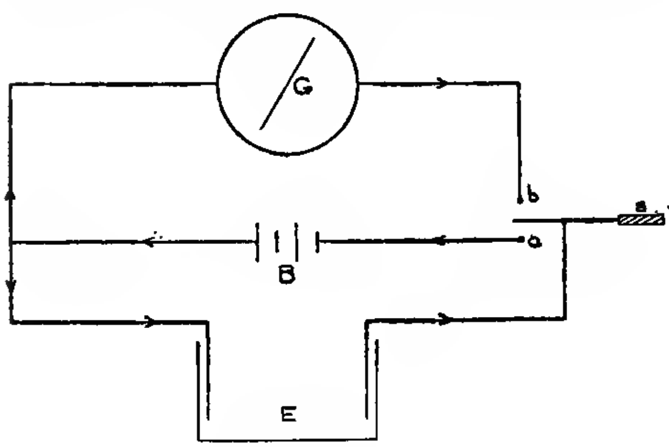

Fig. 52.

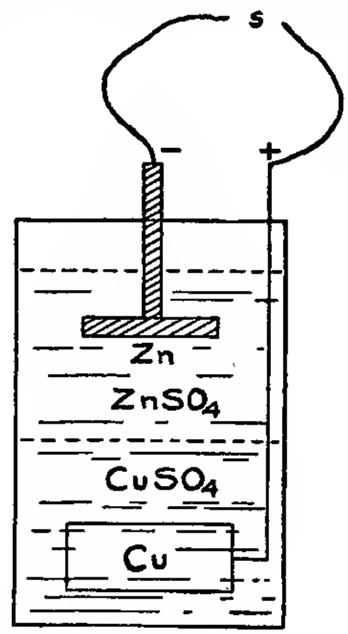

Fig. 53.

by a difference in density as shown in Fig. 53. Here zinc goes into solution at the anode, and copper is deposited on the cathode. This causes a difference of potential between the electrodes, for each atom of zinc that goes into solution carries a positive charge and so the zinc anode is left with an excess of electrons, *i.e.*, is negatively charged. Also, the positively charged ions of copper on reaching the cathode take up electrons which pass over the conducting wire from anode to cathode. Thus a negative current passes over the conductor from zinc to copper, or we may regard it as a positive current from the copper to the zinc. No current as such passes through

the electrolyte except in a sense already explained for an electrolytic cell. Ohm's law and the heat effects are, however, the same in electrolytes as in ordinary conductors.

The style of Daniell cell shown in Fig. 53 is called a gravity cell. Its voltage is about 1.09 and it maintains a very constant current. When not in use the circuit should be closed and enough current allowed to flow to prevent a diffusion of $CuSO_4$ up to the zinc. Otherwise a muddy precipitate will form on the zinc and interfere with the action of the cell.

In another form of the Daniell cell the electrolytes are separated by an unglazed earthenware cup. The zinc may be placed in the cup in a solution of H_2SO_4 or $ZnSO_4$. The outer vessel is filled with a solution of $CuSO_4$ in which is placed the copper electrode.

67. Origin of E.M.F. in an Electric Cell.—According to a theory developed by Nernst a metal in a solution exerts a certain pressure called solution pressure as a result of which it tends to go into solution. On the other hand the solution also exerts a pressure called its osmotic pressure as a result of which it tends to deposit the material of its ions on the metal, *i.e.*, to go out of solution. When these two pressures are in equilibrium there will be no action of either kind. When an atom of metal goes into solution it is positively charged, and the metal which it leaves will therefore be negative. When a metallic ion goes out of solution it receives a negative charge from the plate to which it attaches itself. When electrodes and electrolytes are such that both operations are possible, a conducting wire will convey the excess of electrons from the electrode which possesses large solution pressure to another electrode to which ions are passing from a solution. This constitutes an electric current as produced by a primary cell.

These changes may be illustrated by reference to the cell shown in Fig. 53, where a zinc plate is immersed in a solution of $ZnSO_4$ and the copper plate in a solution of $CuSO_4$. The solution pressure of the zinc plate is greater than that of the copper plate. The zinc plate begins to send zinc ions into the $ZnSO_4$ solution and copper begins to go out of the $CuSO_4$ solution to the copper plate, but, while the circuit is broken at *s*, an inappreciably small change of this kind will make both the $ZnSO_4$ solution and the copper plate positive, thus setting up a positive electrostatic

field which opposes any further change. As soon as the circuit is closed at *s*, electrons will pass on the wire from zinc to copper, thus diminishing the electrostatic stress. Copper ions will then pass out of solution to the copper plate, zinc ions will go over to the SO_4 radicals of the copper sulphate solution, and the zinc plate will then be free to send more positive ions into solutions. Copper ions go out of solution more readily than zinc ions, so that copper only is deposited on the cathode as long as there is copper in solution. In this cell crystals of $CuSO_4$ are kept in the $CuSO_4$ solution, and these will be dissolved as they are needed.

According to the above explanation of a Daniell cell if the $ZnSO_4$ solution is made more dilute or the $CuSO_4$ solution more concentrated, or both, the E.M.F. of the cell should increase, for the osmotic pressure of the zinc solution is thus decreased while that of the copper solution is increased. Experiment shows this to be a fact. There is also, as might be expected, a contact difference of potential at the interface between the two electrolytes, though this is ordinarily small. In fact, a primary cell may have electrodes of the same metal each immersed in a separate solution, the solutions being in contact, or both metals and electrolytes may be the same, but the latter of different concentration.

In all primary cells the general principles of operation are the same as described above. In the simple voltaic cell zinc goes into solution and hydrogen ions go out. In the Daniell cell, when H_2SO_4 is used in place of $ZnSO_4$, zinc ions go into solution and displace $\overset{++}{H_2}$ of the H_2SO_4 solution. The $\overset{++}{H_2}$ and $\overset{++}{Cu}$ are then free to go to the cathode but, since $\overset{++}{Cu}$ goes out more readily, the $\overset{++}{H_2}$ will remain to balance the $\overset{--}{SO_4}$ radical from which the copper has been separated. Hence the collection of hydrogen on the cathode and the consequent polarization which occurs in the voltaic cell is avoided in the Daniell cell.

It has been assumed in the descriptions given above that the zinc is pure or has been amalgamated with mercury. Then no zinc will go into solution unless an equivalent quantity of some other substance carrying a positive charge of electricity can go out. This condition is supplied when a current can flow through a conductor from anode to cathode. If, however, the zinc contains

small particles of carbon or iron and is not amalgamated, a *local action* is set up between these foreign substances and the zinc, whereby zinc may enter an H_2SO_4 solution forming $ZnSO_4$ and setting hydrogen free without any flow of current over the conducting wire of the cell.

68. Energy Relations.—In an electric cell the quantity of energy which is set free by the substance which goes into solution is always greater than that absorbed by the substance which goes out of solution, hence the operation of the cell results in a decrease of potential energy, *i.e.*, in a decrease of the total amount of available energy. This is in accord with the operation of any isolated system in nature.

In a cell where the total decrease of potential energy appears in the electric current and where the quantity of heat, if the energy had appeared as heat instead of an electric current, is known, it is possible to form an equation from which the E.M.F. of a cell may be calculated. The difference between the heat of formation of $ZnSO_4$ and that of $CuSO_4$ for one chemical equivalent of each metal is 25,065 calories. This, expressed in heat units, is the quantity of energy which may be used in producing electricity in a Daniell cell when that quantity of zinc goes into solution and an equivalent quantity of copper goes out. The quantity of electricity involved in this change is, as shown in the preceding chapter, 96,525 coulombs. This quantity times the electromotive force, E, gives the quantity of work in watt-seconds. One watt-second is one coulomb per second under a pressure of one volt. (See § 54.) Hence

$$1 \text{ watt-second} = \frac{(10)^7}{4.187(10)^7} = .2388 \text{ calorie}$$

$$\therefore\ 96525\ E \times .2388 = 25065$$

$$\text{and} \qquad E = 1.087 \text{ volts}$$

This method of calculating the voltage of a cell can be used only when the temperature coefficient is very small, *i.e.*, when the change in E.M.F. due to a change in temperature is a negligible quantity.

69. Other Primary Cells.—Numerous efforts have been made to construct voltaic cells which would not polarize. Polarization

is due to a change in the surface of the cathode resulting from a deposit of hydrogen upon it. Anything that will prevent this is a depolarizer.

In the *Grove cell* the electrodes are platinum and zinc. The platinum is immersed in strong nitric acid, HNO_3, in a porous cup. The cup in turn is immersed in a solution of H_2SO_4 in which the zinc is placed. Zinc goes into solution and hydrogen ions move toward the platinum electrode where it combines with HNO_3, forming water and nitrogen dioxide according to the chemical equation

$$3H_2 + 2HNO_3 = 4H_2O + N_2O_2$$

in which change electrons are taken from the platinum electrode and N_2O_2 escapes into the air where it combines with another molecule of oxygen, forming the noxious red peroxide or nitrogen tetroxide. The advantage of this cell is high E.M.F., about 1.9 volts, and freedom from polarization.

The *Bunsen cell* is the same as Grove's except that a plate of carbon is used in place of platinum.

The *Bichromate cell* is the same as Bunsen's except that a solution of bichromate of potash is used in place of HNO_3 and the porous cup is not used. This is, then, a single-fluid cell with a solution of H_2SO_4 and $K_2Cr_2O_7$. When the former unites with the latter, chromium trioxide, CrO_3, is formed, and this is a strong oxidizing agent which takes up hydrogen ions and prevents polarization. The E.M.F. of this cell is 2 volts or more.

The *Leclanche cell* is made up of zinc and carbon as electrodes, the latter being surrounded by manganese dioxide as a depolarizer. The electrolyte is a solution of ammonium chloride (sal ammoniac). This cell, in some form, is in common use for open circuit work, but it soon polarizes on closed circuit. Its voltage is about 1.5. The advantage of its use is that it needs no attention for a long time and will recuperate during a period of rest. One very convenient form of this cell is the so-called *dry cell* in which the electrolyte is a moist paste composed of ammonium chloride, zinc chloride, zinc oxide, and plaster of Paris. The composition of the paste is varied in different makes of this cell.

70. Standard Cells.—It would be an advantage to have a cell which under specified conditions would always give the same

electromotive force. After its E.M.F. is once determined with accuracy it is possible, by comparison, to determine the E.M.F. of any other cell or the P.D. between two given points on a conductor. One cell that has been selected for this purpose is the Clark cell. As shown in Fig. 54 it consists of an H-shaped glass vessel in which the electrodes are pure mercury on one side, *m*, and an amalgam containing about 15 per cent. of zinc on the other, *a*. On the mercury is placed a paste, *p*, of mercurous sulphate. The cell is then filled with a saturated solution of zinc sulphate, *s*, and is kept saturated by crystals of $ZnSO_4$ which are added. Connection is made with the electrodes by platinum wires which are sealed into the glass. The E.M.F. is 1.4325 volts at 15° C., but there is considerable change in this value when the temperature changes. For any temperature *t*,

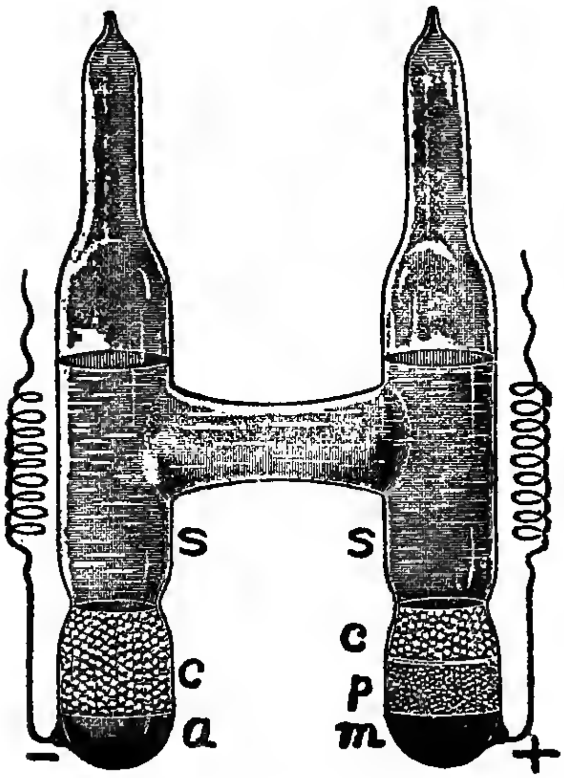

Fig. 54.

Fig. 55.

$$\text{E.M.F.} = 1.4325 - .00119(t-15) - .000007(t-15)^2$$

Another standard which is often preferred because of its small variation with change of temperature is the Weston standard cell. In it cadmium amalgam and cadmium sulphate are substituted for zinc amalgam and zinc sulphate of the Clark cell. In other respects they are alike. This cell is also made in the form shown in Fig. 54 where *m* is pure mercury, *a* is an amalgam of cadmium and mercury, *p* is a paste of mercurous sulphate, *c* is cadmium sulphate in the form of crystals, and *s* is a saturated

solution of cadmium sulphate. A cell of this kind constructed and used in accordance with standard directions will remain constant for a number of years.

The international E.M.F. of the Weston cell is 1.0183 volts at 20° C., and, for any other temperature, t,

$$\text{E.M.F.} = 1.0183 - .0000406(t-20) - .00000095(t-20)^2$$

Standard cells such as the Clark and Weston can be used only in what is called the zero method, *i.e.*, by balancing its E.M.F. against another which is equal and opposite to it. This is explained in the next chapter. If a current is allowed to flow, the cell will soon polarize and become useless.

71. Storage Cells.—A *storage cell*, also called a *secondary cell* or *accumulator*, is one in which the passage of a current from an outside source will effect such changes that the cell will afterwards reverse its action and give out a current. The cell shown in Fig. 52 may be considered as a storage cell, for part of the energy expended was returned. This, however, would be a very poor and inefficient kind of storage cell. The Daniell cell may in a sense also be considered a storage cell, for if a current is passed through it in a direction opposite to that which it would give out, copper goes into solution and zinc is deposited on the zinc plate. Thus potential energy is stored up and will be returned whenever the cell operates in the ordinary manner. Such a cell is said to be reversible.

A common form of storage cell consists of lead grids filled with a paste of lead sulphate. These are the electrodes. The electrolyte is a solution of sulphuric acid. When a difference of potential is maintained at the electrodes, *i.e.*, when, as we say, a current is passed through this cell, hydrogen ions from the H_2SO_4 solution go to the cathode where they act on the $PbSO_4$ according to the equation

$$H_2 + PbSO_4 = H_2SO_4 + Pb$$

Thus this plate, the negative one, is reduced to metallic lead in a very spongy condition. This plate may be recognized by its gray color. On the other hand, SO_4 ions go to the anode and convert it into peroxide of lead, PbO_2, according to the reaction

$$SO_4 + PbSO_4 + 2H_2O = 2H_2SO_4 + PbO_2$$

The PbO_2 remains in the grid and forms the positive plate which

may be recognized by its dark red color. In both reactions H_2SO_4 is added to the solution. In the uncharged cell the density of the solution should be about 1.17 g. per cubic centimetre, but in the process of charging this will increase to 1.21 g. per cubic centimetre. When fully charged, any further current will cause hydrogen to be given off at the cathode and oxygen at the anode just as described for the electrolysis of water.

During discharge the reactions described above take place in a reverse direction and the electrodes are changed again to lead sulphate. The E.M.F. of this cell is very constant. On full charge the voltage rises to 2.5 volts but falls at once to 2.2 volts and during discharge remains steadily at 2 volts until nearly exhausted. The voltage then falls rapidly but should not be allowed to go below 1.8 volts or exceed the normal discharge rate, for an over-discharge will cause the plates to buckle.

In the recent Edison storage cell the electrodes are nickel hydrate, positive, and iron oxide, negative. The electrolyte is a 21 per cent. solution of caustic potash and the container is made of nickel-plated sheet steel. The chemical changes which take place are complex and not yet well understood. The E.M.F. of this cell is 1.2 and its efficiency about 60 per cent., but it can be stored with about twice as much energy as the same weight of lead cell. The efficiency of a new lead battery is about 80 per cent., *i.e.*, it will return 80 per cent. of the energy put into it.

72. Arrangement of Cells.—When a current flows from an electric cell, the electrolyte itself is regarded as forming part of the circuit, and its resistance, called internal resistance, may be represented by r. The remainder of the circuit is called external resistance and may be represented by R. Then by Ohm's law the strength of current, i, is

$$i=\frac{E}{r+R} \qquad (59)$$

where E is the E.M.F.

Two or more cells connected together constitute a battery. A battery may be made up of cells joined in series—*i.e.*, with the negative electrode of the first connected to the positive of the second, negative of the second to the positive of the third, etc., the negative of the last being connected through the external circuit to the positive of the first.

Cells may also be joined in parallel, *i.e.*, one terminal of the external circuit is joined to all the positive electrodes and the other to all the negative electrodes.

Another arrangement is the series-parallel which is a combination of the two above. Here there are several groups of cells, those in each group being joined in series and the groups then joined in parallel. (Fig. 56.)

When cells are in series, the E.M.F. is multiplied as many times as there are cells and the same is true of the internal resistance, but when in parallel the internal resistance is divided as

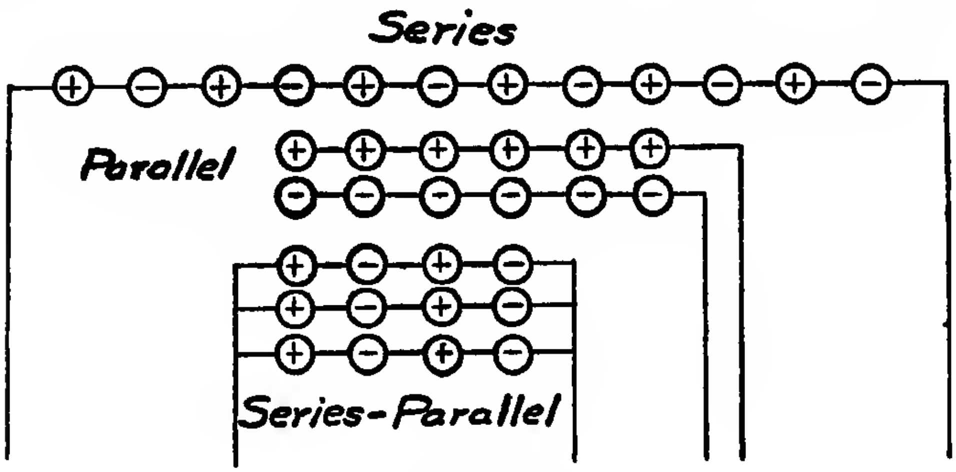

FIG. 56.

many times as there are cells, the E.M.F. remaining the same as for a single cell. A general equation for any possible arrangement would then be

$$i=\frac{pE}{\frac{pr}{\frac{n}{p}}+R}=\frac{pE}{\frac{p^2r}{n}+R} \qquad (60)$$

where p is the number of cells in series in any group, n the total number of cells, and n/p the number of groups.

If $p=n$, then

$$i=\frac{nE}{nr+R} \qquad (61)$$

i.e., there is but one group with all the cells in series.

If $p=1$, then

$$i=\frac{E}{\frac{r}{n}+R} \qquad (62)$$

i.e., all the cells are arranged in parallel.

For a maximum value of i the best arrangement is one where the internal resistance of the entire battery is, as nearly as possible, equal to the resistance of the external circuit. This will appear from a consideration of equation (60) which may be written

$$i = \frac{E}{\frac{pr}{n} + \frac{R}{p}} \tag{63}$$

If the terms of the denominator of the second member of this equation are multiplied together we have

$$\frac{rR}{n}$$

which is a constant quantity, for none of these quantities, r, R, or n, change, whatever the arrangement of the battery may be. But whenever the product of two variables is a constant, their sum is least when they are equal. Consequently i, the strength of current, will be maximum when

$$\frac{pr}{n} = \frac{R}{p}$$

or

$$\frac{p^2r}{n} = R \tag{64}$$

The first term in (64) is the internal resistance of the battery and the second is the external resistance. Hence when p is made such that these terms in equation (60) are equal, or as nearly so as possible, i will be maximum. For example, if $R=3$ ohms and $r=2$ ohms and we have 24 cells, the best arrangement to secure the greatest strength of current is in 4 groups with 6 cells in series in each group, for then

$$\frac{p^2r}{n} = \frac{36 \times 2}{24} = 3 = R$$

Problems

1. What arrangement of 20 cells will give maximum current when for each cell the E.M.F. $=1.09$ volts and internal resistance $= 2$ ohms, the external resistance being 2.5 ohms?

2. The 50 cells of a battery are joined 10 in series, with 5 groups in parallel. The resistance of the entire battery is 4 ohms. What is the resistance of each cell?

3. If the resistance of a lamp is 440 ohms and it requires .25 ampere to operate it, how many storage cells, 2 volts each, will be required, assuming that the internal resistance of the cells may be neglected?

4. The E.M.F. of a cell is known to be 2 volts. A current of .475 ampere is observed to flow through an external circuit of 4 ohms. What is the internal resistance?

5. Six lead storage cells, the E.M.F. of each being 2 volts and internal resistance of each .15 volt, are joined in series. The external circuit contains 3 electrolytic cells in series, each offering a counter electromotive force of 1.5 volts and having a resistance of 2.15 ohms. The resistance of the connecting wires is 1.1 ohms. What strength of current will flow?

Ans. 1. 5 cells in series.
2. 2 ohms.
3. 55 cells.
4. .21 ohm.
5. .887 ampere.

CHAPTER VI

GALVANOMETERS

73. The Tangent Galvanometer.—A tangent galvanometer is one where the strength of an electric current flowing through a coil is proportional to the tangent of the angle of deflection of a magnetic needle at the centre of the coil. This instrument is based on the definition of the *e.m.* unit of current as described in § 47. It is there shown that the strength, F, of the field at the centre of a coil of n turns is

$$F = \frac{2\pi ni}{r}$$

where r is the mean radius of the coil and i is the strength of current in *e.m.* units. If such a coil be set up with its plane parallel to the earth's field, the field of the coil will be at right angles to that of the earth. Let the strength of the former be denoted by F and of the latter by H. Let bc, Fig. 57, be a magnetic needle of pole strength m, and length l placed at the centre of the coil. Then the moment of the couple tending to turn the needle to a position parallel to the field of the coil is Fm times ad. But ad is equal to bc, or l, times the cosine of θ. Thence the moment of the couple is

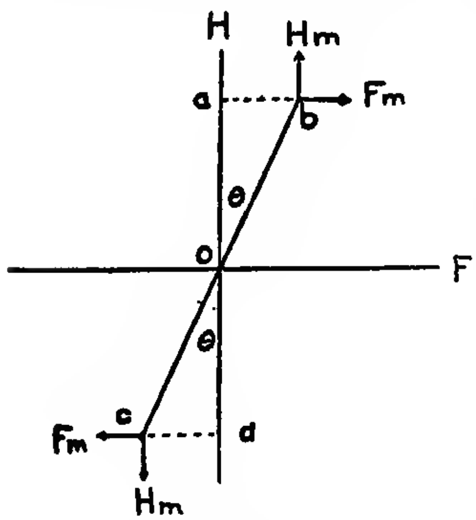

FIG. 57.

$$\frac{2\pi ni}{r} ml \cos\theta$$

There is at the same time a moment tending to turn the needle to a position parallel to the earth's field, and the moment of this couple is $Hm(ab+cd)$. But $ab+cd$ is equal to $l \sin\theta$. Hence the moment is

$$Hml \sin\theta$$

Since the needle is in equilibrium, the two moments are equal and we have the equation

$$Hml \sin \theta = \frac{2\pi ni}{r} ml \cos \theta \tag{65}$$

$$\text{or} \quad H \tan \theta = \frac{2\pi ni}{r}$$

$$\text{or} \quad i = \frac{r}{2\pi n} H \tan \theta \tag{66}$$

The strength, i, of a current may by this method be found in *e.m.* units at any place where H is known. Since i varies as the tangent of the angle of deflection, the instrument is called a tangent galvanometer.

The quantity $\frac{2\pi n}{r}$ contains only terms whose value is fixed in the construction of the instrument and so is called the galvanometer constant, usually denoted by G. Hence (66) may be written

$$i = \frac{H}{G} \tan \theta \tag{67}$$

Since an ampere is one-tenth of an *e.m.* unit, there would be ten times as many amperes in any given strength of current. Hence (67) may be written

$$i = 10 \frac{H}{G} \tan \theta \text{ amperes} \tag{68}$$

It is assumed that the field in which the needle is placed is uniform, and to secure this condition as nearly as possible the length of the needle, Fig. 58, should not be greater than about one-twentieth the diameter of the coil.

The tangent galvanometer is a fundamental instrument, for it is built up on the definition of the *e.m.* unit of current. It was by its use that the definition of the ampere given in § 53 was determined, for by connecting an electrolytic cell in series with the galvanometer, the relation between strength of current and the quantity of metal deposited on the cathode could be found. This method, in turn, may then be used to find H, the value of i being calculated from the deposit of silver, copper, or other substance from the electrolyte.

74. The Astatic Galvanometer.—The coil of a tangent galvanometer must have a large diameter in comparison with length of the needle, consequently the field at its centre is not of great intensity and the needle does not respond to very small changes in current. A very sensitive instrument of this kind was devised by Lord Kelvin and is known as the Kelvin galvanometer. The coils contain many turns of fine wire and are close to the needles

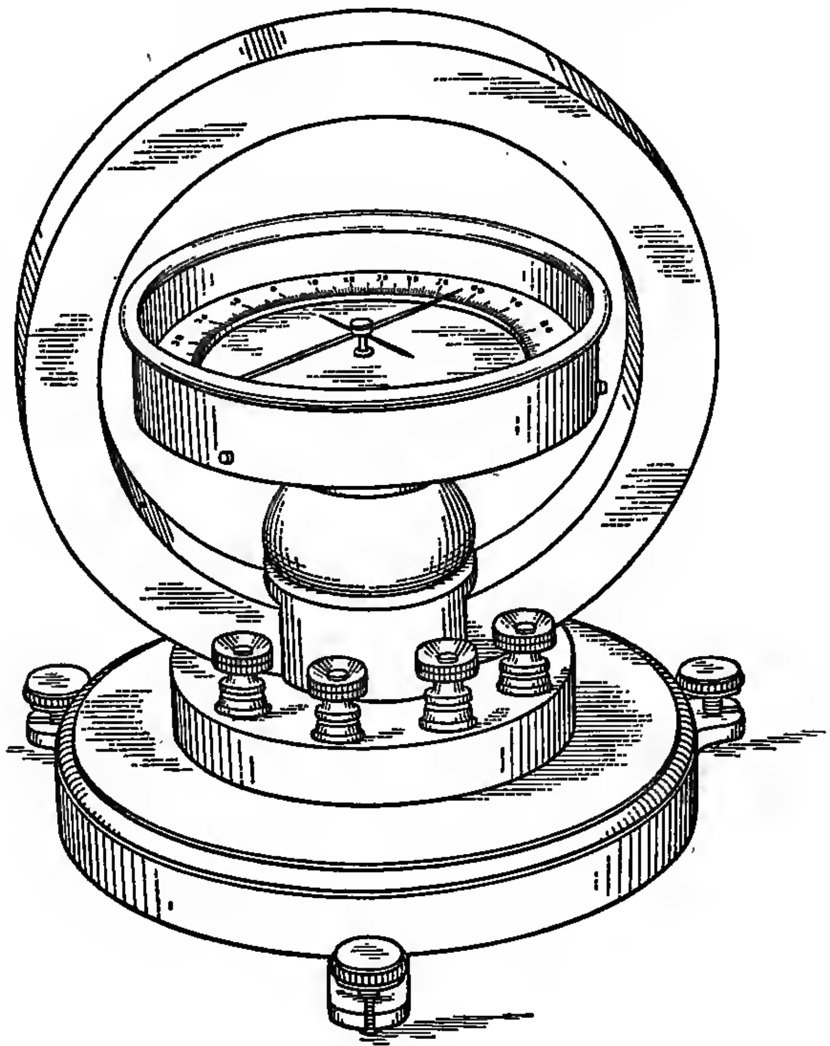

Fig. 58.

at their centres as shown in Fig. 59. The magnetic needles are attached to a very light frame suspended by a delicate fibre. The poles of the needles in one coil point in a direction opposite to those in the other. This is an arrangement known as an *astatic* system, for if the magnetic moments are exactly equal, the tendency of the earth's field will be to turn the upper needles in one direction and the lower in the other. As a result they will not

take a stand in the earth's field, *i.e.*, they will be astatic. A control magnet may be placed above or below the instrument to bring the needles back to a zero position after they have been disturbed. The needle is therefore free to respond to very weak fields produced by the coils, and since the wire in the two coils is wound in opposite directions, the turning moment will be in the same direction for both sets of needles. A small mirror attached to the frame makes it possible, by means of a telescope and scale, to read the deflections of the magnets. The frame is usually suspended by a fine fibre of quartz, and the whole suspended system weighs but a small fraction of a gram. This instrument is valuable for the detection of small currents and the direction in which they flow, but it is not a tangent galvanometer, for the needles do not turn through a uniform field. It may, however, be used to measure strength of current if the relation of current to deflection is determined by previous experiment.

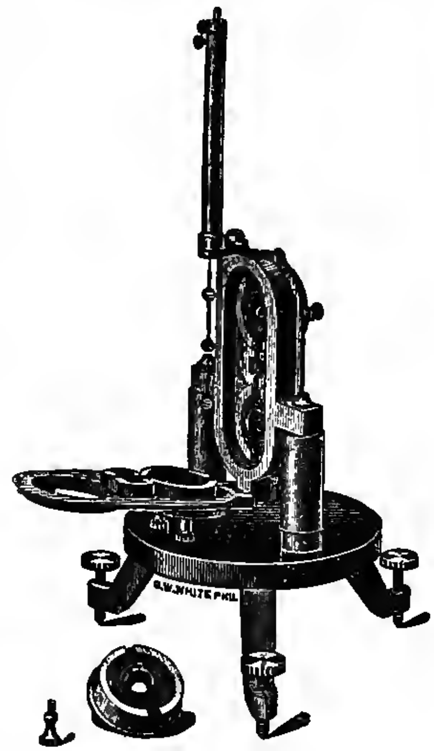

Fig. 59.

75. The D'Arsonval Galvanometer.—The galvanometers described in the two preceding sections are of the moving magnet type, and it is evident they would be easily disturbed by any external magnetic field or by any mass of iron near by. Another type of instrument is the moving coil galvanometer in which the magnets are stationary and the coil turns. These may be made just as delicate as the others and are practically unaffected by outside influences. A common form of such an instrument is known as the D'Arsonval galvanometer. This consists of a strong permanent magnet formed so that its poles are near each other and in this gap is suspended a coil of wire. When a current passes from *e* to *f*, Fig. 60, the coil tends to turn to a position such that its plane will be perpendicular to the field of the magnets and its magnetic field within the coil will be parallel and in the same direction as the field of the magnets.

The same principle applies to the turning of the coil in a galvanometer as we shall see later in the explanation of an electric motor. In Fig. 61 let *ba* and *dc* be conductors carrying electricity

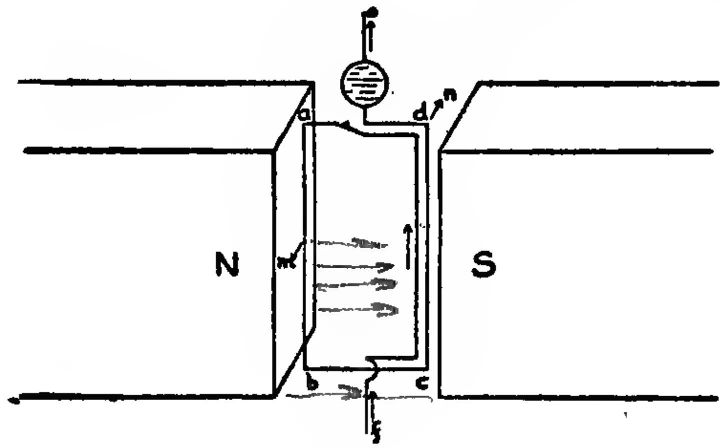

FIG. 60.

through the magnetic field between *N* and *S*. The motor rule is as follows: *Extend the thumb, the first finger, and the second finger of the left hand so that each will be at right angles to the other two, then if the first finger points in the direction of the lines of force, and the second finger in the direction the positive current flows, the thumb will indicate the direction the conductor will move across the magnetic field.*

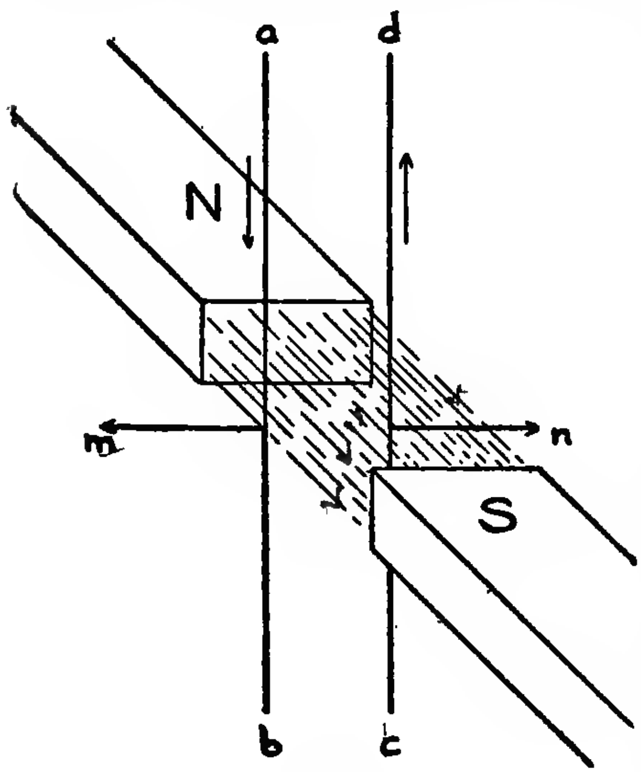

FIG. 61.

Applying this rule to Fig. 61, we find that the conductor *ba* when carrying a current will move across the field in the direction *m* while *cd* will move in the direction *n*. This is just the condition shown in Fig. 60, and when a current flows through the coil in the direction there indicated, the coil will turn in the direction shown by *m* and *n*.

In the definition of the *e.m.* unit of current (§ 46), the current, under conditions there given, will act on unit pole with a force of

1 dyne. But by the law of action and reaction we can as well say that the *e.m.* unit of current is of such strength that each centimetre of its length, when at right angles to the direction of a unit magnetic field in which it is placed, is acted on by a force of 1 dyne. Consequently if the strength of the field, Fig. 60, is H, and the length of the side ab is l, then when a current i flows, the force urging one strand of ab in the direction m is

$$f = ilH \tag{69}$$

An equal force is acting at the other side of the coil in the direction n. The moment of this couple is

$$fh = ihlH \tag{70}$$

where h is the distance between the two sides. The top and bottom of the coil are parallel to the field of the magnets and so do not affect the turning moment. If the coil is turned so that all its sides are at right angles to the field the turning moment would be zero and the only effect of the field on the current in the coil would be a force tending to push out or draw in all four sides.

If there are n turns of wire in the coil, (70) would become

$$Fh = ihlnH \tag{71}$$

Now, hl is the area enclosed by one turn of the coil, and hln is the sum of all such areas. If this is denoted by A,

$$Fh = iAH \tag{72}$$

This equation gives the turning moment no matter what the shape of the coil may be.

When the coil begins to turn it is resisted by the torsion of the suspension fibre and so will come to rest in a position of equilibrium between these two forces. The restoring force exerted by the twisted fibre is its moment of torsion times the angle through which it is twisted. It has been shown (see " Mechanics and Heat," p. 117) that moment of torsion is the ratio of moment of force, Fh, to the angular displacement, θ, which is produced, *i.e.*, it is the moment of force which will cause an angular displacement of 1 radian. The moment of force for θ radians would then be

$$\frac{Fh}{\theta}\theta$$

When the coil has turned through θ radians, the component of its area parallel to the magnetic field is $A \cos \theta$, hence

$$iAH \cos \theta = \frac{Fh}{\theta} \theta$$

or

$$\frac{i}{\theta} = \frac{\frac{Fh}{\theta}}{AH} \cdot \frac{1}{\cos \theta} \tag{73}$$

If θ is very small—*i.e.*, if the coil is made to turn only slightly from its position in Fig. 60—$\cos \theta$ may be taken as unity without appreciable error. Then (73) becomes

$$\frac{i}{\theta} = \frac{\frac{Fh}{\theta}}{AH} \tag{74}$$

Under these conditions the second member of (74) is called the constant of the galvanometer, for all its terms are constant. Hence, if the ratio of i to θ is once determined, the strength of another current is the angular displacement produced times this constant.

Attached at the top of the coil is a mirror, m, so that by use of a telescope and scale set at a known distance, the angle through which the coil turns when a known current flows through it is readily found. Then, knowing i and θ, the ratio of the former to the latter is the constant, k, which is sought.

One convenience in the use of this instrument is that it is practically dead-beat, *i.e.*, the coil promptly turns to its position of equilibrium, or returns to zero, without frequent oscillations before it comes to rest. This is the result of electromagnetic damping which occurs whenever a closed conducting circuit, such as the copper frame seen in front of the coil, Fig. 62, is moved across magnetic lines of force.

The suspension wires of this instrument are usually steel wires or phosphor-bronze ribbons whose moment of torsion is very small. Within the coil is a soft iron core supported from the frame. The core serves to concentrate the lines of force and make a more intense field in the region of the coil.

The sensitiveness of a galvanometer is often designated by what is called the *figure of merit*. This is the strength of current in amperes which will cause a deflection of 1 mm. on a scale 1 m.

distant from the coil. If m, Fig. 63, is the mirror attached to coil and sr a scale graduated in millimetres and 1 m. from m, then a current which will turn m so that a beam of light from f, 1 mm. from o, will on reflection from m return to a telescope at o, is the figure of merit. This is $\frac{1}{2000}$ of the constant, k, as defined above, for the angle turned through by a reflected beam is twice that through which the mirror turns. Hence, when the mirror turns through one radian, the reflected beam will turn two radians, say or. But mo is 1000 mm., hence or is 2000 mm., or 2000 times of.

The term *sensitiveness* also often means the number of megohms —*i.e.* 10^6 ohms—which must be connected in series with the galvanometer to reduce the deflection to 1 mm. on a scale 1 m. distant when the electric pressure is 1 volt. This is the reciprocal of the figure of merit, for by Ohm's law (equation 52), since $E=1$

FIG. 62.

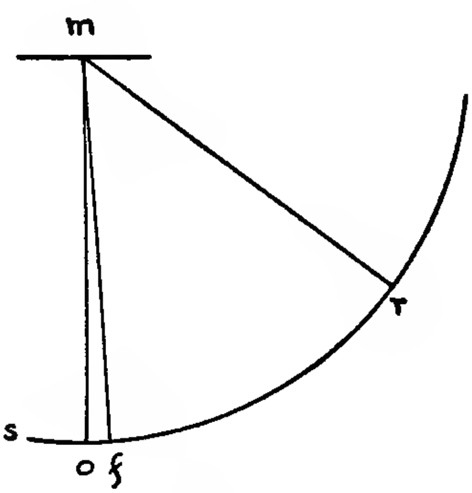

FIG. 63.

and i is the same as for figure of merit, then $R=1/i$. For example, suppose k for a certain galvanometer is $9(10)^{-6}$. Then the figure of merit is $4.5(10)^{-9}$, and the sensibility is 220 megohms.

76. The String Galvanometer.—An instrument of extreme sensitiveness has been invented by Einthoven and is known as the string galvanometer. The fundamental principle involved is the same as that described above for the D'Arsonval, but in this case a single conducting fibre is stretched across the field as shown at ab, Fig. 64. The magnets are pierced so that a bright light may be focused on the fibre by the lenses in the tube through N, and this in turn is focused on a screen by the projection micro-

scope through S. The fibre ab will move across the magnetic field in response to the slightest current which passes through it, and this movement is magnified on the screen. The "string" is made of platinum or quartz coated with silver and is so fine that it can be seen by the naked eye only when brilliantly lighted on a black background, the diameter being only 2 or 3μ. A current as small as about $(10)^{-12}$ amperes may be detected. One use of such an instrument, as described by L. F. Baker of Johns Hopkins University, is in making *electrocardiagrams*, *i.e.*, photographic records of the beating of the human heart. The point where a

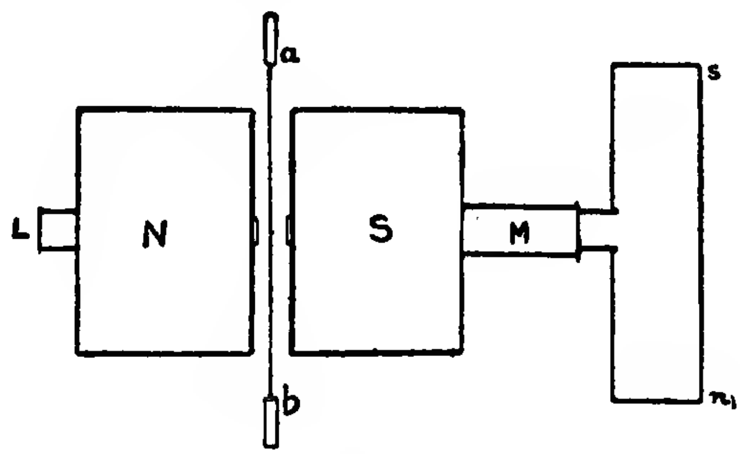

FIG. 64.

muscle is stimulated to activity is found to be negatively electrified, and hence a current will flow on a conductor connecting that point with another point of the body at the moment of excitation. It is sufficient to have the patient grasp the electrodes leading to a and b and then, by moving a photographic film before the eye-piece of the microscope, a record of the action of the heart is recorded.

77. The Ballistic Galvanometer.—In the discharge of a condenser or other charged body it is often desirable to know the quantity of electricity, Q, which passes over a conductor during the very short time required for the discharge. This may be done by use of a galvanometer, preferably the D'Arsonval, in which the coil is made to have a long period and damping is reduced to a minimum. Under these conditions the current has practically all passed before the coil begins to swing. A momentary impulse is given and the swing, or throw, follows. Hence the use of the term *ballistic*. Any delicate galvanometer which is not "dead

beat" may be used in this manner. If, for example, the copper frame in Fig. 62 be removed, the instrument becomes a ballistic galvanometer. After the throw is observed, the coil may be readily brought to rest by pressing a key which closes its circuit and thus, for the time, introduces electromagnetic damping.

The relation between quantity, Q, and the throw n may be approximately expressed by

$$Q = kn \tag{75}$$

where k is a constant which may be determined by passing a known quantity of electricity through the coil and observing the throw n as measured in scale divisions.

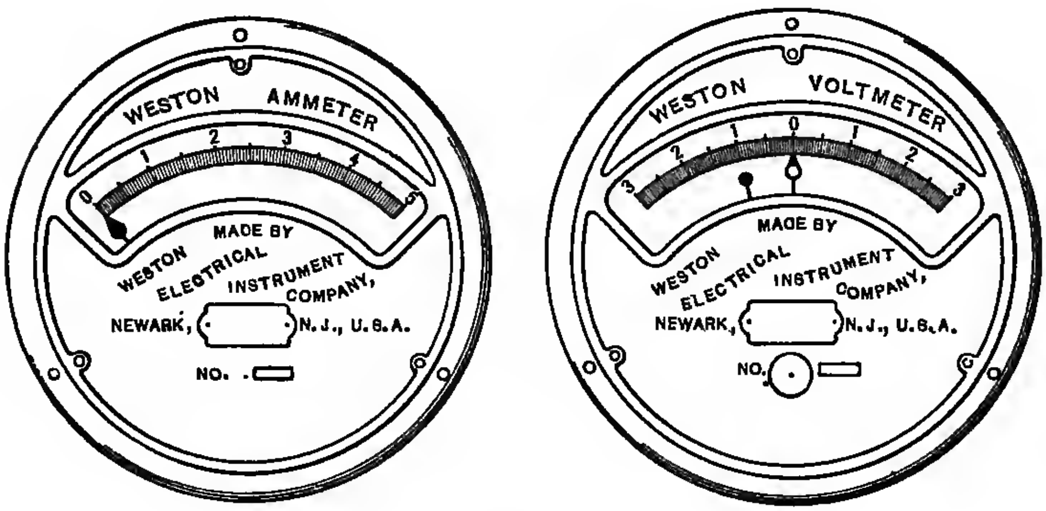

FIG. 65.

78. Ammeters and Voltmeters.—For commercial purposes and also for use in the laboratory it is desirable to have instruments which are so constructed and calibrated that either voltage or amperage will be indicated by a pointer which moves over a graduated scale. The principle of these, for direct current, is the same as that of a D'Arsonval galvanometer, but the coil, instead of being suspended, is pivoted on jeweled bearings and is returned to the zero point by coiled springs which also serve to conduct the current to or from the movable coil. A long pointer attached to the coil moves over the scale (Fig. 65). In the ammeter a low resistance conductor is in series with the main circuit, and only a small fraction of the current is shunted through the coil. In the voltmeter a large resistance is put in series with the coil

and the main circuit so that only a very small current flows. The P.D. is here desired and not strength of current. It would be better if no current whatever passed through the voltmeter, but in instruments of this kind it is always necessary to take off enough current between two points of different potential to operate the coil. The greater the P.D. the greater the current, and hence the greater the deflection of the needle.

For very high voltage, electrostatic voltmeters may be used. These do not require any current. The principle here involved is the same as that of the quadrant electrometer, Fig. 18. As

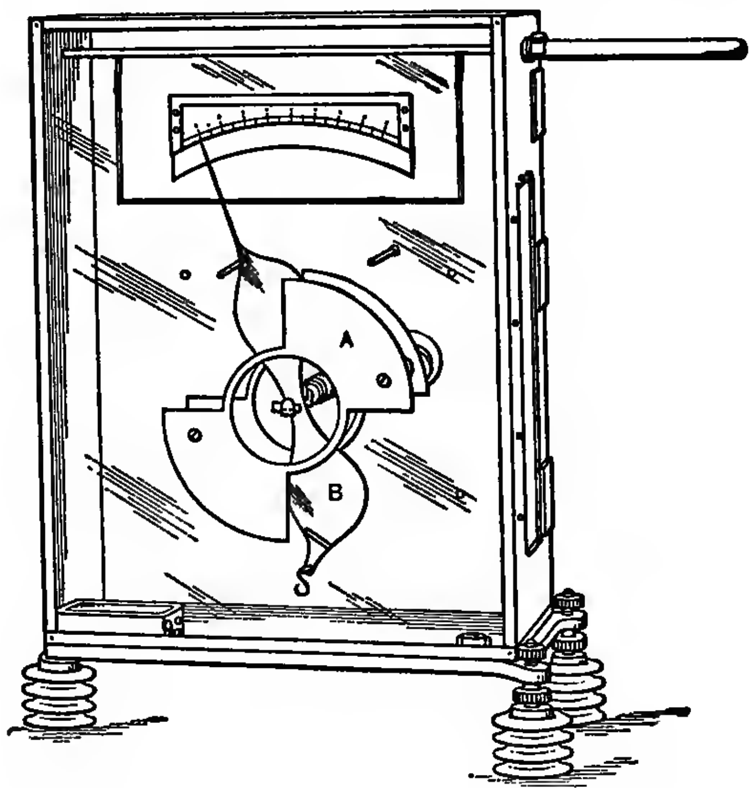

FIG. 66.

shown in Fig. 66, conductors leading from points of different potential are connected, one to *A* which is fixed, and the other to *B* which is movable and carries a pointer. *A* and *B* will thus be charged electrostatically, one positively and the other negatively, and so *B* will turn and move the pointer over a scale which may be graduated so as to read in volts.

79. The Electrodynamometer.—It has been shown (§ 25) that when a current flows through two adjacent coils, they will turn so as to make their magnetic fields coincide in position and direction as nearly as possible. This principle has been applied in the

construction of a dynamometer for measurement of the strength of a current. One form of this instrument is shown in Fig. 67. The central coil is fixed, while the other, consisting of a single heavy wire, carries a pointer which reaches up to a graduated circle at the top of the instrument. The coils are in series so that whatever current passes through one will also pass through the other. A spiral spring connects the movable coil with a torsion head. When a current flows there will be a force tending to make the coils parallel. The number of scale divisions through which the torsion head must be turned to keep the coils at right angles is a measure of the strength of current, but since the coils are in series an increase of current in one means an equal increase in the other, so that their mutual effect produces a force which varies as the square of the current. Hence, if θ is the number of scale divisions moved over by the pointer attached to the torsion head,

$$i = k\sqrt{\theta} \qquad (76)$$

where k is a constant which may be found by observing θ when a known current, i, flows through the instrument.

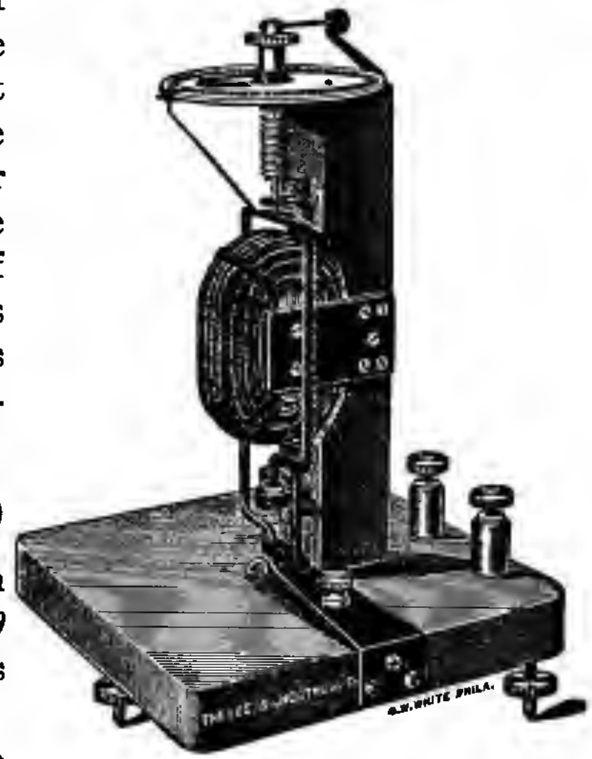

FIG. 67.

This instrument can be used to measure either direct or alternating currents, for when a current is reversed in one coil it is reversed in the other at the same time.

80. The Wattmeter.—The watt (§ 54) is the unit of electrical power. The power is 1 watt when 1 joule of work is done per second. The two factors whose product determines electrical power are strength of current and electrical pressure. The product of the number of volts by the number of amperes gives the number of watts. If, for example, it is desired to know the number of watts needed to operate an incandescent lamp, a voltmeter may be connected, as shown in Fig. 68, so as to show the P.D. at the

terminal of the lamp, and an ammeter in series with the main current will show the strength of current. The product of the reading of these two instruments gives the number of watts. If now an instrument is constructed on the same principle as the electrodynamometer, one coil, called the pressure coil, having many turns of fine wire—*i.e.*, having large resistance like a voltmeter—and the other, the current coil, having few turns of coarse wire; and if these coils are connected, not in series, but as in Fig. 68, the instrument becomes a *wattmeter*. Such coils may be ar-

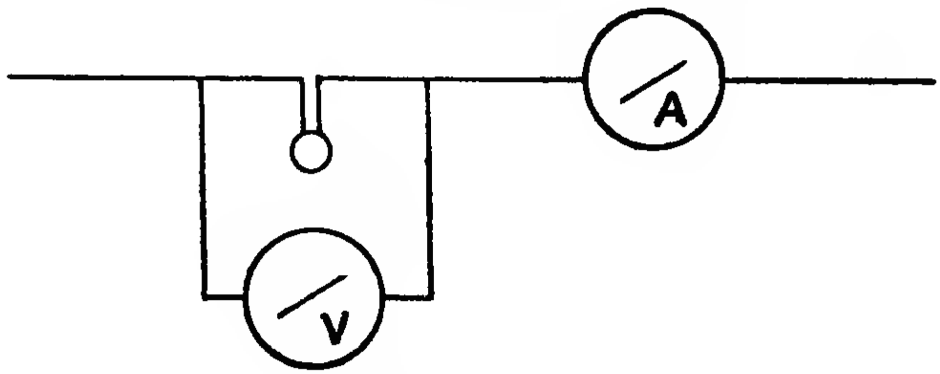

FIG. 68.

ranged as shown in Fig. 69. Then to determine the wattage of a lamp it is only necessary to connect the opposite sides of the lamp to the pressure coil while the current coil forms part of the main circuit.

Problems

1. The coil of a tangent galvanometer, 14 cm. in diameter, contains two turns of wire and is set parallel to the earth's field. What is the strength of current that will deflect the needle 22.4° at a point where the intensity of the earth's field is .2 gauss?

2. A current whose strength is $5(10)^{-7}$ amperes causes, in a D'Arsonval galvanometer, a deflection of 10 cm. on a scale 1 m. distant. What is the sensibility?

3. If the torsion head of an electro-dynamometer must be turned through 16 scale divisions to keep the coils at right angles when a current of 10 amperes flows, what is the strength of current for 49 scale divisions?

4. If 4 lamps are in parallel and each requires 40 watts for their operation, what is the strength of current when the pressure is 110 volts?

5. If 330 watts are required to heat an electric iron on a 110-volt circuit, what will be the cost at 10 cents per K.W. hour, and how many calories of heat will be produced in one hour?

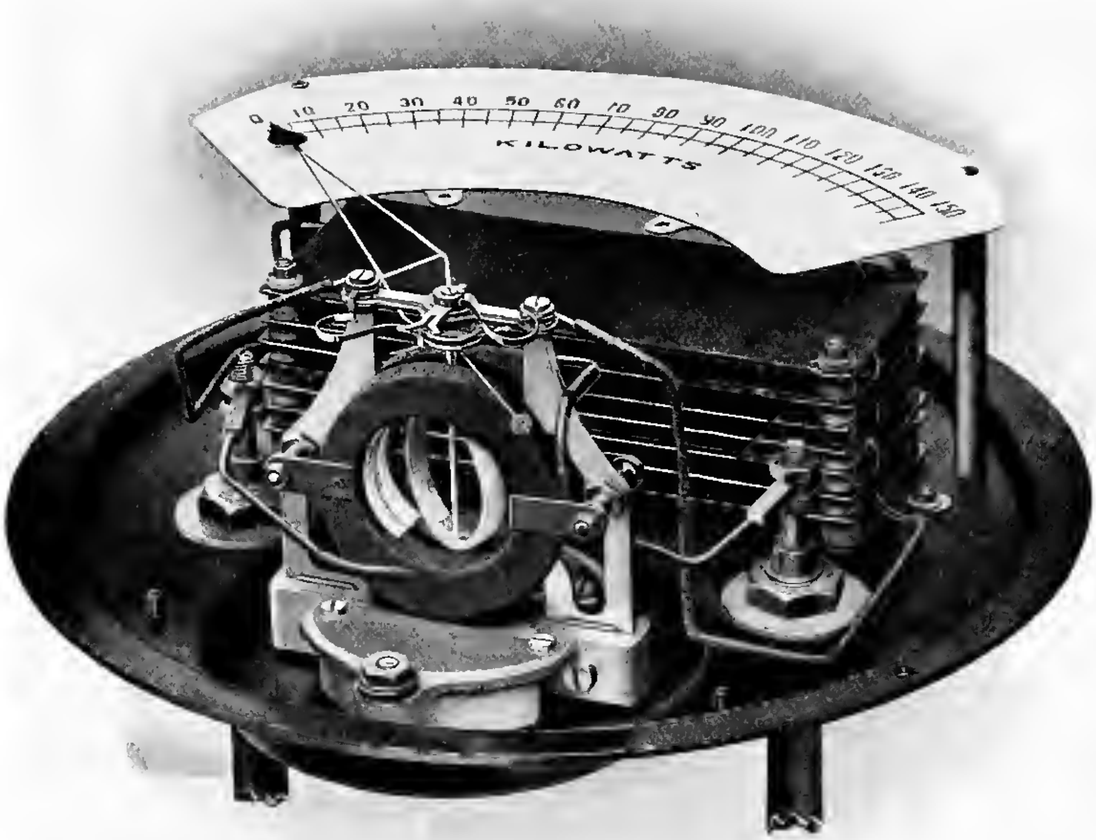

FIG. 69.

6. The coil of a tangent galvanometer is 46 cm. in diameter and contains 150 turns of wire. It is joined in series with a $CuSO_4$ voltameter. What is the strength of the earth's magnetic field if .0275 g. of copper is deposited on the cathode in 30 minutes, the needle of the galvanometer being deflected 45° during that time?

Ans. 1. .451 ampere.
2. 200 megohms.
3. 17.5 amperes.
4. 1.45 amperes.
5. 3.3 cents per hr.
283,735.3 calories.
6. .19 gauss.

CHAPTER VII

MEASUREMENT OF ELECTRICAL RESISTANCE

81. Relation of E, i, and R.—By Ohm's law, stated in § 52, the ratio of the electromotive force E to the strength of current i is a constant quantity R for a given conductor at a constant temperature. Hence the value of E between any two points on a conductor is, from equation (52)

$$E = iR \tag{77}$$

If R becomes large, E must also be large to produce a current of the same strength, and if E remains constant, i may be reduced to any desired strength by increasing R.

82. Specific Resistance.—It is found that the resistance of a conductor varies directly as the length, inversely as the area of cross section, and is different for different materials at the same temperature. This may be expressed by

$$R = k\frac{l}{A} \tag{78}$$

where k is the *specific resistance* or *resistivity* of any given material. If the length l is 1 cm. and the area of cross section A is 1 sq. cm., then $R = k$, *i.e.*, k is the resistance of a 1-cm. cube of the material. This value of k for a number of different materials is given in the appendix. When the value of k is once found, the resistance of any length and area of cross section may readily be calculated.

The resistance of a *mil-foot* is also frequently used as the specific resistance of a conductor. A mil-foot is a wire 1 foot long and one-thousandth of an inch in diameter. The area of cross section of this unit is called one *circular mil*. Then, since the areas of circles are to each other as the squares of their diameters, the square of the diameter of any wire will give the area of cross section in circular mils provided the diameter is taken in *thousandths* of an inch. This is the method used in commercial and construction work, and equation (78) is then written

$$R = k\frac{l}{\text{C.M.}}$$

where l is the length of a wire in feet, C.M. is the area of cross section in circular mills, and k is the resistance of a mil-foot in ohms.

83. Temperature Coefficient of Resistance.—All pure metals and most alloys increase in resistance as the temperature rises. The change in resistance per degree rise in temperature per ohm of the resistance at 0° C. is called the *temperature coefficient of resistance*. If this is expressed by k, then

$$k=\frac{R_t-R_0}{R_0t} \tag{79}$$

and when the value of k is once found for any substance, the resistance, R_t, for any given temperature is found by

$$R_t=R_0(1+kt) \tag{80}$$

The value of k for all pure metals is about .004 ohm. (See appendix.) The value of k for platinum is very constant through a wide range of temperature and so this, with other properties, makes platinum a proper metal for use in resistance thermometers. (See p. 199 of " Mechanics and Heat.")

For alloys this coefficient is much lower than for pure metals, in some cases even zero or negative. For example, manganin is an alloy usually composed of 12 parts nickel, 84 parts copper, and 4 parts manganese. Its temperature coefficient is practically negligible, being about .000001 ohm. This metal is valuable in the construction of measuring instruments not only because k is so small, but also because the thermo-electric current generated when a point of contact of dissimilar metals is heated is also small.

Substances other than metals, notably carbon, decrease in resistance when the temperature increases. The resistance of a carbon filament lamp when hot is about one-half of the cold resistance.

84. Currents in Series and in Parallel. Kirchhoff's Laws.—The battery in Fig. 70 is the seat of an E.M.F. which will cause a current to flow through the circuit *abcdh*. The conductors *ab* and *bc* are joined in series, *i.e.*, the entire current flows through the first and then passes on through the second. Between *d* and *e* the conductors are in parallel. Part of the current will flow on one branch and part on the other.

Before stating the laws of resistance in these two kinds of circuits it may be an advantage to state certain principles implied in Ohm's law but usually stated in what is known as Kirchhoff laws.

1. *The sum of all the currents meeting at a point is zero.*

The currents flowing toward a point are positive, and away from the point, negative. The first law then simply states that no electricity accumulates at any point in a circuit. The strength of current flowing to *c*, Fig. 70, is equal to that flowing away from it. The sum of the currents on the two branches leaving *d* is the same as the current flowing to that point.

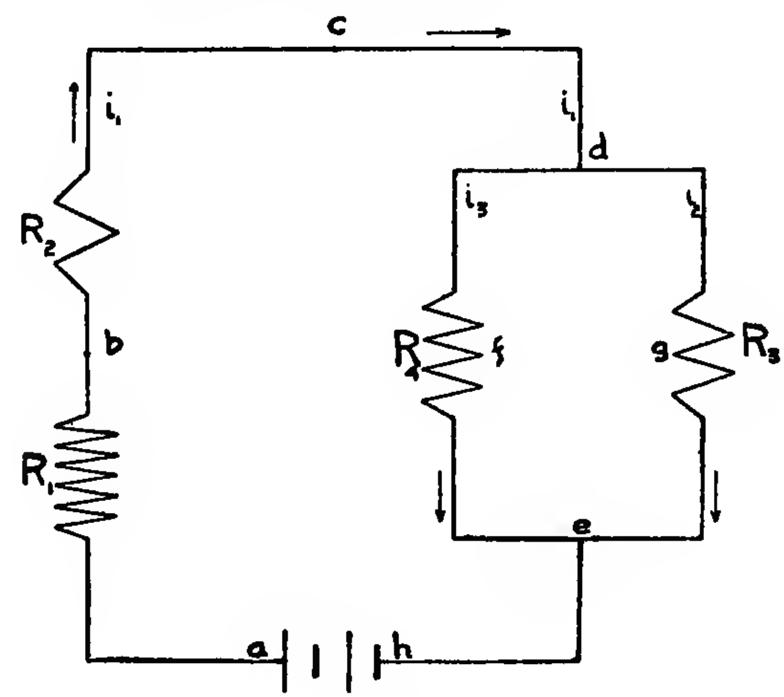

FIG. 70.

2. *In any closed circuit, however complex, the sum of the products obtained by multiplying the resistance of each part of any one path by the strength of current flowing through that part is equal to the sum of the E.M.F.'s in the circuit.*

Thus, in Fig. 70, if a current i_1 flows from a around the circuit, part flowing on each branch of the divided circuit, then $i_1R_1+i_1R_2+i_2R_3+i_1r=$ E.M.F. in the circuit, r being the internal resistance of the battery or other source of E.M.F. The same result is obtained if the other branch of the divided circuit is followed and i_3R_4 is used in place of i_2R_3. If we consider only the path *dgefd*, which does not contain a source of E.M.F., the sum

of i_2R_3 and i_3R_4, taken with the proper signs, is zero. In general, the sum of the iR's of a closed circuit which does not contain an E.M.F. is zero, provided that in following the circuit completely around, the iR's are regarded as positive in the direction the current flows, but negative when opposed to the current.

If we consider only the portion of the circuit from a to c in Fig. 70 and denote its total resistance by R_s, then

$$R_si_1=R_1i_1+R_2i_1$$

$$\text{or} \qquad R_s=R_1+R_2 \tag{81}$$

In general the total resistance of conductors in series is the sum of the individual resistances.

In case of that portion of the circuit from d to e, if R_p is the resistance of the two branches, then, by Ohm's law, equation (52), and since the potential difference E between d and e is the same for either branch,

$$i_2+i_3=\frac{E}{R_p};\ i_2=\frac{E}{R_3};\ i_3=\frac{E}{R_4}$$

Hence

$$\frac{E}{R_p}=\frac{E}{R_3}+\frac{E}{R_4}$$

$$\text{or} \qquad \frac{1}{R_p}=\frac{1}{R_3}+\frac{1}{R_4} \tag{82}$$

In general the reciprocal of the total resistance of any number of conductors in parallel is equal to the sum of the reciprocals of the resistance of the individual conductors, whatever the number of branches in parallel may be. In case there are but two conductors in parallel, from (82)

$$R_p=\frac{R_3R_4}{R_3+R_4} \tag{83}$$

i.e., the total resistance of two conductors in parallel is equal to the product of the individual resistances divided by their sum.

85. Wheatstone Bridge.—The best method of determining the resistance of a conductor is by means of a Wheatstone bridge. This, as shown in Fig. 71, consists of a divided circuit. Part of the current from the battery flows through ABC and a part through ADC. A conductor with a galvanometer in series is

connected between B and D. When these two points do not differ in potential no current will flow through the galvanometer. A coil of unknown resistance X is inserted in the arm AB; a resistance box R is inserted in AD, and by inserting or removing plugs the resistance, R, in this arm may be varied at will; the arms a and b each contain known resistances which may be plugged in or out of the circuit. Let X, R, a, and b represent the resistances inserted in the four branches of the bridge. R is so adjusted that the galvanometer shows no deflection. The bridge is then said to be *balanced*. Now, according to Kirchhoff's second law, the

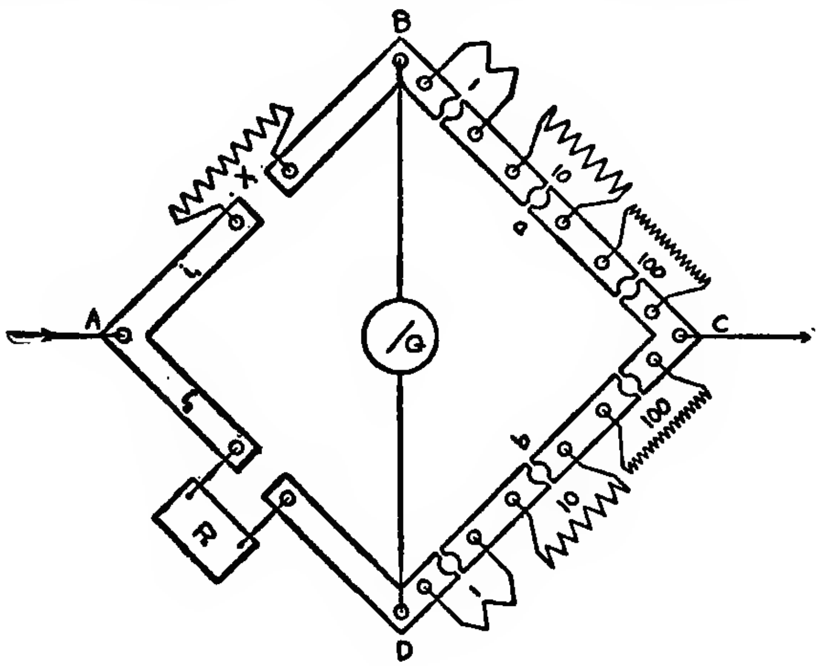

FIG. 71.

sum of the products of resistance by current in all parts of the closed circuit $ABDA$ is, since it contains no source of E.M.F., equal to zero. Hence

$$i_1X-i_2R=0 \tag{84}$$

Likewise in the closed circuit $BCDB$,

$$i_1a-i_2b=0 \tag{85}$$

Dividing (84) by (85),

$$\frac{X}{a}=\frac{R}{b}$$

or

$$X=\frac{a}{b}R \tag{86}$$

Since the resistance of a, b, and R are known, that of X is readily found, and if a is equal to b, X is equal to R. If b is made 100 ohms while a is 1 ohm, then

$$X = \frac{R}{100}$$

For example, if 245 ohms were introduced in R to balance the bridge, the resistance of X is 2.45 ohms.

According to Kirchhoff's first law it is evident that when the bridge is balanced the current which flows through AB will continue with the same strength through BC, and likewise for the other side of the bridge.

The connections of the galvanometer and battery should ordinarily be as shown in Fig. 71. Maxwell's rule for this is that of the two resistances, that of the battery and that of the galvanometer, the greater should connect the juncture of the arms having the two largest resistances to that of the other two arms. The resistance of the galvanometer will nearly always be greater than that of the battery and the resistance of R and b, Fig. 71, will in most cases be greater than that of X and a. Hence the greatest deflection will be obtained when the galvanometer is connected as shown.

The *slide wire* form of the Wheatstone bridge does not differ in principle from the form already explained. As shown in Fig. 72 the ratio coils of Fig. 71 are here replaced by a straight resistance wire. A balance is approximately secured by adjusting the resistance in R while c makes contact with the middle of the wire. Then by sliding c one way or the other along the wire an exact balance is secured. The resistances of a and b are assumed to be in proportion to their lengths, and so, as already shown,

$$\frac{X}{R} = \frac{a}{b}$$

or

$$X = \frac{a}{b} R$$

This form of bridge is convenient but is not so accurate in actual practice as a form to be described later. It is difficult to secure a wire which is exactly uniform, and therefore the assumption that resistance is proportional to length involves a probable

error. Errors also are made in observing the point of contact on the bridge wire and in finding the exact point of balance. Thermo-electric currents are also likely to be set up as a result of heating the points of contact by the hand or otherwise.

These errors may be partially eliminated by exchanging X and R and combining the observations made before and after the exchange. Thus, let the bridge wire, Fig. 72, be 1000 mm. long. Suppose that when X and R are placed as shown in the

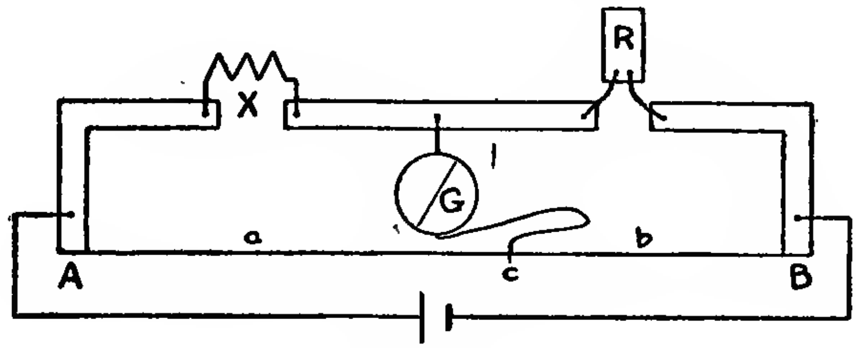

FIG. 72.

figure an error e is included in reading the position of c. Let a_1 be the distance from A to c in the first observation. Then

$$\frac{X}{R}=\frac{a_1+e}{1000-a_1-e} \tag{87}$$

Now let X and R be exchanged. Adjust c for a second balance, assuming that the same error has been made. Let the distance from A to c in this case be a_2. Then

$$\frac{X}{R}=\frac{1000-a_2-e}{a_2+e} \tag{88}$$

Adding numerators and denominators of (87) and (88),

$$\frac{X}{R}=\frac{1000+(a_1-a_2)}{1000-(a_1-a_2)} \tag{89}$$

Thus e is eliminated and the only observations necessary are the lengths a_1 and a_2 of the bridge wire.

The best form of bridge is one where the rheostat, the ratio coils, and the bridge proper are all conveniently arranged in a box, commonly known as the Post Office Box, from its use in the Postal Service of England. The external appearance of such a box is shown in Fig. 73 and the connections, as shown in Fig. 74,

are the same as in Fig. 71 but the battery and galvanometer wires are made to pass through keys on the top of the box. By pressing these at any time, the galvanometer will indicate whether or not the bridge is balanced. By removing plugs from their sockets

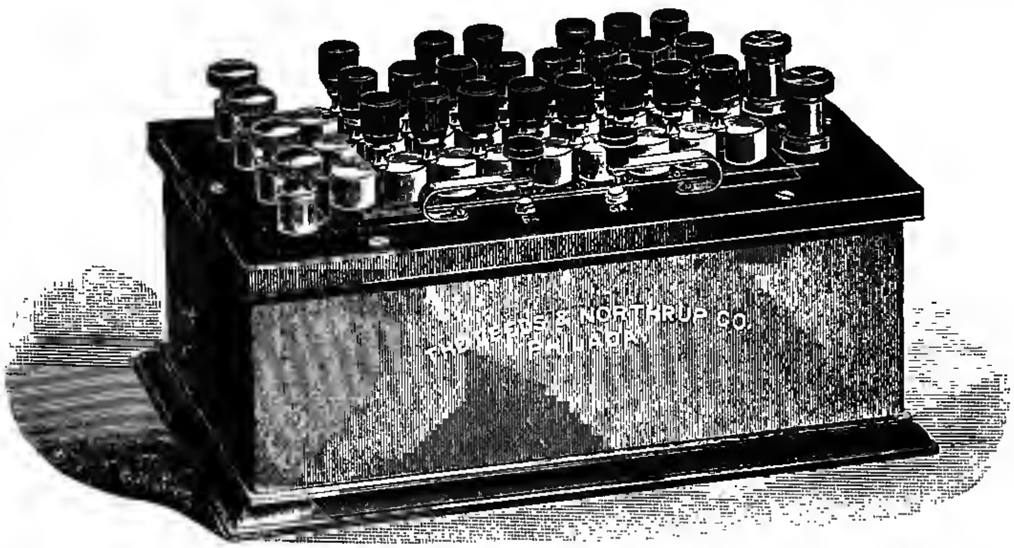

FIG. 73.

between the brass blocks a certain number of ohms of resistance, as marked on the lid of the box, is thrown into that circuit. Suppose that the bridge is balanced when the 1-ohm plug on the right and the 100-ohm plug on the left are removed from the ratio

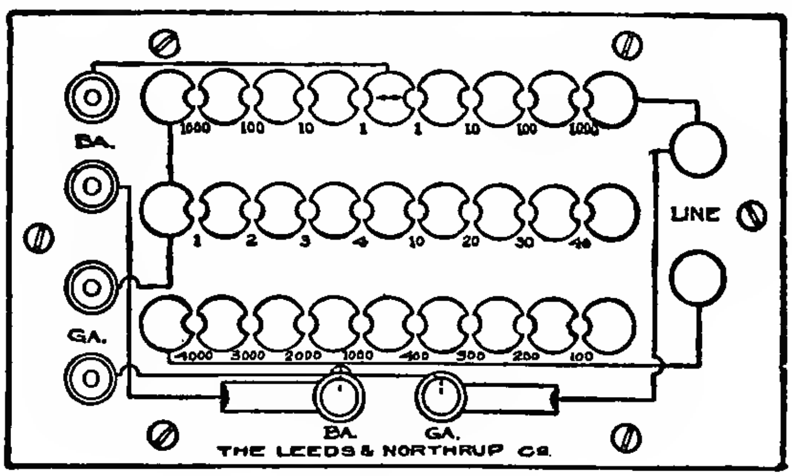

FIG. 74.

coils shown at the top of the diagram, Fig. 74, and the 400, the 30, and the 2-ohm plugs are removed from the rheostat or resistance box. Then by equation (86)

$$X = \frac{a}{b}R = \frac{1}{100}432 = 4.32 \text{ ohms}.$$

86. Shunts.—A shunt is a side path through which a portion of the current in circuit may flow. Thus in Fig. 75 the main current divides at a, part flowing through the shunt S and part through the galvanometer G. This is, then, a divided circuit and, according to Ohm's law, strength of current in each branch is inversely as the resistance of the branches. Let i_g be the strength of current through the galvanometer, i_s through the shunt, and

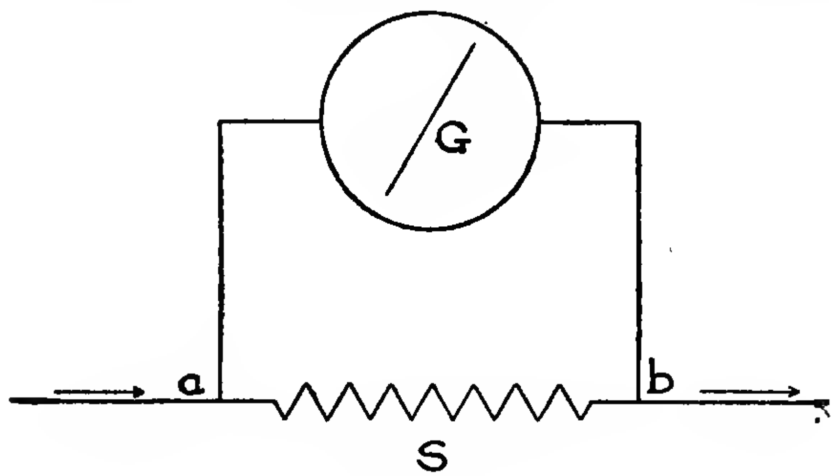

Fig. 75.

i_m the total current on the line. Also let G be the resistance of the galvanometer and S the resistance of the shunt. Then, by the second law of Kirchhoff,

$$i_g G - i_s S = 0 \tag{90}$$

or

$$\frac{i_g}{i_s} = \frac{S}{G} \tag{91}$$

Taking (91) by composition,

$$\frac{i_g}{i_g + i_s} = \frac{S}{G+S} \tag{92}$$

But $i_g + i_s$ is the total current i_m, hence

$$i_g = i_m \frac{S}{G+S} \tag{93}$$

From (93) it is easy to find the portion of a current flowing through a galvanometer or other instrument if the values of S and G are known, or to calculate the resistance of a shunt that will cause any desired fraction of a current to flow through the galvanometer.

If the ratio of S to G is as 1 to 9, equation (93) shows that one-tenth of the current will flow through the galvanometer.

If $$\frac{S}{G}=\frac{1}{99}, \quad i_g=\frac{i_m}{100}$$

If $$\frac{S}{G}=\frac{1}{999}, \quad i_g=\frac{i_m}{1000}$$

Shunt boxes prepared in this manner may be adjusted to any given galvanometer.

The Ayrton universal shunt is, in its operation, independent of the resistance of the galvanometer, but does not indicate directly the portion of the main current through the galvanometer. The external appearance of one form of such a shunt box is shown in Fig. 76. In Fig. 77 is shown a diagram of the connections and plan. A number of resistances are joined in series from E to A. Let the total resistance of these be denoted by S, that of EB by S_1, EC by S_2, and ED by S_3. The several resistances are made so that $BA=\frac{9}{10}S$, $CA=\frac{99}{100}S$, and $DA=\frac{999}{1000}S$. Now let the slide W be moved onto A. The current may then flow on two branches from A, one through the galvanometer and the other through the shunt S back to the other side of the battery. In this position of W we have the ordinary shunt described above and

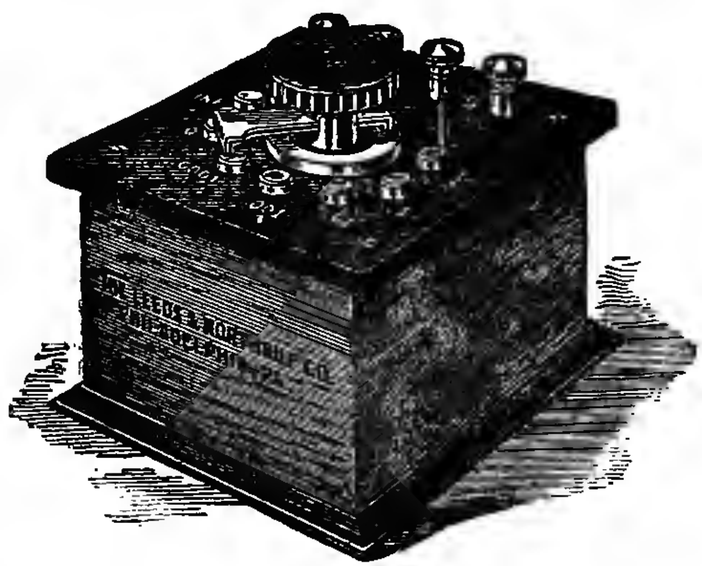

Fig. 76.

$$\frac{i_g}{i_m}=\frac{S}{S+G}$$

as in equation (93). Now let W be moved to B. The current now divides into two branches at B and we have

$$\frac{i_g}{i_m}=\frac{S_1}{S_1+G+.9S}=\frac{1}{10}\frac{S}{S+G} \tag{94}$$

since S_1 is equal to $.1S$.

Comparing equation (93) with equation (94), it is observed that when the slide is on B one-tenth as much current flows through the galvanometer as when it is on A. In a similar manner it may be shown that when the slide is on C, i_g is one-hundredth of i_m, and so on. Thus the Ayrton shunt, the operation of which is independent of G, is adapted to use with any galvanometer.

If the actual strength of current passing through the galvanometer is desired, it may be calculated from equation (93) when the slide is on A. Then any other position of the slide will indicate what fraction of that current is flowing through that galvanometer. A wide variety of current strengths may therefore be compared by noting the deflection of the galvanometer and the relative values of the shunts used. The Ayrton shunt can be used equally well with a ballistic galvanometer.

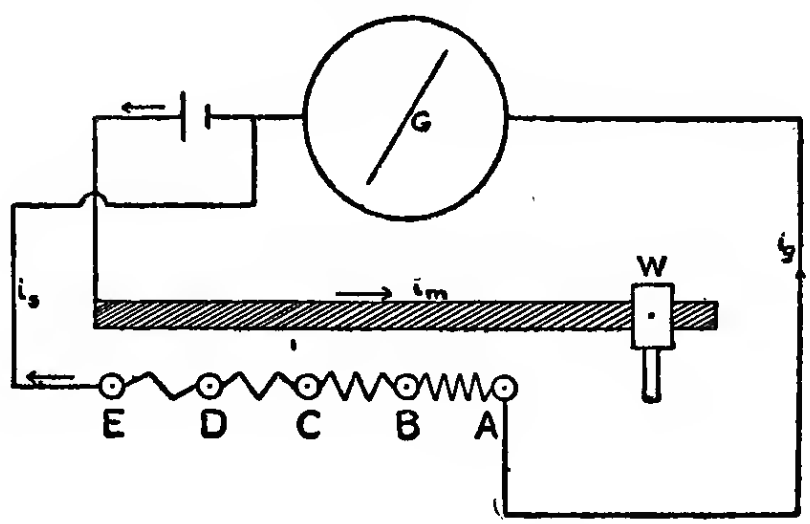

FIG. 77.

87. Potentiometer.—The laws of resistance may be applied in finding the P.D. between two points and in comparing the E.M.F. of cells. An instrument used for this purpose is called a *potentiometer*. Let *ab*, Fig. 78, be a resistance wire the ends of which are kept at a constant difference of potential by the battery B. A constant current will flow from b to a and any point on the wire is at a higher potential than a. Let B_1 and B_2 be the two cells whose E.M.F. is to be compared. Let them be arranged as shown so that either one may be put in circuit by turning a switch. First let the switch be turned to e. Then, since c is at a higher potential than a, a current from B would flow, not only from c to a but also from c through S, G, R, e, B_1 to a. But B_1 in this circuit

would send a current in an opposite direction. By sliding S along ab a point of contact c is found such that the galvanometer shows no deflection. There is then no current through G in either direction, and hence the P.D. between c and a is equal to the E.M.F. of B_1. Now throw the switch on f and move S for a second balance, say at d. Then the P.D. between d and a is equal to the E.M.F.

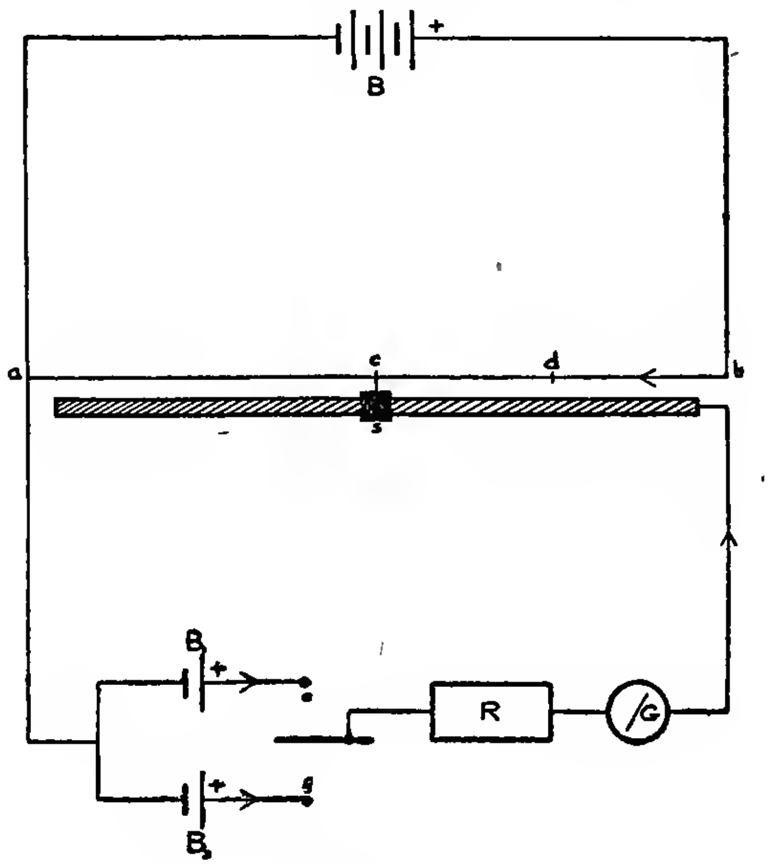

FIG. 78.

of B_2. Since the P.D's between points on a uniform resistance wire are proportional to the lengths of wire between those points,

$$\frac{ca}{da} = \frac{\text{E.M.F. of } B_1}{\text{E.M.F. of } B_2} \qquad (95)$$

If B_1 is a standard cell (§ 70), the E.M.F. of B_2 may readily be found.

A large resistance R is put in circuit with B_1 and B_2 while the proper position of the slide S is being found. This is chiefly to avoid polarization of B_1 or B_2. The usefulness of a standard cell is soon destroyed if it is short-circuited. When a balance is obtained, R may be removed, for then no current will flow.

Problems

1. What is the resistance of a copper wire 200 m. long and 2 mm. in diameter if the specific resistance is $1.584(10)^{-6}$?

2. What must be the area of cross section of a wire 600 feet long that the resistance may not exceed 2 ohms, the resistance of a mil-foot being, for this material, 9.74 ohms?

3. A closed circuit is made up of three conductors in series, their respective resistances being 2, 5, and 6 ohms, and a battery the resistance of which is .3 ohm. What E.M.F. must be impressed by the battery to cause a flow of 2 amperes?

4. If the resistance of a coil of platinum wire at 0° C. is 10 ohms, what will be its resistance when immersed in hot oil at a temperature of 200° C., the temperature coefficient being .00367?

5. What is the total resistance between two points which are joined by 3 conductors in parallel, the resistance of one being 5 ohms, another 10 ohms, and the third being composed of 10 and 20 ohm coils in series?

6. There are 25 incandescent lamps in parallel. The resistance of the conducting wires is .4 ohm. Each lamp when hot has a resistance of 200 ohms and requires a current of .5 ampere. What must be the voltage?

7. If the resistance of a galvanometer is 351 ohms, what must be the resistance of a shunt that .1 of the total current may flow through the galvanometer?

8. If the total resistance of the Ayrton shunt is 4000 ohms and the resistance of the galvanometer is 400 ohms, what portion of the total current will pass through the galvanometer when the slide *W*, Fig. 77, is on *C*?

9. If a galvanometer in use with an Ayrton shunt shows a deflection of 50 mm. on a scale when a current of .025 ampere is flowing in the line and the slide *W*, Fig. 77, is on *A*, what is the strength of current when the deflection is 100 mm. and the slide is on *B*?

Ans. 1. 1.07 ohms.
2. 2922 circular mils.
3. 26.6 volts.
4. 17.34 ohms.
5. 3 ohms.
6. 105 volts.
7. 39 ohms.
8. $\frac{1}{110}$.
9. .5 ampere.

CHAPTER VIII

THERMOELECTRICITY

88. Heat and Electricity.—It has long been suspected, and to some extent has been shown by experiment, that there is an intimate relation between heat and electricity. Heat may be used to produce a current of electricity or electricity to produce heat. Heat will be conducted more rapidly in one direction while a current flows on the conductor, whether with the current or against it depending on the nature of the material. A current in one direction through a juncture of dissimilar metals may heat the joint but in the opposite direction will cool it. These and other relations seem to indicate that many of the phenomena of heat are electrical in character. The fundamental nature of electricity and the electric current is better known to-day than that of heat, and the relation between the two is a fertile field for scientific investigation. The relations given below are statements of some phenomena which have been observed, but the general principle on which they can be explained is not yet known.

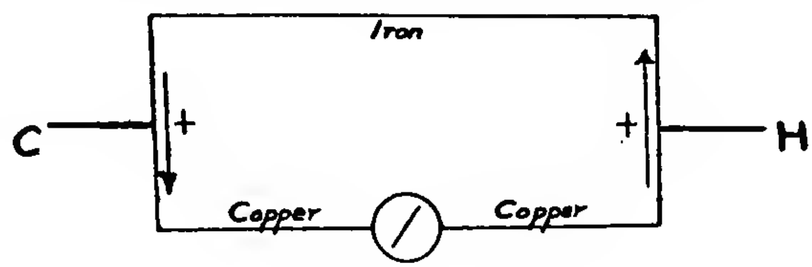

FIG. 79.

89. The Seebeck Effect.—In 1821 while Seebeck was investigating the subject of potential difference he discovered that when a circuit is composed of two dissimilar metals, if one junction is hotter than the other, a current of electricity will flow in the circuit. If iron and copper, Fig. 79, are joined at H and C, the junction H being kept hot while C is cold, a current will flow from copper to iron at H and from iron to copper at C. When the two joints are at the same temperature, no current flows. Another metal inserted in this circuit will not affect the results if the

added junctions have the same temperature. A thermo-couple of iron and copper produces only a small E.M.F.—about one millivolt for a difference of 100° C. when the cold junction is zero. A number of such couples may, however, be joined in series and the E.M.F. is thus multiplied as many times as there are couples. Thermopiles of antimony and bismuth or other metals are constructed in this manner. (See p. 235 of "Mechanics and Heat.")

If the temperature of the hot junction is increased while the other is kept constant, the E.M.F. will also for a time increase, but not in proportion to the increase of temperature. If in case of the copper-iron couple of Fig. 79 the end C is kept at 0° C.

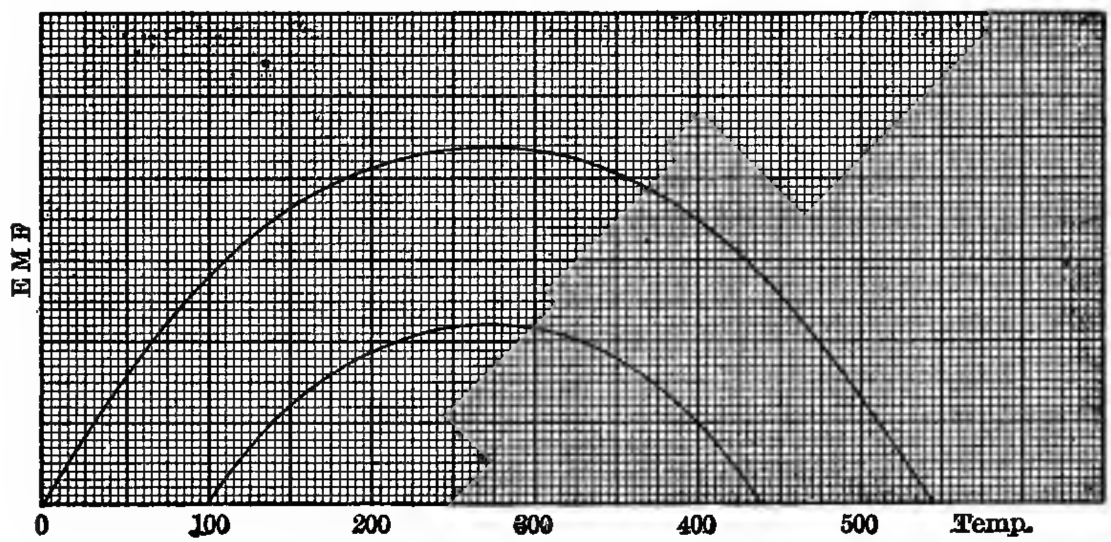

Fig. 80.

while the temperature of H is raised, the curve resulting from plotting the resulting E.M.F. as ordinate and the temperature as abscissa is shown in Fig. 80. This shows that the E.M.F. rises rapidly at first but later at a decreasing rate up to about 270° C. This is called the *neutral point.* A further increase in the temperature of H causes a decrease in the E.M.F., as the curve shows, to about 540° C. when the direction of the current is reversed, *i.e.*, flows from iron to copper at the hot junction. This latter point, discovered by Cumming in 1823, is called the *temperature of inversion* and is always as much above the neutral point as the temperature of the cold junction is below it—*e.g.*, if C, Fig. 79, is kept at 100° C. instead of 0° C., the temperature of inversion will be 440° C. instead of 540° C. Couples of other metals give

different results. With a zinc-iron couple the neutral point is about 200° C. and the temperature of inversion 400° C. With the lead-iron couple, one junction being at 0° C., the numbers are 357° and 714°.

If the temperature t_1 at any point in the curve of Fig. 80 is raised by a small increment to t_2 and the E.M.F. is thereby raised from E_1 to E_2, then

$$p = \frac{E_2 - E_1}{t_1 - t_2} \tag{96}$$

and p is called the *thermoelectric power*. This quantity may then be defined as the rate of change of E.M.F. at any point on the E.M.F.-temperature curve as the difference of temperature of the two junctions is steadily increased. The curve shows that the thermoelectric power is at first large, but decreases to the neutral point where it is zero, and then becomes negative at an increasing rate to the temperature of inversion. Hence, if a curve is plotted with p as ordinates and temperatures as abscissæ, the curve AB is nearly a straight line. The altitude of the ordinate at any point on this curve indicates, not the E.M.F. but the rate of change of E.M.F. as the temperature of the hot junction is raised. From equation (96) the change in E.M.F. is equal to the thermoelectric power times the change of temperature. This product, as plotted in Fig. 81, is an area included between two ordinates, the curve, and the temperature axis. If the difference of temperature is very small, the two ordinates ab enclose an area which is a measure of the E.M.F. for that difference of temperature at the two junctions of the thermo-couple. The area $acdb$ is, likewise, a measure of the E.M.F. when the junctions differ in temperature as much as the distance between these ordinates, for this area is the sum of an infinite number of elementary areas similar to ab but with p having a different value for each successive position on the temperature axis. At the neutral point p is zero, and the lower the temperature of the cold junction the greater the E.M.F. when the neutral point is reached.

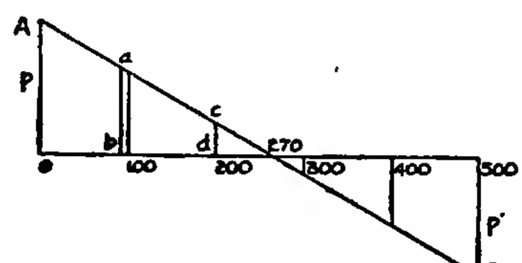

FIG. 81.

Metals are usually arranged in a series in which the thermoelectric power of each is given in microvolts per degree at a given temperature and in reference to some metal, usually lead, taken as a standard of reference. (See appendix.) If, then, it is desired to find the thermoelectric power between any two in the series, say bismuth and zinc, it is only necessary to subtract the numbers. Bismuth is -89 in reference to lead and zinc is $+3.7$, hence their algebraic difference is 92.7. For silver and antimony, 21.

90. The Peltier Effect.—Peltier discovered in 1834 that when a current of electricity flows on a circuit composed of dissimilar metals, the junctions are heated or cooled, depending on the direction of the current. Thus in the copper-iron circuit of Fig. 79, if a positive current flows in a direction indicated by the arrows, H will be cooled and C heated. If the current flows in an opposite direction, C will be cooled and H heated. Thus that junction which when heated produces a current in a certain direction will, when a current is passed through it in the same direction, be cooled, and the cold junction will be heated. This is usually explained by assuming that an E.M.F., called here the Peltier E.M.F., exists at the junctions. These are equal but opposite in direction around the circuit, and so no current flows as long as the temperatures of the two junctions are equal. In the copper-iron couple, Fig. 79, the iron is assumed to have a higher potential than the copper, and so when one junction is heated, thus increasing the P.D., a current will flow on the iron from H to C, and on the copper from C to H. This is the Seebeck effect. The Peltier effect may then be explained as the reverse of this. A positive current in passing from copper to iron must do work against an opposing force, hence heat is absorbed and the junction thereby cooled. At the other junction the same current passes from iron to copper, *i.e.*, in the same direction as the E.M.F. These would cause a current to flow, hence heat is liberated at that junction, *i.e.*, the junction is heated.

While the current is heating one junction and cooling the other it is also creating a difference of potential at the two ends of the iron wire which would cause a current in the opposite direction. The Peltier effect is therefore reversible in character. This must be distinguished from the Joule effect explained in § 54 where a conductor is heated by the passage of a current against resistance

and the quantity of heat varies as the square of the current, while the Peltier heating varies as the current. Also, the Joule process is in no sense reversible, for the heat effect is independent of the direction of the current.

The Peltier effect and its reversible character may be shown by use of the apparatus indicated in Fig. 82. It consists of a thermopile *T* composed of a number of bismuth-antimony couples. Through holes in the side of the caps are inserted two thermometers so that their bulbs lie against the junctions of the couples as shown. By means of a Pohl commutator *C* the current may be reversed either to or from the thermopile. *V* is a millivoltmeter and *B* is a cell of storage battery. To prevent too great a Joule effect, only about 400 milliamperes should be allowed to

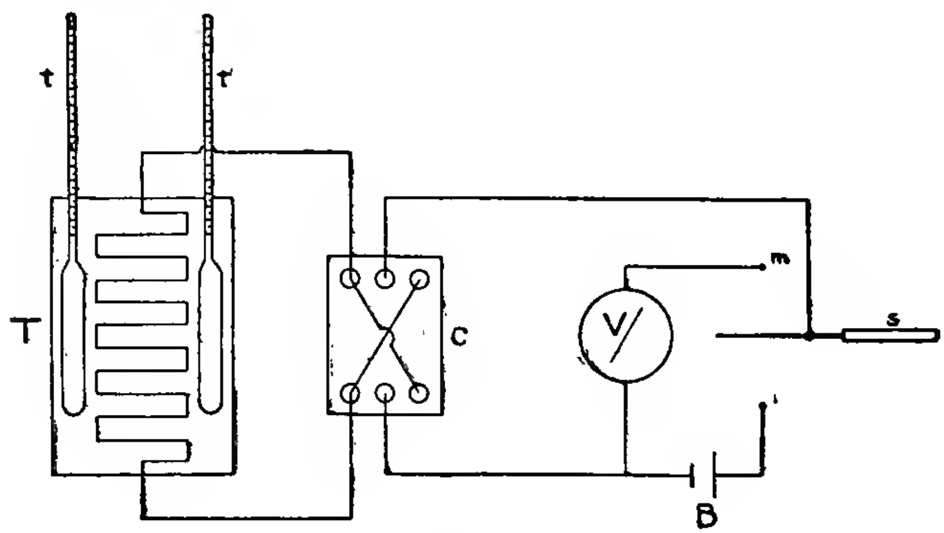

FIG. 82.

flow from the battery. If now the switch *S* is turned to *n*, it will be observed that one thermometer shows a rise of temperature and the other a fall. If then the rocker of the commutator be thrown, thus reversing the current in the thermopile, the indications of the thermometers will also be reversed. If after the current has been flowing for a short time the switch is turned to *m*, the battery will thus be cut out and *V* will be connected in circuit with the thermopile. The deflection in *V* will indicate a considerable current flowing in a direction opposite to that of the battery and continuing as long as there is a difference of temperature on the two ends of the thermopile.

91. The Kelvin Effect.—Lord Kelvin in 1854 showed that since the current in a thermo-couple is not proportional to the difference of temperature at the junctions there must be a potential difference not only at the junctions but also between other points at different temperatures. Let H be a hot point in a conductor, and suppose that the potential here is higher than at the ends C and C', which are cold. Then a current i flowing from C to H must do work against this potential and so heat is absorbed. The conductor between C and H will therefore be at a lower temperature than it would have been if the current had not been passed through it. But from H to C' the current is with the E.M.F. and heat is liberated. This part of the conductor is therefore at a higher temperature than it would have been without the current. If the current is reversed, the heating and cooling effects on the two sides of the hot point are also reversed. In

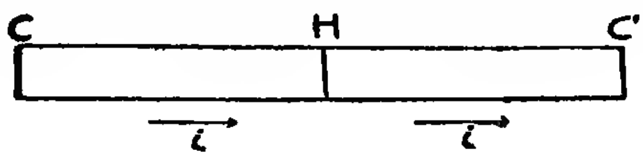

FIG. 83.

both cases there is apparently a greater conduction of heat in the direction the positive current flows. The phenomena just described have been observed in such metals as copper and silver, but the effect is reversed in such metals as iron, bismuth, and platinum.

92. Uses of Thermoelectricity.—The uses of the thermoelectric thermometer and the radiation pyrometer have already been described in "Mechanics and Heat," also the delicate instruments such as the bolometer and radiomicrometer for measurement of radiant energy. All of these involve the principles of thermoelectricity.

Various attempts have been made to use these principles in the production of electric current for commercial purposes but with little success. It would seem possible to construct a large thermopile of many couples, one end of which could be kept hot and the other cold, and thus secure considerable current directly from heat without the use of engines and dynamos. The process, however, is not an efficient one, for heat is rapidly conducted from the hot to the cold junctions and so is lost as far as the pro-

duction of current is concerned. Also, it is not possible to maintain a great difference of temperature at the junctions, for the thermoelectric power grows less and less to the neutral point, after which the E.M.F. begins to decrease. But even as an inefficient device it is found that after continued use a deterioration of some kind occurs at the junctions and the current produced by a given difference of temperature grows less.

The relation between heat and electricity cannot yet be said to be known, but is an interesting field for future investigation. It is not impossible that the dream of inventors and scientists in regard to the direct conversion of heat into electrical energy may come true, but the discovery will most likely be made by careful researches in the field of thermoelectricity.

CHAPTER IX

CONDENSERS

93. Capacity of Condensers in e.m. and Practical Units.—The general nature of condensers has already been described in §'s 18 and 19, and capacity C has been defined by equation (6) as the charge Q per unit difference of potential V. The numerical value of C will plainly be different for different systems of units. If Q and V are in *e.m.* units, the value of C will be the capacity in that system. If Q is taken in coulombs and V in volts the value of C will be the capacity in practical units called *farads*. A condenser has a capacity of 1 farad when it will hold 1 coulomb of electricity for each P.D. of 1 volt between its terminals.

Since the coulomb is $(10)^{-1}$ *e.m.* units and the volt is $(10)^8$ *e.m.* units, the value of a farad is

$$C=\frac{Q}{V}=\frac{(10)^{-1}}{(10)^{8}}=(10)^{-9}\ e.m.\ \text{units}$$

The farad is inconveniently large and so one-millionth of it, called the *microfarad*, is used as the practical unit. This is $(10)^{-15}$ *e.m.* unit.

94. Standard Condensers.—Condensers carefully adjusted and enclosed in boxes as shown in Fig. 84 are used as standards of capacity. The condenser may be subdivided, *i.e.*, may contain a number of separate condensers that can be united in various ways by connections on the top of the box. In the diagram, Fig. 85, A and B are brass strips which are the terminals of a condenser like that in Fig. 16. B and C are the terminals of another, and so on, each condenser being composed of several layers of tin-foil insulated with mica. In the diagram as shown the total capacity is one microfarad, subdivided into .5, .2, .2, .05, and .05 microfarad. If plugs be now inserted in a and b, the

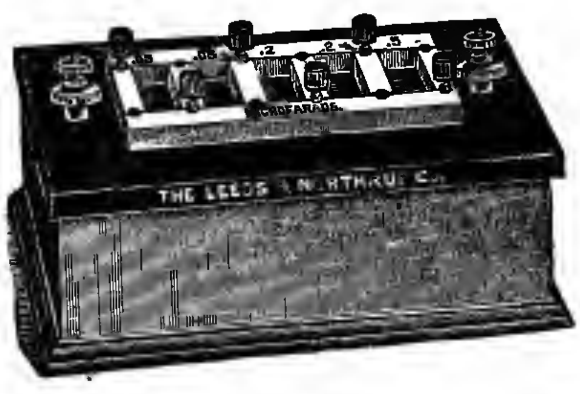

FIG. 84.

binding posts p and p' become the terminals of the condenser AB. If these posts are then connected to the terminals of a battery or other source of E.M.F., the condenser AB will be stored with electricity, the quantity of which will depend on the capacity of the condenser and the E.M.F. of the battery. If the plug c is added, both AB and BC are stored. If d, e, and f are added, the entire condenser is in use.

Condensers may be joined in parallel or in series.

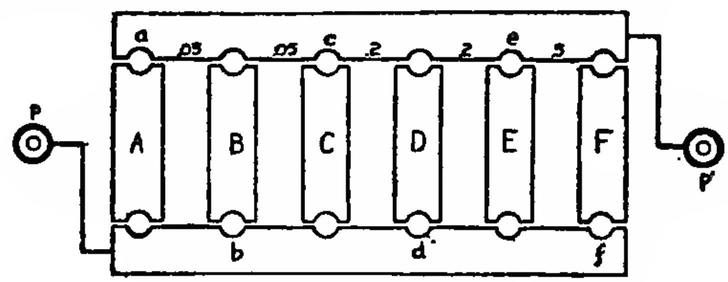

FIG. 85.

When in parallel, as just described, there is an equal difference of potential V at the terminals of each condenser. Also, the total quantity of electricity Q in all the condensers in parallel must be the sum of the quantities in each, hence

$$C=\frac{Q}{V}=\frac{Q_1}{V}+\frac{Q_2}{V}+\frac{Q_3}{V}, \text{ etc.} = C_1+C_2+C_3, \text{ etc.} \tag{97}$$

i.e., the total capacity of cells in parallel is the sum of the individual capacities.

When in series, the total potential is the sum of the potentials of each, *i.e.*, the P.D. between O and P, Fig. 86, must be the sum of the P.D.'s between s and t, and m and n. Also, since the number of electrons leaving m must equal the number that comes to t, and since n and s, connected by a conductor and acted on inductively, must have as great an excess of electrons on n as there is a deficiency on s, there is the same charge in each condenser. Hence for condensers in series the total difference of potential V is

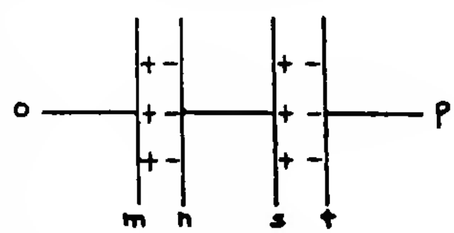

FIG. 86.

$$V=V_1+V_2+V_3, \text{ etc.} \tag{98}$$

and the total charge Q is

$$Q=Q_1=Q_2=Q_3, \text{ etc.} \tag{99}$$

Then, since in any condenser

$$V=\frac{Q}{C}$$

substituting the corresponding value of V for each condenser in equation (98),

$$\frac{Q}{C}=\frac{Q_1}{C_1}+\frac{Q_2}{C_2}+\frac{Q_3}{C_3}, \text{ etc.}$$

$$\text{or} \quad \frac{1}{C}=\frac{1}{C_1}+\frac{1}{C_2}+\frac{1}{C_3}, \text{ etc.} \tag{100}$$

Thus for condensers in series the reciprocal of the total capacity is the sum of the reciprocals of the several capacities. If, for example, plugs are inserted at c and f, Fig. 85, the condensers CD, DE, and EF are placed in series and their total capacity is one-twelfth microfarad.

95. Measurement of Capacities.—It is often necessary to know the capacity not only of condensers as such but also of electrical lines and combinations which act as condensers. Submarine cables, for example, may have a capacity of more than a microfarad per mile, the insulation forming the dielectric between the conducting wires at the centre and the steel covering and water on the outside. Also, telephone wires in air or in conduits, particularly if close together, have a considerable capacity which must be considered in the transmission of currents.

One method of finding the capacity of a line or other condenser is by comparison with a standard condenser. A scheme of connections for this purpose is shown in Fig. 87. If the switch S is turned to n, the battery B is connected to the condenser C. The switch is then turned to m. This cuts out the battery and puts the ballistic galvanometer G in circuit with the condenser C. The discharge of C causes a throw of the galvanometer coil, which is proportional to the quantity of electricity and to the capacity of the condenser. This throw is observed by means of a telescope and scale. Then the condenser whose capacity is sought is removed and a variable standard condenser put in its place. The capacity is then so adjusted that, using the same battery as before,

the throw of the galvanometer will be nearly the same. Since the capacities are proportional to the deflections produced and three of the terms are known, the unknown capacity can readily be calculated.

Another method of measuring capacity is by means of a Wheatstone bridge. Let R_1 and R_2, Fig. 88, be resistances in two arms of the bridge, two condensers whose capacities are C_1 and C_2 being inserted in the other two arms. An alternating current is furnished by an induction coil or a small magneto I. A telephone receiver T will indicate when b and c are at the same potential. When the resistances R_1 and R_2 are adjusted so that no sound is heard in T, the bridge is balanced. Then a current divides

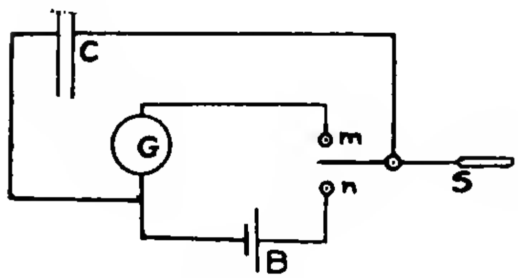

FIG. 87.

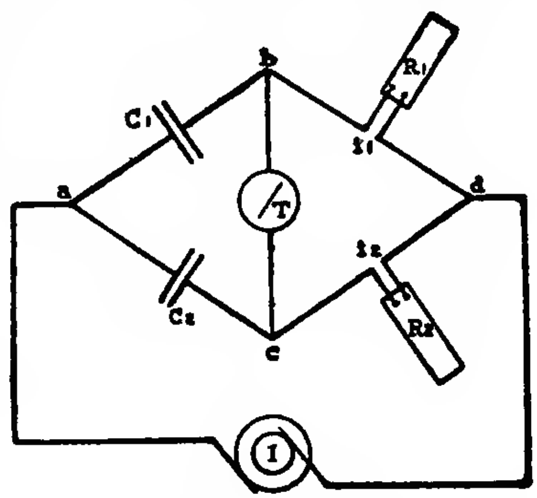

FIG. 88.

at d, part flowing through R_1 to charge C_1 and part through R_2 to charge C_2. When the current is reversed or stopped, the condensers will be discharged through these same branches, but none will cross on bc. Since in a divided circuit the strength of current in each is inversely proportional to the resistance in those branches,

$$\frac{i_1}{i_2}=\frac{R_2}{R_1} \tag{101}$$

The time of charging and discharging must be the same for both condensers for otherwise some current would cross on bc, hence

$$\frac{i_1 t}{i_2 t}=\frac{Q_1}{Q_2}=\frac{R_2}{R_1} \tag{102}$$

for the product of strength of current i by time t during which it flows gives the quantity Q of electricity. But the difference of potential between a and b and between a and c remains constant

during the whole process of charging and discharging and consequently, from equation (6), C must vary directly with Q. Hence, from (102), substituting C for Q,

$$\frac{C_1}{C_2} = \frac{R_2}{R_1} \tag{103}$$

i.e., the capacities are inversely as the resistances. One of the condensers is a standard of known capacity and the two resistances are known, hence the capacity of the other condenser is readily found.

96. Ratio of e.m. to e.s. Unit of Quantity.—By determining the capacity of a given condenser in microfarads by the method just described, and then calculating the capacity of the same condenser by equation (17), it is possible to find the ratio of the electromagnetic unit of quantity to the electrostatic unit. For this purpose a condenser like that shown in Fig. 89 should be used. The plates may be separated by a uniform sheet of mica or may be used as an air condenser, the plates being kept apart by very small pieces of mica near the edge. The capacity of any condenser may be expressed by

$$C_{e.m.} = \frac{Q_{e.m.}}{V_{e.m.}} \tag{104}$$

or

$$C_{e.s.} = \frac{Q_{e.s.}}{V_{e.s.}} \tag{105}$$

the subscripts *e.m.* and *e.s.* indicating, respectively, electromagnetic and electrostatic units. Dividing (105) by (104),

$$\frac{C_{e.s.}}{C_{e.m.}} = \frac{\dfrac{Q_{e.s.}}{Q_{e.m.}}}{\dfrac{V_{e.s.}}{V_{e.m.}}} \tag{106}$$

If the unit by which any given quantity of electricity is measured is small, the number of such units will be proportionately large. If the *e.s.* unit is small as compared with the *e.m.* unit, $Q_{e.s.}$ will be numerically large as compared with $Q_{e.m.}$ The numbers expressing the same quantity will be inversely proportional to the size of the units used in measuring the quantity. The number expressing the potential, however, will be directly proportional to the magnitude of the units, for the greater the unit the greater the number of ergs of work that must be done in moving it from

one point to another of different potential. Hence, in equation (106), $V_{e.m.}$ bears the same ratio to $V_{e.s.}$ that $Q_{e.s.}$ does to $Q_{e.m.}$ The product of extremes and means in the right-hand member of this equation will therefore be the square of the ratio of the units. Then if the value of $C_{e.s.}$ is found by calculation and $C_{e.m.}$ is found, for the same condenser, in microfarads and reduced to absolute units by multiplying by $(10)^{-15}$, the ratio of these two capacities will by equation (106) be equal to the square of the ratio of the units of quantity. The value obtained for the ratio of $C_{e.s.}$ to $C_{e.m.}$ is very nearly $9(10)^{20}$. Hence the ratio of the *e.m.* to the *e.s.* unit of quantity is very nearly $3(10)^{10}$. This, in centimetres, is the velocity of light, a fact which has been significant in the development of the electromagnetic theory of light.

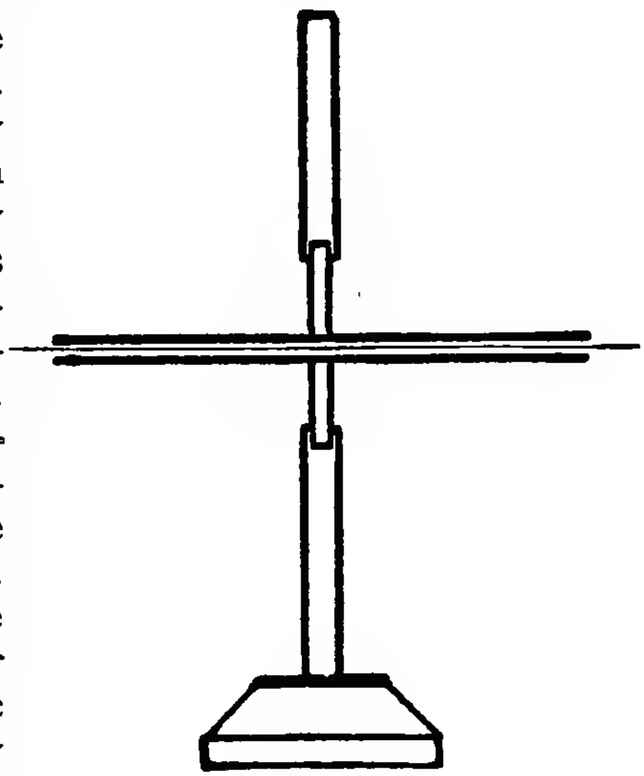

FIG. 89.

97. Dielectric Constant.—The dielectric constant or specific inductive capacity has been defined (§ 21) as the ratio of the capacity of a condenser when any given dielectric is used to the capacity when air is the dielectric. Hence, by use of the apparatus shown in Fig. 89 it is possible to introduce various dielectrics between the plates and then, by charging with the same battery and discharging through a ballistic galvanometer, to compare capacities. These would be in proportion to the throws of the galvanometer if the plates are separated by the same distance in each case.

The charge in a condenser depends in some measure on the time of charge. As explained in § 22 a certain quantity of electricity is absorbed by the dielectric, and this increases with the time of charging. The total charge is therefore not given back

on the first discharge, but will appear on subsequent discharges. Mica is comparatively free from this objection and so is used as the dielectric in standard condensers.

Problems

1. What quantity of electricity will a condenser of .5 microfarad capacity hold when the P.D. is 4 volts?

2. What is the capacity of a condenser that holds .0033 of a coulomb when the pressure is 110 volts?

3. How long a time will be required for a current of 1.5 amperes under a pressure of 110 volts to charge a condenser of 30 microfarads?

4. A cable two miles long is charged by a battery and then discharged through a galvanometer, causing a deflection of 120 divisions on a scale. A condenser of .5 microfarad charged by the same battery causes a deflection of 150 divisions. What is the capacity of the cable?

5. Three condensers having capacities .5, .2, and .2 microfarad, respectively, are connected in series. What charge will be needed to store them under a pressure of 120 volts?

6. Three condensers are in parallel. Their respective capacities are .5, .2, and .1 microfarad. What quantity of electricity measured in *e.s.* units will charge them at a pressure of 20 volts?

Ans. 1. $2(10)^{-6}$ coulombs.
2. 30 microfarads.
3. .0022 sec.
4. .2 microfarad per mile.
5. $10(10)^{-6}$ coulombs.
6. $4.8(10)^{5}$ *e.s.* units.

CHAPTER X

ELECTROMAGNETS

98. Solenoids.—A solenoid is a continuous conductor wound in form of a helix. From principles already explained in § 25 we see that, when a current of electricity flows on the conductor, a magnetic field is produced within the solenoid which then exhibits the properties of an ordinary bar magnet as shown in Fig. 24. The direction of the field within the solenoid depends on the direction in which the current flows. A conventional rule for this is as follows: Grasp the conductor at any point with the right hand so that the thumb points in the direction the positive current flows, and the fingers will then encircle the conductor in the direction of the lines of force, which direction within the solenoid is the direction of the field there. The end from which these lines emerge is the north-seeking pole of the solenoid.

Fig. 90.

The strength of the field within a solenoid may be calculated, as shown in Appendix 2, by use of the equation

$$H = \frac{4\pi ni}{10} \text{ gausses} \tag{107}$$

where n is the number of turns per centimetre and i is the strength of current in amperes. The strength H may be made large by increasing n and i. The best method of magnetizing a bar of steel or a magnetic needle is by placing it in such a strong field. When a core of soft iron is placed in the solenoid, the number of magnetic lines is enormously increased. This arrangement constitutes what is called an electromagnet. The extensive use of the electromagnet results from the fact that the iron core is a magnet while a current flows in the solenoid, but ceases to be a magnet when the electric circuit is broken. Hence its use in operating sounders of telegraph instruments, ringing bells, producing magnetic fields of dynamos,

shifting heavy masses of iron by means of the electric crane, and numerous other uses of this character.

If the core of an electromagnet is short, the number of lines of force induced in it will not be so great as when longer and in a longer solenoid, for the poles of the magnet exert a force within the coil opposite to that of the coil. When the poles are farther apart their effect in this respect is not so great. If the helix of the solenoid is bent in form of a circle as shown in Fig. 91, the lines of force form a closed circuit and there are no poles. The lines of force are then found only in the space within the helix, as iron filings show. The field is nearly uniform and an iron ring in this field will be magnetized equally in all parts.

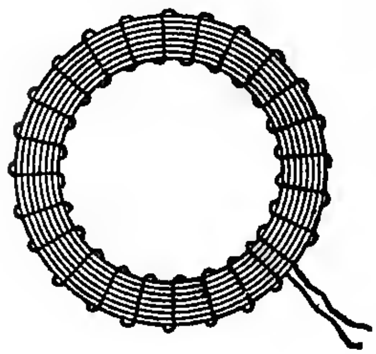

FIG. 91.

99. Magnetic Flux.—The total number of lines of magnetic force passing through any given area is called the *magnetic flux* through that area. The letter H has been used to represent strength of magnetic field, and strength of field is defined as the number of lines per square centimetre. Hence, if ϕ stands for magnetic flux, then for a given area, A, across a uniform field, such as in a long solenoid or a ring solenoid,

$$\phi = HA \tag{108}$$

or if, as explained in § 34, the field contains a paramagnetic substance, the total induction per square centimetre is B, which includes H, then

$$\phi = BA \tag{109}$$

The number of lines per square centimetre is the *flux density*. In air this is H, but in other material it is B and the relation of B and H is

$$B = \mu H$$

where μ is the permeability of the substance. (See § 37.) Hence by calculating H from equation (107) and observing B, the permeability of any substance may be found, or if μ is known, the induction B is readily calculated. In this manner the various values of H which were plotted as abscissæ in Figs. 31 and 32 were found, the resulting induction being plotted as ordinates.

One turn of the wire of a solenoid carrying one ampere is called one *ampere-turn*. In equation (107) it is seen that the magnetizing force H, or the flux density in air, varies directly as ni, *i.e.*, as the number of ampere-turns per centimetre. Thus, in one centimetre, a single turn of wire in which five amperes flow will produce the same value for H as five turns in which one ampere flows.

100. Magnetomotive Force.—Just as in electricity we speak of an electromotive force as that which causes a current to flow, so in magnetism, although nothing actually flows, yet a force is required to set up that condition known as a magnetic circuit. This is called the magnetomotive force. Electromotive force is defined as the number of ergs of work performed in carrying unit charge around the electric circuit, and in a similar manner the magnetomotive force is defined as the number of ergs of work required to carry unit pole around a magnetic circuit. The former is often called electric pressure and is measured in volts, while the latter is called magnetic pressure and is sometimes expressed in *gilberts*. In case of a magnet the magnetomotive force (*m.m.f.*) is the work required to move a unit N pole from the S pole to the N pole. In a ring solenoid such as in Fig. 91, it is the work done in moving unit pole once around the circuit in opposition to the field within the helix.

If equation (107) is multiplied by the length of the solenoid, L, then

$$HL = \frac{4\pi Lni}{10} \tag{110}$$

and since n is the number of turns per centimetre, Ln is the total number of turns and may be represented by N. Then

$$HL = \frac{4\pi Ni}{10} \tag{111}$$

Now H is the intensity of the magnetizing force—*i.e.*, the force with which unit pole would be urged—and L is the distance, hence HL is the *m.m.f.* of the circuit. Then

$$m.m.f. = \frac{4\pi Ni}{10} = 1.257\,Ni \tag{112}$$

i.e., the *m.m.f.* is equal to the total number of ampere-turns times 1.257.

101. Magnetic Reluctance.—Ohm's law as expressed by equation (52) simply states that the strength of an electric current is directly proportional to that which causes it and inversely proportional to that which resists it. This law is, then, a general principle which has been shown to be rigidly applicable to the electric current.

Likewise, in the magnetic circuit we have magnetic flux, ϕ, which corresponds in a sense to i of the electric circuit, *m.m.f.* corresponds to *e.m.f.*, and magnetic reluctance R_m corresponds to electric resistance R. Hence we may speak of Ohm's law of the magnetic circuit and express it by

$$\phi = \frac{m.m.f.}{R_m} \qquad (113)$$

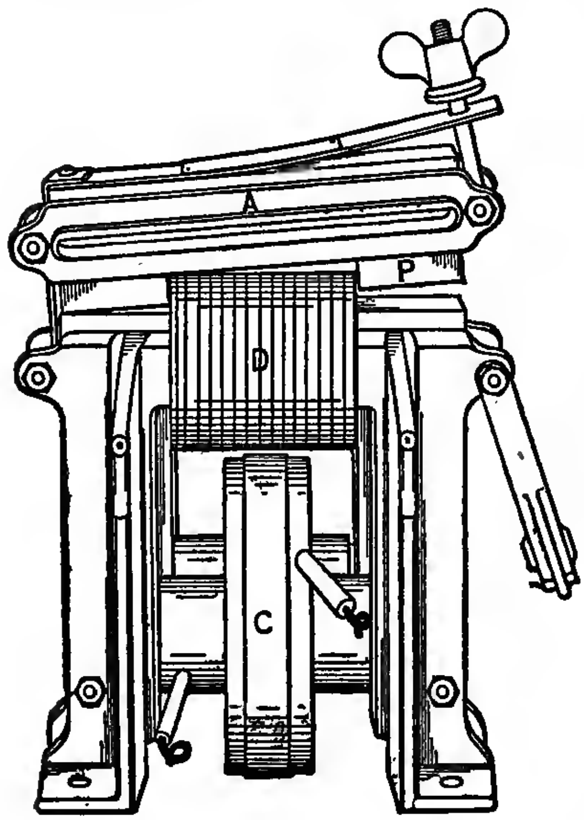

FIG. 92.

Reluctance may be found when the dimensions and specific reluctance of a given substance are known, by use of an equation similar to (78), thus

$$R_m = K\frac{L}{A} \qquad (114)$$

where L is length, A is area of cross section, and K is the specific reluctance, *i.e.*, the reluctance of one cubic centimetre. But since reluctance is the reciprocal of permeability just as resistance is the reciprocal of conductivity, (114) may be written

$$R_m = \frac{L}{\mu A} \qquad (115)$$

Reluctances in series and in parallel are calculated as in resistances, and in shunted magnetic circuits the flux is inversely proportional to the reluctances of the branches. If a current of electricity is passed around the iron core D, Fig. 92, lines of force will be set up through C and A, but if a gap is made in the iron

circuit at P, the reluctance of the upper branch will be increased and there will be an increased flux through C.

The apparatus here shown is a transformer which raises the voltage of an alternating current from 110 to 20,000 volts and the strength of the induced current is regulated in the manner described above.

Problems

1. What is the intensity of the magnetic field within a solenoid when the current strength is .2 *e.m.* unit and there are 6 turns per centimetre?

2. A solenoid 50 cm. long and having 300 turns is bent in form of a ring and filled with iron for which $\mu=450$. What flux density will be caused by a current of 5 amperes?

3. What is the reluctance of a rod of iron 2 cm. in diameter and 100 cm. long, when $\mu=1000$?

4. If a solenoid 40 cm. long has four turns per centimetre, what will be the *m.m.f.* when the current is 2 amperes?

5. If a magnetizing force of 5 gausses is needed to cause an induction of 9000 gausses in a certain specimen of iron, how many ampere-turns are needed to set up 100,000 lines in a ring of this metal 200 cm. long and 40 cm.^{2} in cross section?

6. An iron rod 3 m. long and 75 cm.^{2} in cross section is bent in form of a circle but with an air gap of 3 cm. between the ends. If $\mu=800$, what is the reluctance of the entire circuit?

Ans. 1. 15.08 gausses.
2. 16969 gausses.
3. .032 unit.
4. 402.24 units.
5. 220.98.
6. .045 unit.

CHAPTER XI

ELECTROMAGNETIC INDUCTION

102. Induction.—Electric induction, in a general way, refers to the effects produced on bodies in an electric or magnetic field. Electrostatic induction has already been defined as the appearance of positive and negative charges in a conductor when placed in an electrostatic field, and, likewise, magnetic induction is the phenomena observed when a magnetic substance is placed in a magnetic field.

In 1819 Oersted discovered the fact that a current of electricity would influence a magnetic needle or any form of magnet. Not only would the current affect a magnet but would cause such substances as iron to become magnets so that strong electromagnets could be made. A current of electricity therefore produces a magnetic field about its path. If therefore a current produces magnetism, it seemed very probable that in some manner a magnetic field could be made to produce electricity. An investigation of this subject by Faraday in England and Henry in America resulted in the discovery of electromagnetic induction. Faraday published his results in 1831 while Henry, although his discovery antedated that of Faraday, delayed publication till 1832.

103. Nature of Electromagnetic Induction.—As long as a current flows on a conductor, a magnetic field is maintained in the region of that conductor, but, on the other hand, the existence of a steady magnetic field in the region of a conductor will not cause any flow of electricity. This may be illustrated by a water analogy. Suppose numerous slender elastic rods are fastened at one end and that the free ends project into a stream of running water. As long as the water flows the rods will be bent and their strained condition may be taken to represent the magnetic field which accompanies the flow of an electric current. If, however, the water is not in motion and the rods are all bent in the same direction by some outside force, then, although the water may be momentarily set in motion, the fact that the rods are kept in that strained condition will not cause a current to flow. Similarly we would not expect that a strained condition of the ether would

cause a steady flow of an electric current, for such a condition would be in direct conflict with all principles of conservation of energy.

Faraday and Henry discovered a variety of ways by which a current may be induced in a conductor, the principle common to all being that *any change in the number of magnetic lines enclosed by a conducting circuit will induce a current in that circuit.* If two coils, A and B of Fig. 93, are placed near one another, the terminals of A being connected to a source of electricity, and the coil B being closed through a galvanometer, then whenever the circuit of A is made or broken, the galvanometer will show a momentary current in B. Either starting or stopping the current in A causes a change in the number of magnetic lines passing through B. If an iron core is placed within the coils, the induction will be greater, for there will be a greater change in the number of magnetic lines

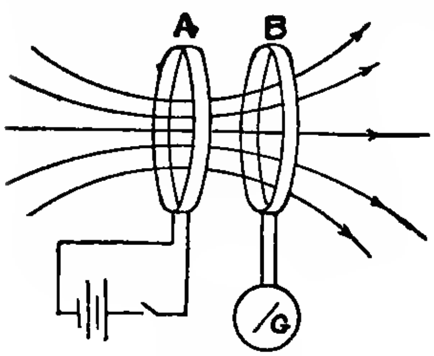

FIG. 93.

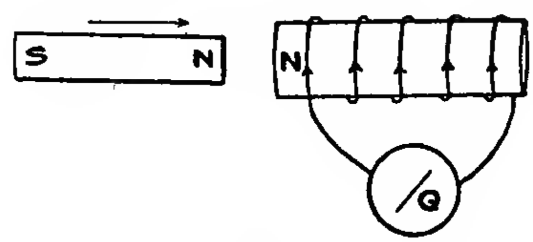

FIG. 94.

when the current is started or stopped. A steady flow of current in A will not cause any current in B, for then the magnetic lines enclosed by B will be neither increasing nor diminishing, but any change in the relative position of the two coils will induce a current in B. If A is replaced by a bar magnet, whenever a pole of the magnet is advanced toward B or withdrawn, induction follows. The numerous kinds of changes by which a current is induced in a closed conducting circuit may all be explained in accordance with the general principle stated above.

The direction of the induced current is always such as to oppose the motion which produces it. This is known as Lenz's law and is illustrated in Fig. 94, where a north-seeking pole of a magnet moved toward a coil induces a current in such a direction as to produce a north-seeking pole at that end of the coil, and thus oppose the approach of the magnet. The withdrawal of the magnet produces

a current in the opposite direction, thus making that end of the coil a south-seeking pole and again resisting the motion which causes the induction. Similar phenomena may be shown in case of the two coils in Fig. 93. Any change which causes induction requires, according to Lenz's law, that force must be exerted through a distance, *i.e.*, that work must be done. When a conductor is moved across lines of force the direction of the induced current may be conveniently found by what is known as the *dynamo rule, viz.*, extend the thumb, the first, and the second fingers of the right hand so that each is at right angles to the

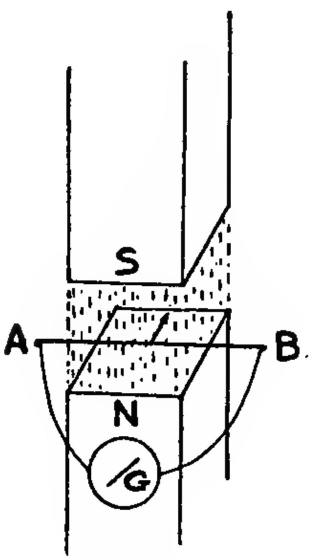

Fig. 95.

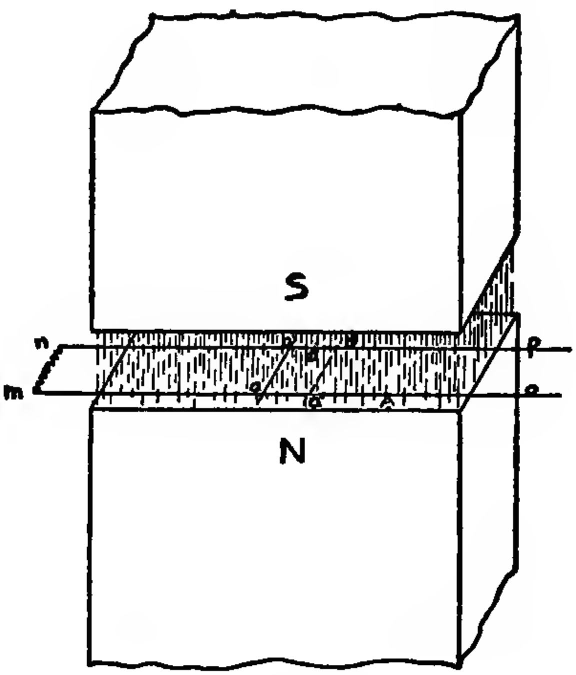

Fig. 96.

other two, then if the first finger points in the direction of the magnetic lines (from N to S) and the thumb in the direction the conductor is moved, the second finger will indicate the direction in which the current flows. Thus in Fig. 95 a current will flow from A to B if the conductor AB is moved in the direction of the arrow. This rule applies only to the conventional current. For the electron or negative current the left hand must be used.

The electromotive force of the induced current is proportional to the rate at which a conductor cuts lines of force, or to the rate of change of flux included in a closed circuit. The conductor may be only a straight rod or wire, and if it is moved across lines of force a difference of potential will be produced at the two ends. If this

conductor forms part of a closed circuit, a current of electricity will flow. To show that, in accordance with principles already stated, the *e.m.f.* is proportional to the change of flux, suppose that a uniform magnetic field exists between the poles N and S, Fig. 96. Let two conducting rods *mo* and *np* be placed in this field and connected at *mn* by a wire. A sliding rod *ab* laid on the two rods then closes the circuit *abnm*. Suppose now that this circuit contained an *e.m.f.* which would cause a current to flow around in the direction *bamnb*, then, by the motor rule, the slide would be urged toward *mn* with a certain force F. But if the circuit does not contain an *e.m.f.* and a force F is exerted to move the slider from the position *ab* to *a'b'*, then by the dynamo rule a current will flow around the closed circuit in the direction *bamn*. Let the intensity of the magnetic field be H gausses, the strength of current i electromagnetic units, and the length of the slider l cm. The current here is that which results from moving the slider across the lines of force. Then, by the definition of the *e.m.* unit of current, the force applied in moving the slider is

$$F = iHl \text{ dynes} \tag{116}$$

Let this force be applied through a distance d, then the work, W, done is

$$W = Fd = iHld \text{ ergs} \tag{117}$$

This is the amount of energy expended in moving the slider across the field and thus producing the current i. The current should receive all the energy expended in producing it. It has already been shown that the energy of a current is equal to the product of its strength i, the electromotive force E, and the time t during which it flows. If, then, t is the time occupied in moving the slider from *ab* to *a'b'*, the energy of the current produced may be represented by

$$W = iEt \text{ ergs} \tag{118}$$

We therefore have from (117) and (118)

$$iHld = iEt$$

or

$$E = \frac{Hld}{t} \tag{119}$$

Now H is the number of lines per square centimetre and ld is the area $abb'a'$. Hence Hld is the change of flux through the closed circuit. This divided by the time gives the change per unit of time, *i.e.*, the rate of change. Hence the *e.m.f.* is equal to the rate of change of flux, or the rate at which lines of force are cut. From equation (119) we see that the *e.m.f.* in *e.m.* units is unity when one line of force is cut per second. Since the volt is 10^8 times this unit, the *e.m.f.* is 1 volt when a conductor cuts lines at the rate of 100,000,000 per second, or when that is the rate of change of flux.

It should be noted in Fig. 96 that the movement of the slider causes a current which tends to move the slider in an opposite direction. This is in accordance with Lenz's law, and therefore force must be used in moving the slider and energy must be expended in producing the current.

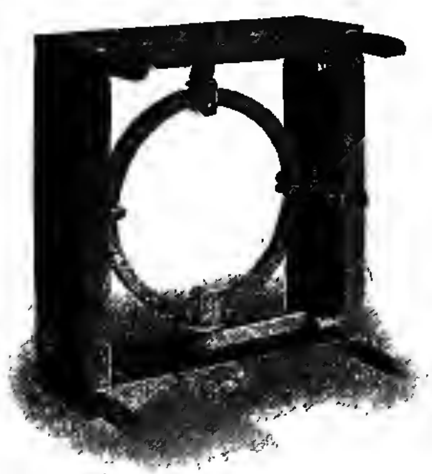

FIG. 97.

An interesting illustration of induction by cutting lines of force is found in the use of the earth inductor. This, as shown in Fig. 97, consists of a coil of insulated wire having several hundred turns mounted so that it may be rapidly turned through 180° in the earth's magnetic field.

As shown in Fig. 39, the earth's field may be regarded as having a horizontal and a vertical component, each of which may be found separately. Let the inductor be set so that it will when rotated cut only the horizontal component. Let a be the area in square centimetres of each turn of the coil and H the number of lines per square centimetre in the field. If there are n turns in the coil, the total area is na. The coil is first set so that its plane is at right angles to the field. When it turns through 90° the total change of flux is Han. The same change of flux will occur in the next quarter revolution. The total change is therefore $2Han$. If the half revolution is accomplished in time t the *e.m.f.* induced is

$$E = \frac{2Han}{t} \qquad (120)$$

for this expresses the rate of change of flux. Therefore by Ohm's law, equation (50),

$$i = \frac{2Han}{Rt} \tag{121}$$

and this is the strength of current, as measured in electromagnetic units, that would flow through any circuit connected to the terminals of the coil, R being the resistance of the entire circuit including the wire on the coil.

Since the quantity of electricity is $Q = it$, then

$$Q = \frac{2Han}{R} \tag{122}$$

and
$$H = \frac{QR}{2an} \tag{123}$$

where H is the strength of the horizontal component of the earth's field in gausses. The vertical component may be found in the same manner by placing the inductor so that it will cut lines only in that direction.

To find Q the terminals of the inductor coil may be connected to a ballistic galvanometer. The commutator of the inductor is so made that the circuit is broken the instant the coil completes its turn through 180°. The throw of the galvanometer may then be taken as proportional to the quantity Q of electricity sent through it. A standard condenser may then be made to cause the same throw, and from a knowledge of the capacity C of the condenser and the *e.m.f.* used in charging it, all reduced to *e.m.* units, the value of Q is found by equation (6).

104. The Principle of the Dynamo.—A dynamo is a device for producing electric currents according to the principles of electromagnetic induction. In any dynamo, then, conductors are made to cut lines of force either by moving the conductors across the lines or by moving the magnets so that the lines will cross the conductors. Let the coil *abdc*, with terminals at *e* and *f*, Fig. 98, be rotated in a clockwise direction in a magnetic field. The ends *bd* and *ac* will not cut any lines and so we need consider only *ab* and *cd*. Since *ab* is moving downward across lines of force, and *cd* is moving upward, the direction of the induced *e.m.f.* will,

according to the dynamo rule, be from b to a and from c to d—*i.e.*, it would, if e and f were joined by a conductor, cause a current to flow around the loop. It is evident, however, that, although the speed of rotation is uniform, the rate of cutting lines is different at different points in the rotation. Let the plane of the coil be in such a position that it makes an angle θ with a perpendicular to the field. Let the velocity of ab be indicated by v and represented in Fig. 98 by the vector ay. This vector may be resolved into the components xy perpendicular to the field and ax parallel to the field. But

$$xy = ay \sin\theta = v \sin\theta \tag{124}$$

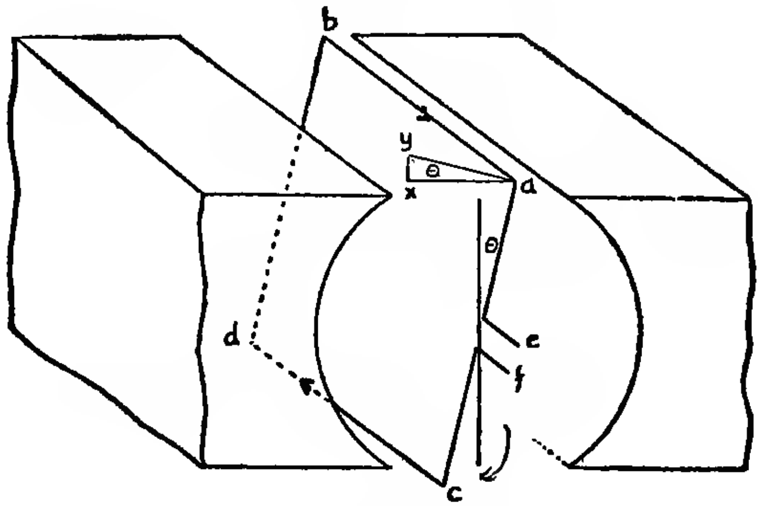

FIG. 98.

Hence the velocity at right angles to the field is, for any position of the coil, $v \sin\theta$. If l is the length of ab and H is the strength of the field, then for both ab and cd,

$$e.m.f. = 2Hlv \sin\theta \tag{125}$$

When the plane of the coil is parallel to the field, $\theta = 90°$ and $\sin\theta$ is unity, and lines are then being cut at the maximum rate. Equation (125) then becomes identical with (119), for v is the distance moved directly across lines of force in unit of time. When the plane of the coil is at right angles to the field, θ becomes zero and so the *e.m.f.* is zero, *i.e.*, no lines are being cut when the coil is in that position.

During the first quarter of a rotation the *e.m.f.* increases, for both ab and cd are moving into positions where they cut lines at a maximum rate. During the second quarter the *e.m.f.* decreases

to zero. At the beginning of the third quarter the direction of the *e.m.f.* is reversed in the coil, as an application of the dynamo rule will show, and will increase to a maximum in that direction, returning again to zero at the end of the last quarter. These changes when plotted give a sine curve as shown in Fig. 99, where the abscissæ are the successive angles made by the coil with a plane perpendicular to the field and the ordinates are the corresponding values of the *e.m.f.* as calculated by use of equation (125).

If now a metal ring is attached to each terminal *e* and *f* of Fig. 98 and made concentric with the axis of the coil but insulated from it, then the circuit of the coil may be closed by an external conductor terminating in brushes that rest on the rings.

The *e.m.f.* induced by rotating the coil will now cause a current of electricity to flow in the circuit in one direction during the first half of the rotation and in an opposite direction during the second half, *i.e.*, the current through the coil and the external circuit will be alternating, changing direction twice in each rotation of the coil.

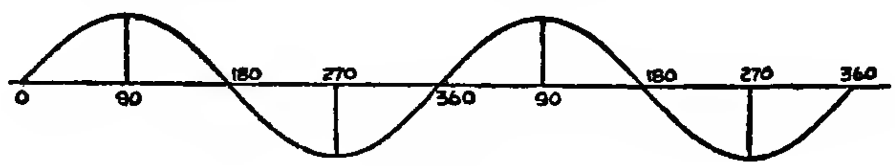

FIG. 99.

A complete set of changes is called a *cycle*. The curve shown in Fig. 99 is for two cycles. In the arrangement shown in Fig. 98 there would be as many cycles as there are rotations of the coil. Sixty rotations per second or 3600 R.P.M. would give 60 cycles per second. The number of cycles per second is called the frequency of the alternating current.

A very common frequency in use for both power and light is sixty, and where the current is used for power only, twenty-five cycles per second is in common use.

To secure a frequency of 60 it would be necessary to rotate the coil or armature of Fig. 98 very rapidly. This would be objectionable for mechanical reasons. The same frequency may be obtained at a lower rate of rotation by using more poles to produce the field through which the coils of the armature are made to pass. This is shown in Fig. 100 where the electromagnets

producing the field are so wound that they are alternately N and S poles when a direct current is made to flow through them. The armature is a drum of soft iron with a slotted surface. In these slots coils of wire are wrapped in manner shown in Fig. 100, B, there being as many coils as there are poles in the field. The coils are a continuous conductor wrapped back and forth on the surface of the drum, the direction of the wrapping being reversed in each successive coil so that, when one is passing an N pole of the field and the next an S pole, the directions of the induced *e.m.f.* will not oppose each other but will produce a current in the same

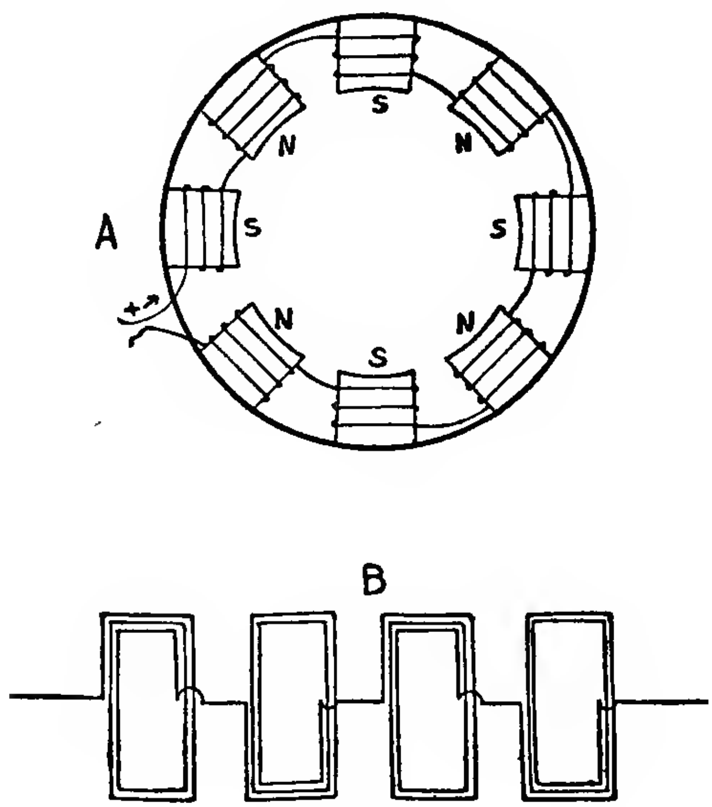

FIG. 100.

direction throughout the wire of the armature coils. The direction of the current in the armature will be reversed each time the coils pass opposite poles and so there will be as many alternations as there are poles and as many cycles as there are sets of N and S poles. For example, if there are eight poles there would be four cycles in each rotation of the armature and sixty cycles per second would be produced by 900 R.P.M.

In the machines just described the field magnets are stationary and the coils into which the current is induced are moved rapidly

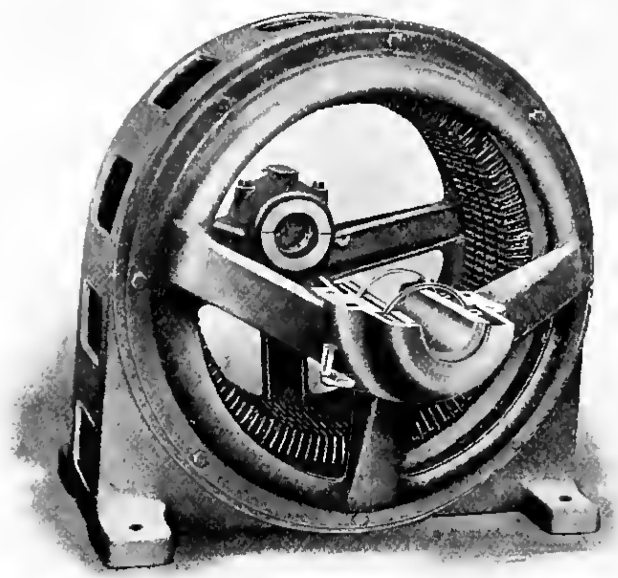

Fig. 101.

in the magnetic field. All that is required, however, for induction is that there be relative motion of coils and field. The coils may be stationary and the field magnets in motion. It is found to be of considerable advantage both in construction and operation to build alternating generators in this manner. In this case the armature coils are placed in the slots of a heavy iron frame as shown in Fig. 101. This part of the machine is called the *stator*. The *rotor* is shown in Fig. 102 and consists of the field coils mounted on an axis. This is placed within the stator and magnetized by a direct current from a separate dynamo called the exciter. This current passes around the poles in such a direction as to make them alternately N and S poles. When the rotor is turned we have, then, the same relative motion of magnets and armature as in the previous case. The induced current flows directly from

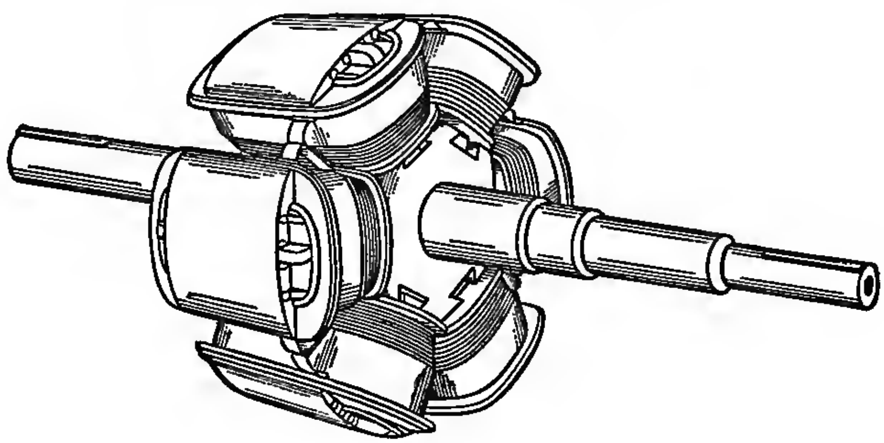

FIG. 102.

the stator and the coils of the armature can be more effectively insulated and are not so liable to mechanical injury as when they constitute the moving part.

105. The Direct Current Dynamo.—In all dynamos the current induced in the armature coils is an alternating one. This current, however, may be made unidirectional on the line leading out from the dynamo. This is done by use of a commutator, the principle of which is shown in Fig. 103. Here a copper or brass ring is split into equal parts and insulated. This is mounted on the shaft of a rotating armature.

Let the terminals e and f of Fig. 98 be attached one to each segment of the commutator. Then, as the coil rotates, the brushes a and b, Fig. 103, will slide from one segment to the other each

time the current is reversed, and so the current which passes out on the line will always be in the same direction, *i.e.*, the current will be *direct*, usually indicated by D.C. while the *alternating current* is indicated in writing by A.C.

Although the commutator just described produces a direct current, it by no means produces a constant current, *i.e.*, a current having the same intensity at all times. A plot such as that of Fig. 97 would show that through 180° the current rises from zero to maximum and then back to zero, but during the next 180° this is repeated, producing a curve like that in Fig. 104.

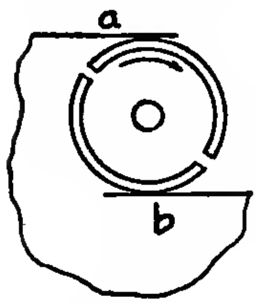

FIG. 103.

The two most common kinds of armatures are called the ring armature and the drum armature. The former is often known as Pacinotti's ring or the Gramme ring. It consists of a bundle of wires or laminated iron upon which coils of insulated wire are wound as shown in Fig. 105. There are as many segments of the commutator as there are coils on the ring, and the coils are connected to each other so that they form a continuous circuit.

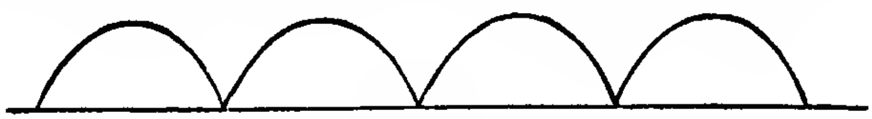

FIG. 104.

Because of the permeability of the iron ring the lines of force from N to S pass through the material of the ring and there are practically none in the space within the ring. When, therefore, this armature is rotated, only that part of the coils on the outside of the ring will cut lines of force. Let the rotation be in the direction indicated by the arrow. Then, applying the dynamo rule, it will be seen that a current will flow through the coils on both the right and left sides of the ring toward the brush a and from the brush b. This causes a difference of potential between a and b and consequently a flow of electricity on any circuit terminating in these brushes.

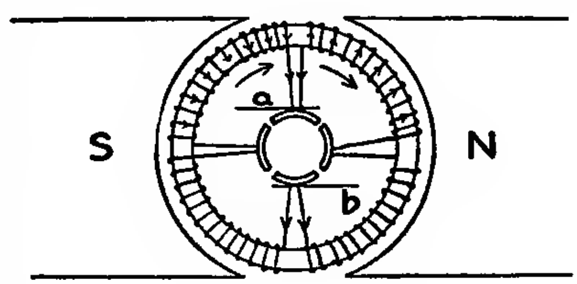

FIG. 105.

Fig. 106.

Coils at the upper and lower points of the ring are not cutting lines of force, while those coils 90° from these points are cutting lines at the maximum rate. Intermediate coils are approaching or receding from these maximum points. Since there are many coils on the ring, one scarcely has passed the point of maximum cutting before the next has come into that position. The *e.m.f.* will therefore never fall to zero as in Fig. 104. The current will be direct and at the same time more nearly constant.

The drum armature is the one now in common use. It consists of a laminated iron core in form of a cylinder upon which coils of wire are wrapped from end to end on the outside of the drum only. Thus all strands of the wire, except where they cross the ends of the drum, will cut lines of force. An armature of this kind is shown in Fig. 106. Its operation is exactly the same as that of the ring.

106. Eddy Currents.—We have seen that whenever a conductor is moved across magnetic lines of force a current is induced in it. This is true not only of copper wires but of any conducting material. In a mass of metal of considerable size currents are thus made to circulate within the body of the metal. These are called Eddy or Foucault currents and may be illustrated by use of the apparatus in Fig. 107, which consists of an aluminum disc mounted so that it may be rotated between the poles of an electromagnet. Consider only a radial line from the centre of the disc down to the bottom. This line may be regarded as a conducting wire which is made to cut lines of force when the disc is rotated. Under conditions indicated in the figure, a current flows on this radial line toward the periphery of the disc and will return to the centre on other parts of the disc which are not passing the magnet. Thus an eddy current is made to circulate in the disc. Such an arrangement as this was first constructed by Faraday and is known as the disc dynamo, for a copper wire connecting the axis

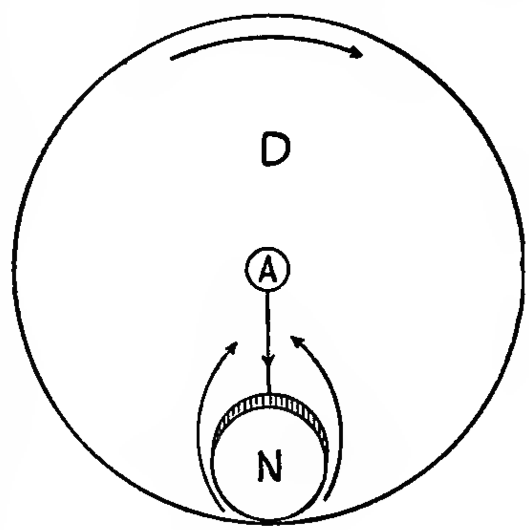

FIG. 107.

to the periphery will contain a current when the disc is rotated. It will also be noticed that it requires considerable force to turn the disc in the magnetic field. This is in accord with Lenz's law that the current produced opposes the motion producing it. Herein lies the principle of electromagnetic damping used to advantage in many instruments, *e.g.*, in D'Arsonval galvanometers to render them " dead beat " and in watt-hour meters to make the speed of rotation proportional to the energy measured.

In dynamos, however, eddy currents in the iron cores of the armatures are a great hindrance to the efficiency of a machine for they heat the iron and resist the force turning the armature. All the energy expended in doing this is a complete loss for these currents are circulating in the body of the core and do not enter the coils. It is for this reason that cores of armatures are made of numerous thin laminæ placed side by side and separated by shellac, thin paper, or iron rust. This does not interfere with magnetic lines but prevents eddy currents which tend to flow at right angles to the planes of the lamina.

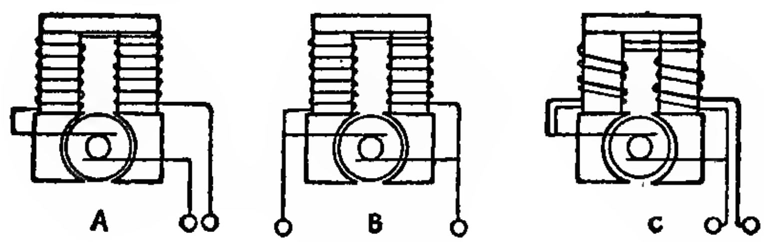

Fig. 108.

For reasons given above, this laminated iron structure will be found wherever a changing magnetic field would cause objectionable eddy currents, for example, in armatures of dynamos and motors, in pole pieces of the same, and in cores of transformers and induction coils.

107. Winding of D.C. Dynamo.—The field magnets of a direct current dynamo are usually excited by the current from the dynamo itself. For this purpose one of three different styles of winding may be used, depending on the conditions under which the dynamo is to be used. (1) Series winding, Fig. 108, *A*, where the coil on the field magnet is in series with the line. This is suited to a line on which the load is constant, for it is evident that any change in the quantity of current flowing in the line would also change the strength of the magnetic field, and consequently

the voltage would not be constant. (2) Parallel or shunt winding, Fig. 108, *B*, where the circuit is divided, part flowing through the field and part on the line. By means of a rheostat placed in series with the field coil it is possible to regulate the strength of the magnet and thus to maintain a nearly constant voltage. (3) Series-parallel winding which is a combination of the first two, Fig. 108, *C*. This arrangement will automatically regulate the voltage, for any change in the line current that would raise or lower the P.D. of the brushes is compensated by the few turns of the line on the field magnets.

The current in the armature causes a magnetic field at right angles to that of the field coils. The combination of the two produces a resultant field. If *Ox*, Fig. 109, is the strength of field in which the armature rotates and *Oy* that produced by the armature, then *Or* is the resultant field and the brushes should be set so that a line joining them is perpendicular to this resultant.

108. Polyphase Generators.—The alternating generators which have been described are single-phase machines. In them one cycle is completed before another is begun. By phase is meant the position of any point in a cycle of operations. The cycle is usually regarded as divided into 360°, so we may speak of a 30° phase or 90° phase, etc. In this way the progress of a point in a cycle may be indicated, or, in comparing two cycles, the difference in phase may be denoted by the same method, *e.g.*, if one cycle differs from another by 90°, then one is one-fourth completed before the other begins and we say there is a phase difference of 90°.

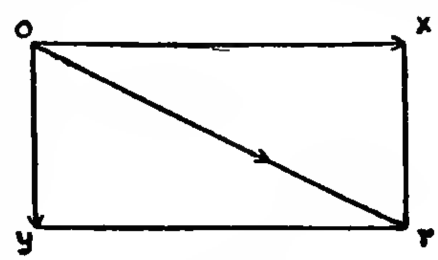

FIG. 109.

In single-phase generators, Fig. 98, the difference of phase of successive cycles may be said to be 360°, each beginning where the preceding one stopped, but in polyphase generators separate coils are placed in such position on the armature that the cycle of one is not completed before that of a second or third coil has begun. It is possible to so wrap the armature that the cycles may differ by only a few degrees, but in practice there are but two kinds of polyphase machines—the two-phase and the three-phase—the phase difference in the former being 90° and in the latter 120°.

In a two-phase, two-pole generator there are two independent coils on the armature placed at right angles to each other. This condition would be represented by Fig. 98 if the armature contained another loop at right angles to the one already there. There would then be four separate terminals and four rings on the shaft. While one coil is cutting lines of force at a maximum rate and consequently producing maximum *e.m.f.*, the *e.m.f.* in the

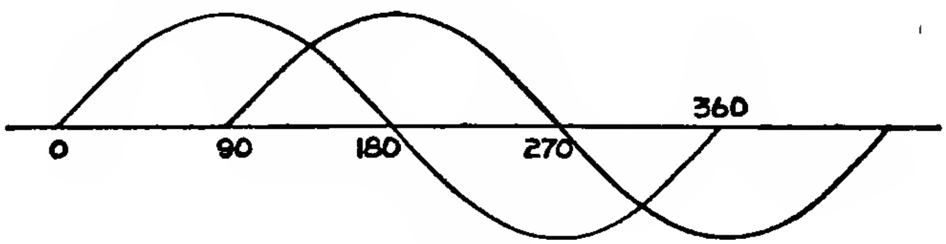

FIG. 110.

other coil is zero, but after the armature has turned 90° the *e.m.f.* of the first coil becomes zero and that of the second maximum. This is represented by the curves in Fig. 110, where it is observed that any point in one cycle differs by 90° from a corresponding point in the other, *i.e.*, the phase difference is 90°.

The coils of the two-phase generator may be represented by the lines *ab* and *cd* in Fig. 111, *A*, where a conductor leads out from each of the four collector rings. A single-phase alternating current is produced by each coil and the two currents are related as explained in Fig. 110.

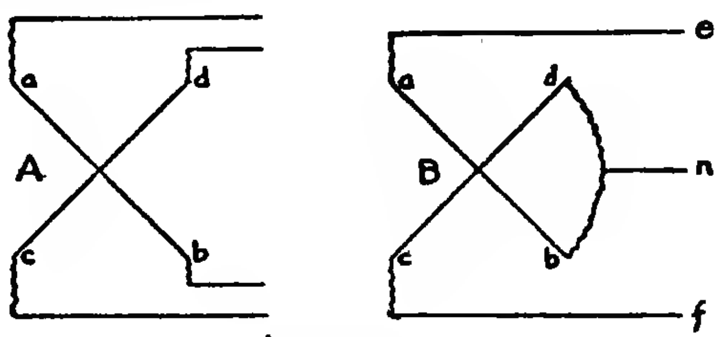

FIG. 111.

Instead of the four-wire system just explained it is more common to connect the two coils in series as shown in Fig. 111, *B*, where *d* and *b* are connected to a common ring. This gives a three-wire system in which one wire serves as a common return for the other two, the difference of potential between *e* and *n* or *f* and *n* being the same as if the coils were separate.

In the three-phase generator three coils are so placed on the armature that the *e.m.f.* of the first passes through 120° of its cycle before the second coil begins its cycle, and the second through 120° before the third begins. Thus the difference of phase is 120°. This is represented in Fig. 112. Since there would be six terminals of the three coils, there would have to be six rings and six brushes

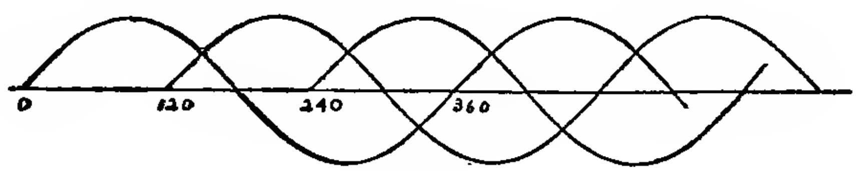

FIG. 112.

if these three currents are to be conducted away and treated as separate single-phase currents. This, however, is not the purpose in the construction of polyphase generators, as will be shown later in describing induction motors.

Let the coils of a three-phase machine be represented by Fig. 113, *A*, and let the like terminals, as *b*, *d* and *f*, be joined and con-

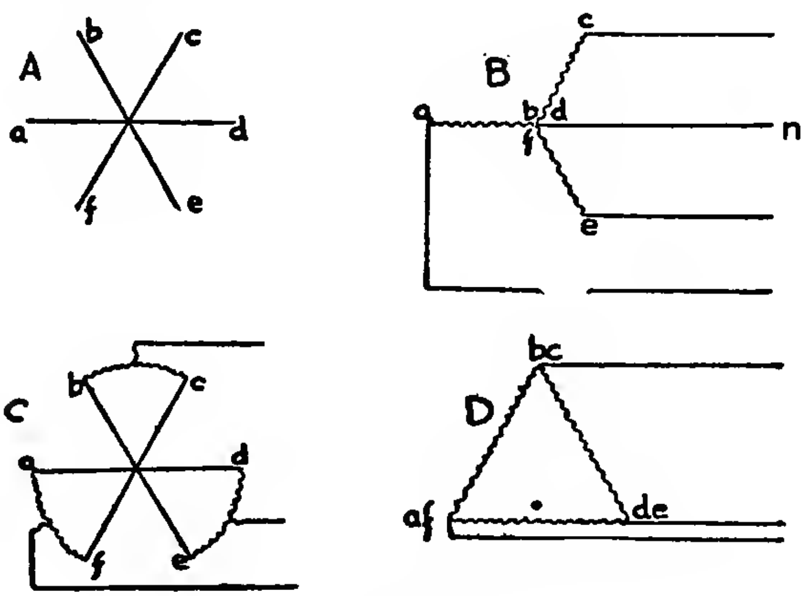

FIG. 113.

nected to a ring, the other terminals being connected each to a separate ring. There would thus need to be but four rings and four distributing wires, the wire *n* serving as a common return for the other three. If there is the same load on each of these three circuits, *i.e.*, if the system is balanced, no current will flow

on *n* and so this wire need not be used. We then have a three-phase, three-wire system. This is often called the Y connection because of the form of the diagram, Fig. 113, *B*.

Another arrangement called the *delta* connection is shown in Fig. 113, *C*, where the coils are connected so as to form a closed circuit, as *a* to *f*, *c* to *b*, and *e* to *d*. This again gives a three-phase, three-wire system.

109. The Induction Motor.—The chief use of the dynamo current in its early applications was in the production of electric light. For this purpose the single-phase current is as good as any, but there soon arose a demand for electric current for power purposes as well as for light. The operation of an induction motor requires that there should be a rotating magnetic field and such

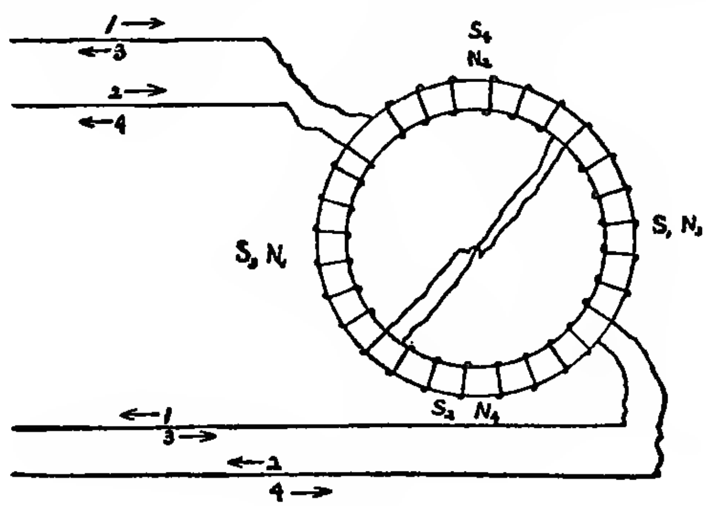

FIG. 114.

a field can be produced by use of a polyphase current. To explain such a field let us make use of a four-wire two-phase current explained above. A ring of iron, Fig. 114, is wrapped with four coils and the terminals connected as shown in the figure. There are two independent coils on the generator and when the current in one is maximum that in the other is zero, and while the current in one is increasing that in the other is decreasing. Also the direction of the current in each coil is reversed at the middle and end of each cycle. Keeping these facts in mind, suppose the current is maximum on the line 1, Fig. 114, and flowing as the arrows show. The current in line 2 is at that instant zero. An inspection of the figure will show that the iron ring is thus magnetized so that its north-seeking pole is at N_1 and the opposite pole at S_1.

As the armature of the generator continues to turn, the current in 1 decreases and that in 2 increases, thus moving the north-seeking pole N_1 along the ring to N_2. Current 2 then decreases and 1 is reversed on the first line, becoming 3. This moves N_2 to N_3. The current 3 then decreases and 2 is reversed, becoming 4. Thus N_3 is moved to N_4. This operation is then repeated and the N and S poles rapidly rotate on the iron ring, producing what is called a rotating field. By wrapping the iron ring with three coils and connecting to the wires from a three-phase generator, a similar rotating field is produced. Such is the field required for an induction motor. The ring and its coils are known as the *stator*. The armature or *rotor* is in form of a cylinder composed of two metal discs with copper strips from one disc to the other at regular intervals around the entire circumference. A simple form of such rotor is shown in Fig. 115. Because of its form it is often spoken of as a squirrel cage armature. The interior of this cylinder is filled with laminated iron to increase the density of the magnetic flux in the region of the cross bars. If, now, this armature be mounted in the rotating field of Fig. 114, a current will be induced in the copper bars in accordance with the principles of electromagnetic induction, for a moving field is just as effective in producing an induced current as the movement of a magnet which carries its field with it. The direction of the induced current is, according to Lenz's law, such as to oppose the relative motion of the field and rotor and so the rotor will turn with the field but at less speed, for if there were no relative motion there would be no induction. In a motor of this kind no current from an outside source is sent into the armature.

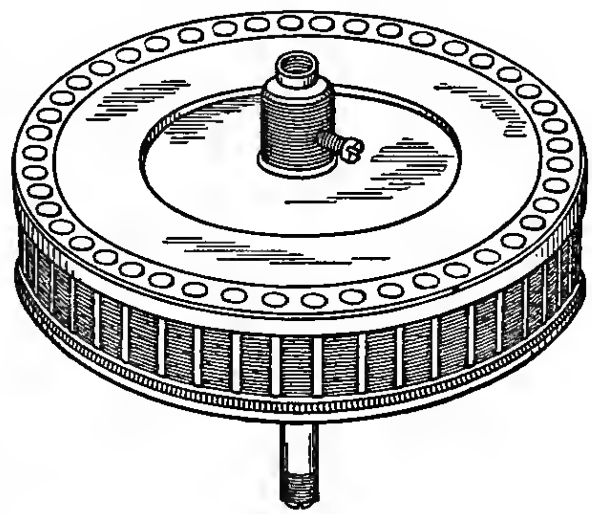

FIG. 115.

110. Synchronous and D.C. Motors.—A single-phase current cannot be made to produce a rotating magnetic field, hence if a single-phase generator is to be operated as a motor it is necessary

that both the generator which is producing the current and the motor which is receiving it be running at the same speed—*i.e.*, the speed of rotation of the armature in the motor must be such that it would produce the same number of cycles per second, if it were operating as a generator, as are being produced by the generator that is supplying current to the motor. A motor running in this way is said to be in *synchronism* with the generator and is called a synchronous motor. Such a motor is not self-starting. Some outside source of power is needed to first give the armature the proper speed, and any excessive load that causes the motor to lose step will stop it, hence synchronous motors are best suited to continuous and unchanging loads.

In regard to D.C. motors little need be said, for they do not differ in principle from D.C. generators. The same machine may often be put to either use. A current passed through the coils of an armature, in the same direction as that in which the current flows when the machine is driven as a generator, will turn the armature in the opposite direction. All motors operate as dynamos and produce a counter electromotive force which opposes the current that operates the motor. This, however, is only another statement of Lenz's law.

111. Self-induction.—It has been shown that a conductor in which a current flows is always surrounded by a magnetic field. Let the diagram Fig. 116 represent a cross section of such a conductor and let the direction of the lines of force, as shown, be that of a current entering the paper. When a current is started in the conductor in this direction a magnetic field moves out in all directions. The effect is the same as if the field were stationary and the conductor moved in the opposite direction. Now apply the dynamo rule by holding the first finger tangent to a circle of the field and pointing in the direction of the arrows, the thumb pointing in the direction the conductor may be assumed to have moved relative to a growing field. The second finger will then point toward the reader, *i.e.*, in a direction opposite to that in which the current flows. If the current is now stopped, the field will gradually collapse and move in from all sides to the conductor. An application of the above rule, with the thumb pointing in the opposite direction, will show that the induced *e.m.f.* tends to cause

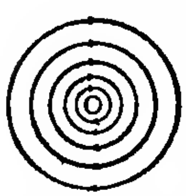

Fig. 116.

a current in the same direction as that which is already flowing. Thus when a current is being started or stopped the electromagnetic effect is in opposition to the change, checking a starting current and prolonging a stopping one. After a current once reaches its full strength no such effect is observed as long as there is a steady flow. These phenomena do not differ in principle from others already described, but here the induction is in the conductor itself and so is called *self-induction.*

In a short wire self-induction is slight. In a long, straight wire it is greater than in the short wire. If the long wire is formed into a compact coil of many turns, self-induction may become very strong, for the field built up around each strand overlaps the other strands of the coil. An iron core within the coil still more increases the induction. In any coil, then, the self-induction will depend on the strength of current, i, the dimensions of the coil, and the presence or absence of an iron core. For any given coil without iron core the number of magnetic lines set up is proportional to the strength of current. A unit current in the given coil will cause a certain number of lines which we will designate by L. If the strength of current is i units, the number of lines will be iL. We may then write the equation

$$N = Li \tag{126}$$

where N is the total number of lines in the coil and L is, for that coil, a constant called the *inductance* or the *coefficient of self-induction.* We may write (126) in the form

$$L = \frac{N}{i} \tag{127}$$

and define inductance as the number of lines of induction caused by unit change of current.

If both members of (126) be divided by t, the time during which the current changed, we have

$$\frac{N}{t} = \frac{Li}{t} \tag{128}$$

The first member of (128) is the rate of change of flux and hence is equal to the *e.m.f.* produced, hence if i is unity and t is one

second, $e.m.f. = L$. Inductance may then be defined in terms of units of *e.m.f.* caused by unit change of current per second. In formula

$$e.m.f. = L\frac{i}{t} \tag{129}$$

If i is one *e.m.* unit of current then L is equal to one unit of *e.m.f.*

The practical unit of inductance is that self-induction of a circuit which will cause a counter *e.m.f.* of one volt when the current changes at the rate of one ampere per second. This unit is called the *henry*. Since the volt is 10^8 *e.m.* units and the ampere 10^{-1}, it is seen from (129) that the henry is 10^9 *e.m.* units of inductance. The henry is for some purposes inconveniently large and so the millihenry is often used instead.

If a coil contains iron the change in induction is not proportional to the change in current, for the permeability of iron is not constant (equation 32). Hence equation (129) will not hold for this condition.

A good illustration of self-induction may be given by use of apparatus shown in Fig. 117. A large electromagnet is put in circuit with a battery and key. A suitable electric lamp L is shunted across the terminals of the magnet and will be heated to a dull glow while the circuit is kept closed but glows brightly at the moment of opening or closing the circuit. While the key is being held down the strength of current in the magnet and lamp will be inversely as the resistance in their respective circuits, but at the moment of closing the circuit the counter *e.m.f.* of self-induction, which is large in the magnet, checks the flow there, and consequently more current flows through the lamp from a to b during the short time required for the current to set up a magnetic field in the coils.

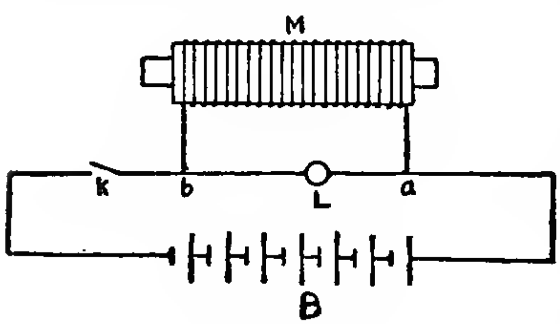

FIG. 117.

When the current is broken, the energy of the field is converted back into that of a current flowing in the same direction as the original current, and so will pass through the lamp from b to a.

The energy of a magnetic field may be estimated if L and i are known. The quantity of electric energy W is

$$W = QE \tag{130}$$

where Q is the quantity of electricity and E is the *e.m.f.* If the current begins at zero and rises uniformly to its maximum value i in time t, the quantity is

$$Q = \tfrac{1}{2} it \tag{131}$$

Also if L is the inductance of the coil, then the number of lines induced would be Li and the rate of change in these lines, *i.e.*, the *e.m.f.* would be

$$E = \frac{Li}{t} \tag{132}$$

Substituting these values of Q and E in equation (130),

$$W = \frac{Li^2}{2} \tag{133}$$

112. Measurement of Inductance.—There are various methods of determining by experiment the inductance of a circuit. One method is illustrated in Fig. 118. The principle involved is that of a balanced Wheatstone bridge. The lines R_1, R_2, and R_3 represent resistance boxes in which the coils are so wound that they are practically without self-induction and without capacity. L_1 is a fixed standard of self-induction, the number of millihenrys being known. L_2 is a variable standard and is constructed, as shown in Fig. 119, of two concentric coils one of which may be turned to any angle with the other, and the top is so graduated that the pointer indicates the number of millihenrys.

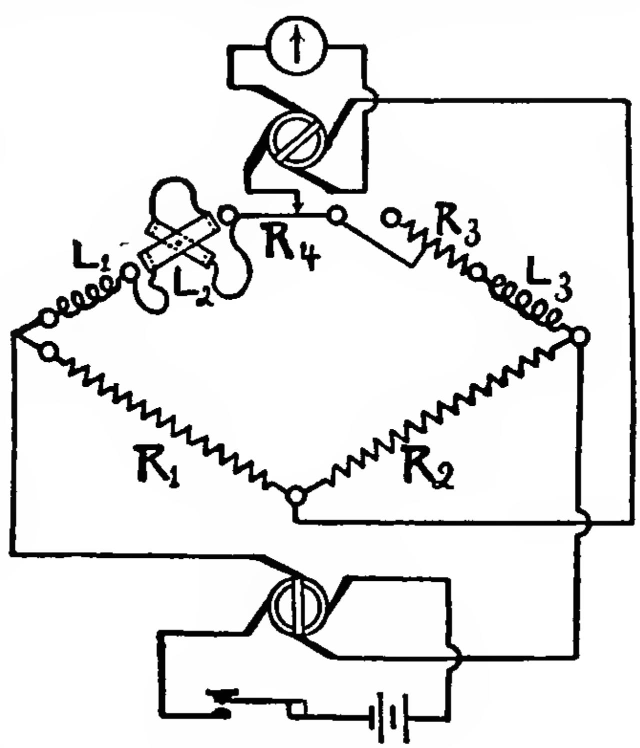

FIG. 118.

Now if a steady current be sent through the bridge, there will be no self-induction as long as the strength of the current remains unchanged, and the bridge will be balanced when $R_1 : R_2$ as the ohmic resistances of the two corresponding upper arms are to each other. The slide wire R_4 facilitates the exact adjustment of this balance. If now the current be made and broken or rapidly reversed in direction there will be self-induction in L_1, L_2, and L_3 which will

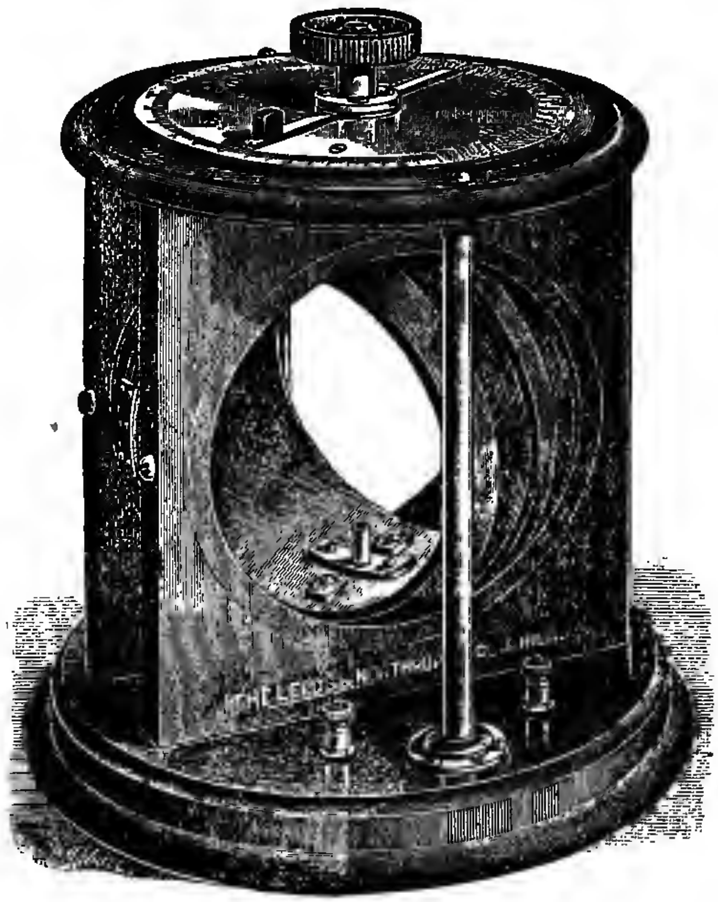

FIG. 119.

set up a counter *e.m.f.* and check the passage of the current. Therefore, unless the self-induction in these two arms of the bridge bear the same ratio to each other as $R_1 : R_2$, the balance will be disturbed. The operation then consists in adjusting L_2 until the galvanometer shows no deflection, *i.e.*, until the bridge is again balanced. Then

$$\frac{R_1}{R_2} = \frac{L_1 + L_2}{L_3} \tag{134}$$

$$\therefore L_3 = \frac{R_2}{R_1}(L_1 + L_2) \tag{135}$$

A reversal of current through the bridge may be obtained by use of a sechometer shown in Fig. 120. Two commutators, one on each side, are attached to the same shaft and upon each of these rest four brushes in the manner shown in Fig. 118. A study of the diagram will make it clear that while one commutator will cause an alternating current in the arms of the bridge, the other will rectify and make unidirectional any current that passes through the galvanometer. By rapidly turning the sechometer to make the pulsations very frequent, the effect on the galvanometer will be practically the same as that due to a direct current,

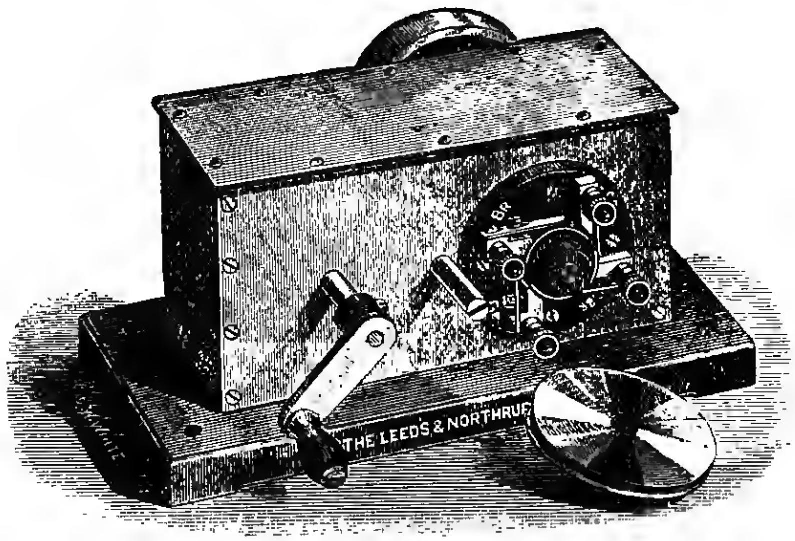

FIG. 120.

and when a balance is secured there will be no deflection. The inductance in L_3 may then be calculated by equation (135), the other terms being known.

The inductance of a coil may also be calculated by use of the equation

$$L = \frac{4\pi^2 r^2 N^2}{l 10^9} \tag{136}$$

where N is the total number of turns, r is the radius of the coil in centimetres and l is the length in centimetres. The value of L will be in henrys. This is rigidly correct only for long thin coils but is useful in finding the approximate value for shorter coils. If the coil contains an iron core the value found by the equation above must be multiplied by μ, the permeability.

113. Mutual Inductance.—The discussion in § 112 relates only to induction in a coil resulting from the variation of the strength of current in that coil and so is called self-induction. When, however, there are two coils and a change of current in one induces a current in the other, the phenomenon is called *mutual induction.* The coil in which the current flows is called the primary and that in which a current is induced is called the secondary. The induction depends on the dimensions, number of turns, and relative position of primary and secondary, but for any fixed combination of two coils without iron core the induction in the secondary is proportional to the change of current in the primary.

The *coefficient of mutual induction*, or *mutual inductance*, is also measured in henrys, and the henry is here defined as the mutual induction between two coils when a change of one ampere per second in the primary induces an *e.m.f.* of one volt in the secondary.

114. The Induction Coil.—An explanation of the induction coil involves a consideration of both self-induction and mutual induction. In structure the coil consists of a bundle of soft iron wires on which are wound one or two layers of coarse wire. Wires are used in the core instead of solid iron to prevent eddy currents. Comparatively few turns of wire are used on the primary coil to avoid excessive self-induction. The battery current must pass in circuit through the contact point *p*, Fig. 121, and the primary coil. When a current begins to flow the iron core is magnetized and pulls the hammer *h* toward it. This breaks the circuit at *p*, the core loses its magnetism, and the hammer returns to its first position, thus again making contact at *p*. The circuit is thus automatically made and broken, and as a result a high *e.m.f.* is induced in the secondary which consists of many turns of fine wire wrapped in form of a coil concentric with the primary.

We have seen that the magnitude of mutual induction depends on the *rate* of change of current in the primary. When the circuit is closed at *p* a short time will elapse before the current will reach its maximum strength because of the counter *e.m.f.* of self-induction in the primary coil. Likewise, for the same reason, a certain time must elapse before the current will fall to its zero value when the circuit is broken. A bright spark is observed at the contact *p*

when the circuit is broken. This was called by Faraday the "extra current." It should be noted, however, that a certain amount of energy was required to create the magnetic field when the current was started, and this energy is transformed into current again when the circuit is broken, thus producing the so-called "extra current." The spark prolongs the time of break and consequently diminishes the *e.m.f.* of mutual induction. If, therefore, a long jump-spark is desired between the terminals of the secondary coil, it is necessary to devise some method by which the magnetic field of the primary can be very suddenly changed. One way of doing this is to connect the terminals of a condenser to the opposite sides of the contact point. Then the "extra current," instead of jumping the gap at *p*, is stored in the condenser. The battery alone will charge the condenser while the circuit at *p* is open and the "extra current" adds to this charge. The *e.m.f.* of the battery is not sufficient to maintain such a charge and, before contact is again made at *p*, the condenser is partly discharged through the battery and primary in a direction opposite to that in which the battery current was flowing. Thus the magnetic field falls as a result of the broken circuit and at the same time is swept out by the reversed current from the condenser. The condenser therefore serves the double purpose of shortening the time of break and of sending a reversed current through the primary before the circuit is again closed. The rate of change of lines of force through the secondary is therefore very great when the circuit is broken but not when it is closed. The induced current is therefore practically unidirectional.

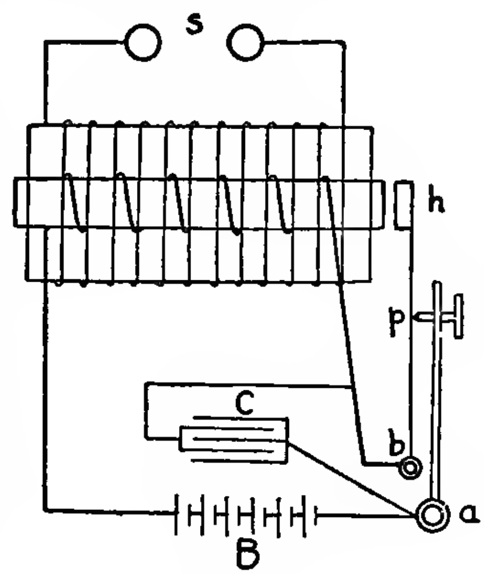

Fig. 121.

115. Alternating Current Transformer.—The action of an alternating current transformer is similar in principle to that of an induction coil, *i.e.*, the principle of mutual induction. Here the two coils are embedded together in laminated iron, and the lines of force set up by the primary do not reach out into air but have

a complete circuit in iron. The alternating current is constantly changing so that the magnetic field produced rises and falls during a half cycle, then reverses in direction, rising and falling in the second half. This change of flux in the primary also occurs in the secondary coil for they are near, one to the other, and on the same iron core. Hence if the secondary forms a closed circuit an alternating current will be induced in it.

In a primary coil of this kind the counter *e.m.f.* of self-induction is very large and, since there are many alternations per second, there is not time for the line current to set up a magnetic field in one direction before the current is reversed. The passage of an

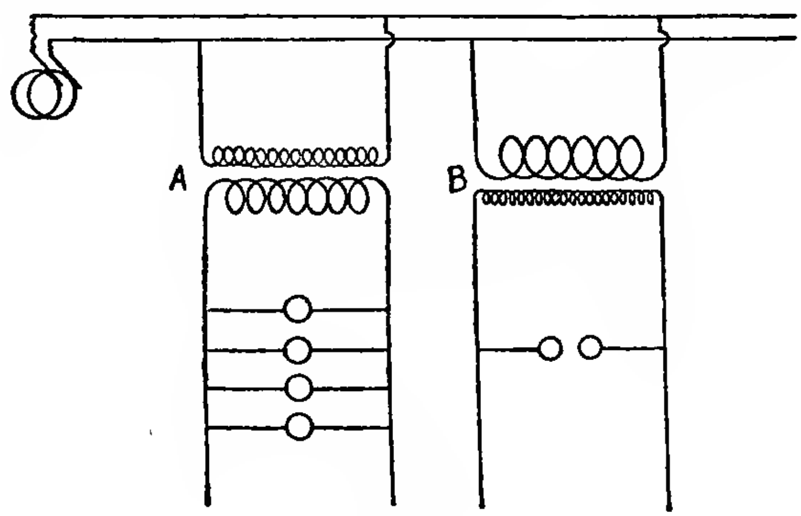

FIG. 122.

alternating current may thus be completely blocked. A coil made for this purpose is often called a *choke coil* or *impedance coil.* The primary of a transformer acts as a coil of this kind and is practically an open circuit as far as the passage of an alternating current is concerned provided the secondary circuit is open.

If, however, the secondary is closed in any manner, as by turning on incandescent lamps, Fig. 122, *A*, then the magnetic field which the primary coil would set up if it were not for the counter *e.m.f.* is at once converted into current in the secondary coil, and this current flows in such a direction that its magnetic field would be opposite in direction to that of the primary. Consequently, just in proportion to the amount of resistance removed from the secondary lines, as by inserting more lights or motors in parallel across the lines, the counter *e.m.f.* in the primary

will be reduced. Hence the flow of current in the primary is automatically adjusted to meet the demand for current on the secondary.

Modern transformers are very efficient in operation, as much as 97 per cent. (in those of the best construction) of the energy delivered to the primary being converted into current in the secondary. The remainder appears as heat in the iron due to eddy currents and hysteresis. The energy therefore remains practically constant in this transformation. But the energy of a current is the product of its *e.m.f.* by the *strength of current.* If either of these two factors is increased, the other must decrease in the same proportion if their product is to remain constant. Herein lies the chief value of a transformer, for the *e.m.f.* of the induced current may be raised to any desired value by increasing the number of turns of wire in the secondary coil, but the strength of the induced current is reduced in the same proportion. For example, if the number of turns in the secondary is ten times as great as in the primary the induced *e.m.f.* will be ten times as great as that of the current in the primary, but the strength of the current will be one-tenth as great. Such are called step-up transformers. On the other hand, if the secondary contains fewer turns than the primary the *e.m.f.* is reduced and the strength of current increased. This arrangement is known as a step-down transformer.

The use of transformers is further discussed in the paragraph on " Distribution of Electricity," § 121.

116. Tesla Transformer.—A high frequency transformer, often called the Tesla transformer, operates on the principle of mutual induction as described above, but the current sent through the primary is one which results from the discharge of a condenser. This discharge is oscillatory. If, for example, one end of a wire is touched to the outer coating of a Leyden jar and the other end is brought near to the knob connected to the inner coating, a bright spark will be seen at the gap and during the short time of the spark an electric charge will surge back and forth many times. The rate of oscillation may be thousands or even millions per second.

The primary coil, *P*, Fig. 123, consists of a very few turns of heavy copper wound on a wooden spool. Within the spool is the secondary coil which consists of a single layer of fine wire

wound on a wooden or paper cylinder. These coils must be well insulated from each other either by an air space or by enclosing the secondary in oil. The primary coil is connected in series with a spark gap *G* and a condenser *C*. A high tension transformer, *T*, which may be an induction coil, or an alternate current transformer which steps the current up to, say 20,000 volts, is joined to the condenser. The condenser will thus be continuously charged and so will continuously discharge itself through the spark gap *G* and the primary *P*. Thus a current of high tension and very great frequency is induced in the coil *S*. This current may be used in a variety of ways such as the excitation of X-ray tubes and the production of many phenomena peculiar to a current of this character.

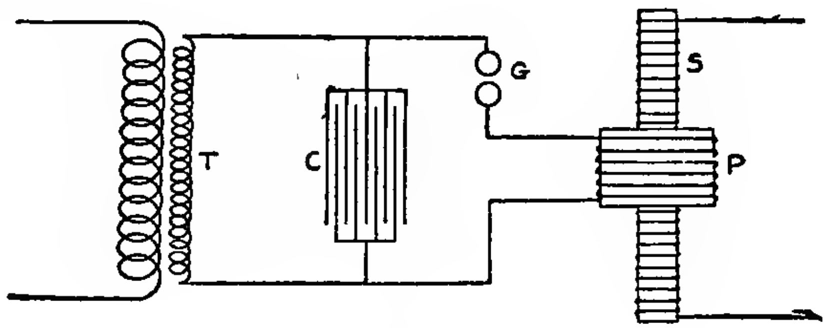

FIG. 123.

117. Effective Value of Alternating Current.—The alternating current is not measured by units based on effects which it as such produces. It is constantly changing in strength and reverses its direction many times per second, hence its unit of strength cannot, for example, be based on an electrolytic effect as in case of the ampere for direct current (§ 53). It is possible, however, to select certain properties common to both the direct and alternating current and, by comparison of the effects produced, to speak of the amperes, volts, etc., of an alternating current. It is then understood that certain effects produced are the same as would have been produced by a direct current of the value given. Direct current units thus used to designate an alternating current are called *virtual* or *effective* units. The properties chosen for this comparison are either the electrodynamic (§ 79) or the heat effects (§ 54). Both of these are proportional to the square of the current i. If then values of i are found at each successive instant throughout, say, one cycle of an alternating current (Fig. 99) and each of

these values are squared, then the square root of the mean of the squares is the virtual current. Likewise the *e.m.f.* that would produce this current is called the virtual *e.m.f.* and is the square root of the mean square of the instantaneous values of *e.m.f.*

A curve plotted in the manner shown in Fig. 99 may be regarded as a harmonic or sine curve, hence $y = r \sin \theta$—*i.e.*, the ordinates representing *e.m.f.* or strength of current are sine functions of the phase, θ, or of the time. By taking the square root of the mean of the squares of all the ordinates from o to 2π—*i.e.*, through one cycle—we obtain the virtual current or the virtual *e.m.f.* It may thus be shown that if i_v represents the virtual current and i_m the maximum current attained in the cycle, then

$$i_v = \frac{i_m}{\sqrt{2}} = .707 i_m \tag{137}$$

or

$$E_v = \frac{E_m}{\sqrt{2}} = .707 E_m \tag{138}$$

where E represents *e.m.f.*

The derivation of these equations is given in Appendix 3.

Alternating current voltmeters and ammeters give the virtual values.

Ohm's law is not applicable to alternating currents for reasons given in the next paragraph, but we may speak of a virtual resistance, R_v, and write

$$i_v = \frac{E_v}{R_v} \tag{139}$$

which bears some resemblance to Ohm's law.

118. Impedance.—The virtual resistance alluded to above is called *impedance*. It includes both the ohmic resistance and that resulting from self-induction.

It is shown in Appendix 4 that for an alternating current having a frequency n and strength i in a circuit of inductance L,

$$\text{counter } e.m.f. = 2\pi n L i \tag{140}$$

The impedance, R_v, is the square root of the sum of the squares of ohmic resistance, R, and the quantity $2\pi nL$, hence

$$R_v=\sqrt{R^2+(2\pi nL)^2} \tag{141}$$

Impedance may, because of self-induction, be so great as to practically prevent the flow of any current, and such is the case in the primary of a transformer and also in a choke coil (§ 114).

It should be noted in equation (141) that impedance increases with increase in frequency n, hence in the oscillatory discharge of a condenser where the frequency may be very great a current will leap across an air gap a, Fig. 124, rather than pass around a single turn of a good conductor.

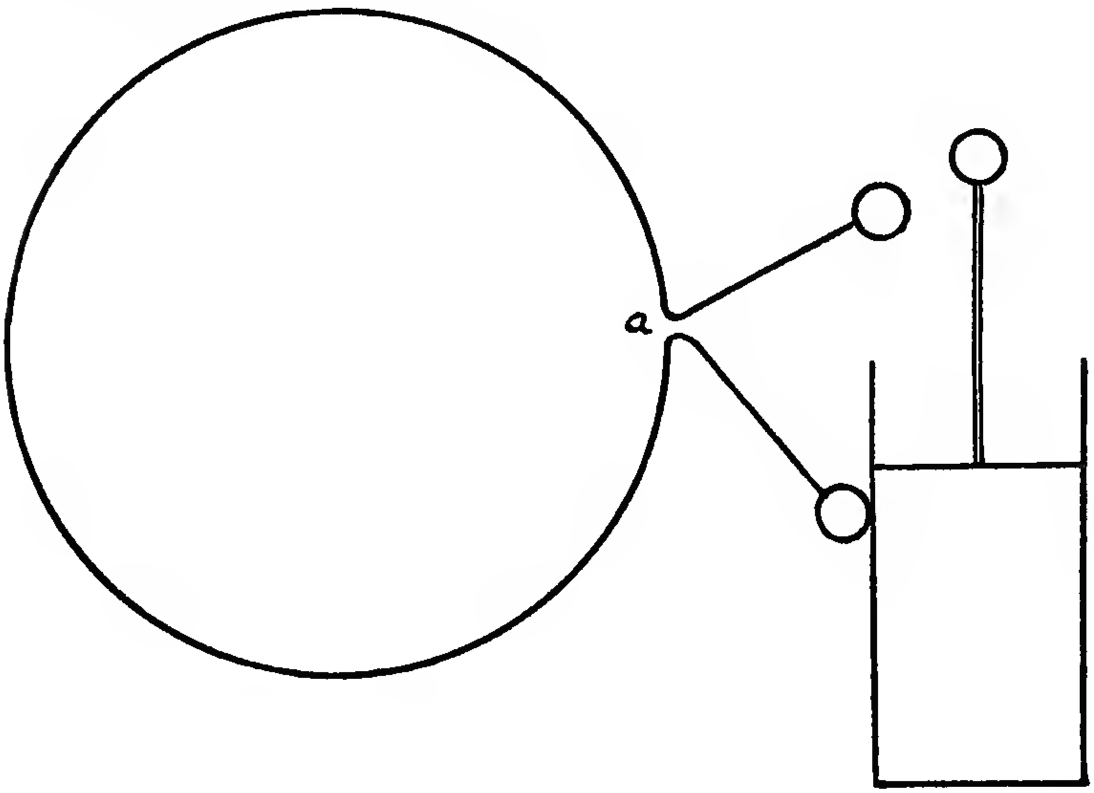

FIG. 124.

The origin of equation (141) will be more clearly seen in the next paragraph.

119. Lag and Lead.—When a circuit contains only ohmic resistance the current and the *e.m.f.* are in the same phase—*i.e.*, as shown in Fig. 125, A, the phase of the current curve i is at all points the same as that of the *e.m.f.* curve e. But when the circuit also contains inductance the current is retarded while it is increas-

ing and prolonged while it is decreasing (§ 111). The effect is to cause the current to lag behind the *e.m.f.* impressed on the circuit. If the circuit contains inductance only—*i.e.*, if the ohmic resistance is negligible—the lag will be 90°, for the *e.m.f.* in this case need be only as great as the counter *e.m.f.* of self-induction as this is greatest when the current is *changing* at the greatest rate, *i.e.*, when the current is zero. Likewise, when the current is maximum its rate of change is zero, hence the impressed *e.m.f.* is zero; thus the difference of phase between the *e.m.f.* and the current is 90°. This, however, is not the case when we have to consider both ohmic resistance and inductance.

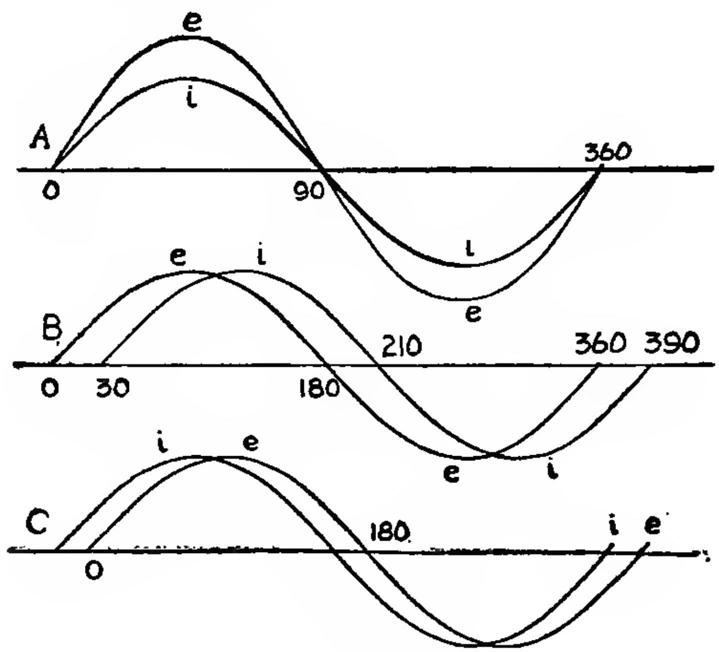

FIG. 125.

A concrete example may assist in understanding the principles here involved. Suppose that a current of 30 amperes and 60 cycles per second is to be sent through a circuit of 2 ohms resistance and an inductance of .002 henrys. What *e.m.f.* must be impressed on the circuit? As far as the ohmic resistance is concerned we see from equation (52) that

$$E = Ri = 2 \times 30 = 60 \text{ volts}$$

The counter *e.m.f.* of self-induction is by equation (140),

$$2\pi n Li = 2 \times 3.1416 \times 60 \times .002 \times 30 = 22.6 \text{ volts}$$

It might appear at first thought that the necessary impressed voltage would be the sum of 60 and 22.6. Account must be taken of the fact, however, that *e.m.f.* is in phase with the current when only ohmic resistance is considered, but 90° ahead of current as far as inductance is involved. Hence vectors representing these two quantities must be drawn at right angles to each other and the impressed *e.m.f.* is then the sum of these vectors. In Fig. 126, Ri is by Ohm's law the *e.m.f.* needed to overcome the simple resistance and $2\pi nLi$ is the *e.m.f.* needed because of self-induction. Hence, the necessary impressed *e.m.f.* is

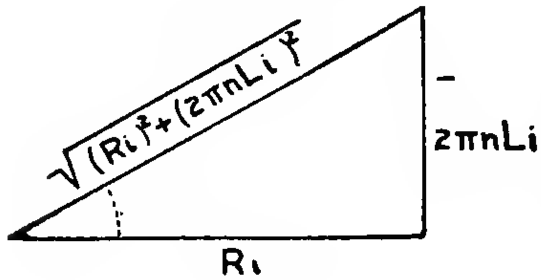

FIG. 126.

$$E=\sqrt{(Ri)^2+(2\pi nLi)^2} \tag{142}$$

Hence, the number of volts required in the problem given above is

$$\sqrt{60^2+22.6^2}=64.1$$

From equation (142) we obtain

$$i=\frac{E}{\sqrt{R^2+(2\pi nL)^2}} \tag{143}$$

The denominator in this equation is the quantity called *impedance* (§ 117).

If the circuit contains a condenser or considerable electrostatic capacity for any cause, the current will *lead* the *e.m.f.* as shown in Fig. 125, *C*. If the circuit contains capacity only, the lead will be 90°, for current will flow into the condenser most rapidly when the *e.m.f.* is changing most rapidly, *i.e.*, when the *e.m.f.* curve is passing through points such as 0, 180, etc. Current will continue to flow into the condenser but at a decreasing rate until the *e.m.f.* becomes maximum. As soon as the *e.m.f.* begins to decrease, the condenser will begin to discharge a current in the opposite direction and so will cause the current to reach a zero value before the *e.m.f.* becomes zero—*i.e.*, when the *e.m.f.* is at its maximum value no current is flowing, for the back pressure of the condenser is equal to

the *e.m.f.* impressed on it. The current, therefore, is zero when the *e.m.f.* is maximum, and the back discharge of the condenser, which begins as soon as the *e.m.f.* begins to decrease, causes a reversal of the current at that point.

It is thus seen that capacity and inductance have opposite effects on an alternating current and, if both are introduced into a circuit and properly adjusted in value, one may neutralize the effect of the other and the current will then flow as though only ohmic resistance were present.

The effect of inductance or capacity on an alternating current is often called *reactance*. These, like resistance, are measured in ohms. It may be shown that capacity reactance is expressed by $1/2\pi nC$, where C is the capacity in farads and n is the frequency. The inductive reactance has already been shown to be $2\pi nL$. Since the former causes the current to " lead " and the latter causes it to " lag," their combined effect would be their difference. When, therefore, a circuit contains resistance, inductance and capacity, equation (141) for impedance must be written

$$R_v = \sqrt{R^2 + \left(2\pi nL - \frac{1}{2\pi nC}\right)^2} \tag{144}$$

120. Power of Alternating Current.—Power or the rate at which a current is capable of doing work is for the direct current (§ 54) found in watts by multiplying volts by amperes. Likewise, for an alternating current in phase with the *e.m.f.* the power is the product of virtual volts and virtual amperes, or the power at any instant is the product of the *e.m.f.* and strength of current at that instant, for whatever positive, negative, or zero values the *e.m.f.* may have, there is at the same time a corresponding value of the same kind for the current (Fig. 125, *A*). When there is " lag " or " lead " the product of amperes by volts as measured by ammeters and voltmeters will no longer give the true number of watts (Fig. 125, *B* and *C*). The correct equation then is

$$P = Ei \cos \theta \tag{145}$$

where P is the power and θ is the angle of " lag " or " lead." When θ is 90°, P is zero and the current is said to be *wattless*, as would be inferred from the curves in Fig. 125, *B* and *C*. When

$\cos \theta$, called the *power factor*, is known, power is readily found. When θ is zero $\cos \theta$ is unity and this is simply the case when the current and *e.m.f.* are in phase.

121. Advantage in Use of an Alternating Current.—A great advantage of the alternating current is that its use makes possible an extensive and economical distribution of electric current from a central plant. Numerous water-falls in various parts of the country contain an enormous quantity of energy. Hydro-electric plants placed at these points can be made to convert potential energy of the water to energy of the electric current and transmit the same to distant points. Many such plants have been established and electric energy is being distributed to points distant 100 miles or more—in a few places as far as 260 miles.

In any line there is resistance, R, and hence a drop in voltage. The rate at which energy is consumed in the line is Ri^2 (§ 54). The power delivered at the end of the line will be the product of i and the voltage E at that point. Hence the power P used both in heating the conductor and in doing useful work is

$$P = Ri^2 + Ei \tag{146}$$

It is desirable, of course, to employ as much of the power as possible in doing useful work, *i.e.*, to reduce Ri^2 and increase Ei. The former may be accomplished by increasing the size of the conductor, thus diminishing R and increasing the voltage at the end of the line, for if R is less the drop in voltage will be less. This, however, is not desirable because of the cost of copper and the added difficulty of construction.

The latter—*i.e.*, the increase of Ei relative to Ri^2—is accomplished by raising the voltage E. For the same value of Ei the value of i is decreased in the same proportion as E is increased, their product remaining constant. The value of Ri^2 is thus greatly reduced and a larger proportion of power delivered at the end of the line.

Herein lies the great advantage of the alternating current, for by use of a transformer the voltage can be "stepped" up or down as desired. The pressure at the terminals of a generator may be 2000 or 3000 volts, more or less, and this may at once be raised by transformers to many thousands of volts, then transmitted over the lines and finally "stepped" down to a voltage which is safe and convenient for use.

Fig. 127.

In the ordinary city plant the dynamo produces a sufficiently high *e.m.f.*—say 1100 volts—and transformers are then used only at the end of the line to " step " the voltage down to, say 110 or 220 volts. In long lines, however, where a great quantity of energy is to be sent out from large central plants, 100,000 volts is a common potential and as high as 140,000 volts are in use.

With such enormous electrical pressure the old distinction between static and dynamic electricity disappears and the dynamic current exhibits all the properties once supposed to belong only a current produced by a friction or influence machine. The construction and insulation of a high tension line are difficult as compared with a line of low voltage, but there is great advantage in the use of the high voltage. As the voltage is raised, however, a point is reached where the conductor is surrounded by a glow of light called the *corona*. This indicates rupture in the dielectric strength of air and any higher voltage results in a serious corona loss of energy.

122. Rectifiers.—For reasons given above the alternating current is in very common use, but for some purposes such as the storage of batteries, electric plating, etc., a direct current must be used, hence the need of some device for converting the alternating to a direct current. The two most effective ways of doing this is by means of (1) a rotary transformer and (2) a mercury arc rectifier.

The former consists simply of a direct current generator driven by an alternating current motor. The latter needs a more extended explanation as follows: A vacuum tube is provided with four electrodes. *A* and *A′*, Fig. 128, are carbon anodes which are joined to the alternating current lines *H* and *G*. *B* is a mercury cathode which is joined to the positive pole of battery *J*, the other pole being joined to a point, *D*, between two reactance coils *E* and *F* connected as shown. *C* is a starting electrode.

Mercury vapor is a very poor conductor of electricity but when it is ionized it becomes a good conductor in one direction. To start the operation the tube is rocked so that the mercury in *C* and *B* will be joined and then separated, thus producing an electric arc. This produces the excitation necessary to make a good conducting path of the conventional current from either *A* or *A′* to the cathode *B*. The starter may then be switched out of circuit.

The lines H and G are alternately positive and negative. Suppose that at a given instant H is positive. A current will then pass through A, B, J, E, and over to the negative wire G, but when H changes from positive to negative it has for an instant a zero value at which time neither A nor A' are sending current to the cathode and the operation of the tube would on this account cease.

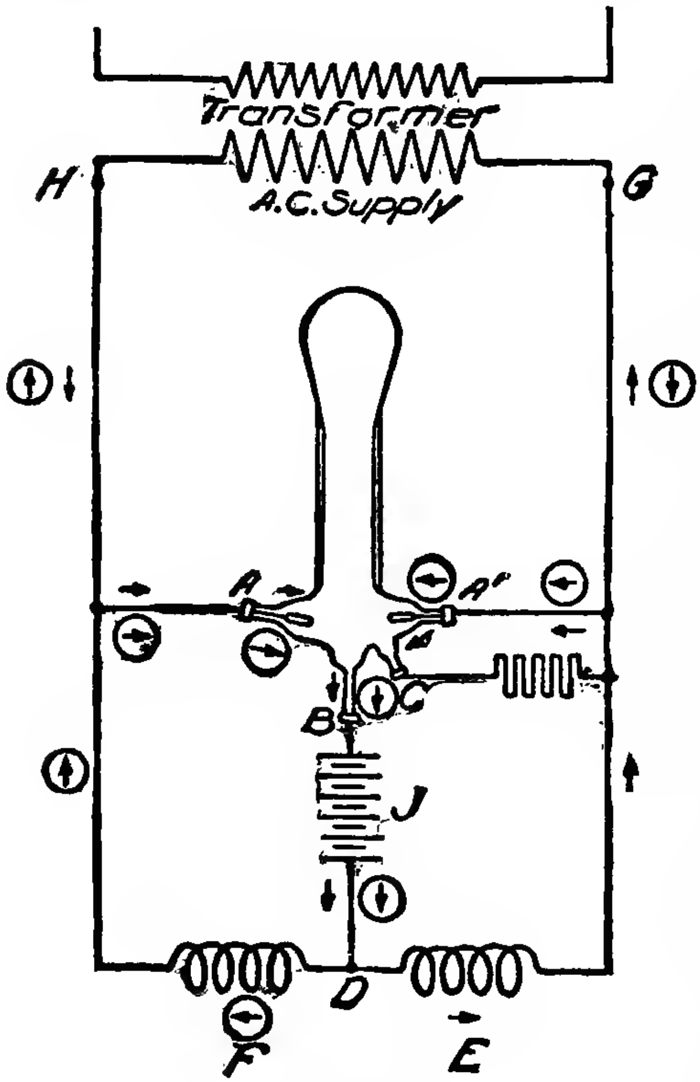

FIG. 128.

The purpose of the reactance coils is to maintain the arc from A to B while H is decreasing and G is increasing to a sufficiently high potential to send a current from A' to B against the counter *e.m.f.* of the battery. The principle of this action of the reactance coil has been explained in the paragraph on self-induction. When the current from A flowed through the coil E, it stored the coil with a quantity of energy in form of a magnetic field. When the current ceased the energy of this field was converted into current flowing in the same direction. Thus the current is maintained from A to B while G is rising to sufficient voltage to send current from A' to B. Then the same operation takes place from that side and through reactance F. No current flows directly from the lines through E and F, for these coils are at that instant discharging in an opposite direction.

By this arrangement the entire wave form is used and a continuous current is sent through the battery or other load.

123. Dimensions of Electromagnetic Units.—The electrostatic system of units, as we have seen, is based on the definition of unit quantity of electricity. The electromagnetic system, however, is based on the definition of unit magnetic pole. The force exerted between two poles is expressed by

$$F=\frac{mm'}{\mu r^2}$$

In definition of unit pole m and m' are equal, hence their product is the square of either one. Then substituting the dimensions of force for F, distance for r, and retaining μ in the equation,

$$m=[M^{\frac{1}{2}}L^{\frac{3}{2}}T^{-1}\mu^{\frac{1}{2}}]$$

The strength of a magnetic field at any point is measured by the force which would be exerted on a unit pole at that point, hence, if a pole m is urged by a force F, the strength of field H is

$$H=\frac{F}{m}$$

and substituting dimensions for F and m

$$H=[M^{\frac{1}{2}}L^{-\frac{1}{2}}T^{-1}\mu^{-\frac{1}{2}}]$$

Unit current strength is that current, unit length of which at unit distance will produce unit field, then for a length $2\pi r$—*i.e.*, circumference of a circle—at a distance r from the centre, and a strength of current i, the strength of field at the centre of the circle is

$$H=\frac{2\pi i}{r}$$

Hence, since r is length and the dimensions of H are given above,

$$i=[M^{\frac{1}{2}}L^{\frac{1}{2}}T^{-1}\mu^{-\frac{1}{2}}]$$

Since quantity of electricity is equal to it we have only to multiply the dimensions of i by $[T]$ to get

$$Q=[M^{\frac{1}{2}}L^{\frac{1}{2}}\mu^{-\frac{1}{2}}]$$

Potential difference between two points is defined as the work required to move unit charge from one point to the other. Work is the product of a force by a distance, hence if V is potential or *e.m.f.*

$$V = [MLT^{-2}]\,[L] \div [M^{\frac{1}{2}}L^{\frac{1}{2}}\mu^{-\frac{1}{2}}]$$
$$= [M^{\frac{1}{2}}L^{\frac{3}{2}}T^{-2}\mu^{\frac{1}{2}}]$$

Resistance, R, is by Ohm's law equal to *e.m.f.* divided by current, hence

$$R = [LT^{-1}\mu]$$

Capacity, C, is equal to quantity divided by *e.m.f.*, hence

$$C = [L^{-1}T^{2}\mu^{-1}]$$

Thus from the definition of any unit in a system the dimensions of that unit may be written.

Both the *e.m.* and the *e.s.* units are absolute units based on the centimetre, gram, and second as units of length, mass, and time.

The value of a few of the more common practical units in terms of *e.m.* and *e.s.* units is given in the table below.

	Practical unit.	*e.m.* units.	*e.s.* units.	Symbol.	Derivation of number of *e.s.* units.
e.m.f.	Volt	10^8	$\frac{1}{3}\,10^{-2}$	V	$10^8 \div 3(10)^{10}$
Current	Ampere	10^{-1}	$3(10)^9$	i	$10^{-1} \times 3(10)^{10}$
Quantity	Coulomb	10^{-1}	$3(10)^9$	Q	$10^{-1} \times 3(10)^{10}$
Resistance	Ohm	10^9	$\frac{1}{9}\,(10)^{-11}$	R	$R = \frac{V}{i}$
Inductance	Henry	10^9	$\frac{1}{9}(10)^{-11}$	L	$L = \frac{V}{i}$
	Millihenry	10^6	$\frac{1}{9}(10)^{-14}$	..	$L \div 1000$
Capacity	Farad	10^{-9}	$9(10)^{11}$	C	$C = \frac{Q}{V}$
	Microfarad	10^{-15}	$9(10)^5$	..	$C \div (10)^6$
Work or energy	1 Volt – Coulomb = 1 joule	$10^8 \times 10^{-1} =$ 10^7 ergs	$\frac{1}{3}10^{-2} \times 3(10)^9 =$ 10^7 ergs		
Power	Ampere – Volt = 1 watt	$10^{-1} \times 10^8 =$ 10^7 ergs per sec.	$3(10)^9 \times \frac{1}{3}(10)^{-2}$ 10^7 ergs per sec.		

If a given quantity of electricity is measured in *e.s.* units and also in *e.m.* units, it has been shown in § 48 that the ratio of the number of *e.s.* units to the number of *e.m.* units is very nearly $3(10)^{10}$.

If, now, we take the ratio of the dimensions of quantity in the two systems, regarding μ and k as unity in air, we have

$$\frac{L^{\frac{3}{2}}T^{-1}M^{\frac{1}{2}}}{L^{\frac{1}{2}}M^{\frac{1}{2}}}=\frac{L}{T}$$

Distance per unit of time is velocity v and this would suggest the equation

$$v=3(10)^{10}$$

The velocity of light is $3(10)^{10}$ cm./sec.

If we equate the dimensions of quantity in the two systems and include μ and k we have

$$L^{\frac{3}{2}}T^{-1}M^{\frac{1}{2}}k^{\frac{1}{2}}=L^{\frac{1}{2}}M^{\frac{1}{2}}\mu^{-\frac{1}{2}}$$

$$\therefore \frac{L}{T}=\frac{1}{\sqrt{k\mu}}=v$$

Experimental determinations of the value of this ratio also give very nearly $3(10)^{10}$.

Theoretical considerations of such relations as these led Maxwell in 1873 to suggest that light and all other ether radiations are electromagnetic in character. Many experiments since that time strongly confirm Maxwell's theory. This subject is more fully discussed in the next chapter.

Problems

1. A straight wire kept in a horizontal position in an east and west direction is 15 m. long. It is let fall from a height of 50 m. at a place where the horizontal intensity of the earth's magnetism is .2 dyne. What is the maximum *e.m.f.* generated in the wire and which end of the wire has the higher potential?

2. A rectangular coil of wire 20×50 cm. contains ten turns and makes 400 revolutions per second on its longer axis, in a magnetic field perpendicular to the axis. The strength of the field is 10,000 lines per square centimetre. Find the maximum *e.m.f.*, the average *e.m.f.*, and the factor which multiplied by the former will give the latter.

3. If the inductance in a circuit is 70 millihenrys, what is the consequent reactance of an alternating current of 60 cycles?

4. If the virtual value of an alternating current is 17.675 amperes, what is its actual value at phase 210°?

5. An alternating current whose maximum value is 40 amperes lags 25° behind the voltage. What is the value of the current at the instant when the voltage is in the 80° phase?

6. If a virtual current of 25 amperes lags 40° behind a virtual *e.m.f.* of 110 volts, what is the power?

7. An electric line having a resistance of 20 ohms carries a current of 50 amperes under a pressure from the dynamo of 2000 volts. How much power will be saved for useful work at the end of the line by raising the voltage to 50,000 volts?

8. The coil of an earth inductor is 20 cm. in diameter and contains 100 turns of wire. The terminals of the coil are attached to a ballistic galvanometer and the resistance of the entire circuit is 100 ohms. When the coil is placed so that its axis is vertical and turned rapidly through 180°, the galvanometer deflection as read on the scale is 20 cm. It is then found that a condenser of 3 microfarads capacity, charged by a battery of 2 volts, will cause the same deflection. What is the strength of the horizontal component of the earth's magnetism?

Ans. 1. 9.35 millivolts.
The east end.
2. 2513.28 volts.
1600 volts.
.636.
3. 26.39 ohms.
4. 12.5 amperes.
5. 32.76 amperes.
6. 2106.5 watts.
7. 49,920 watts.
8. .19 dyne.

CHAPTER XII

ELECTROMAGNETIC WAVES

124. Electric Oscillations.—It has already been shown that an electric charge is surrounded by an electrostatic field and that points in this field may differ in potential by a certain number of volts. When the charge is set in motion its field is carried with it and also there is set up a magnetic field at right angles to the direction of the motion. The strength of this magnetic field depends on the rate of flow of electric charges, *i.e.*, on the amperage.

If the direction in which the charge moves is suddenly reversed, the direction of motion of the electrostatic field is reversed and also the direction of the magnetic lines of force. If these reversals are repeated regularly and at short intervals, the ether surrounding the charge is subjected to alternate strains in one direction and then in the other, thus causing ether waves which may move out in all directions from the point of disturbance.

The length of such waves is determined by the rapidity of the alternations or, as here called, the oscillations of the current. The greater the number of oscillations the shorter the wave, just as the greater the number of vibrations of a tuning fork the higher the pitch.

125. Length of Ether Waves.—Since an electromagnetic pulse moves with a velocity of about $3(10)^{10}$ cm. per second, if the number of oscillations are comparatively few the wave will be of enormous length. The ordinary alternating current of 60 cycles per second causes waves several thousands of miles long. It is possible to construct generators having many poles and then rotate the armature with such speed as to produce a frequency of 10,000 or 15,000 cycles per second. The length of the wave is thereby proportionately reduced.

The best method of producing oscillations of great frequency is by the discharge of a condenser. The nature of an oscillatory discharge has already been described in § 116. The circuit through which the discharge takes place, however, may be such as to prevent oscillations and the discharge is then simply a diminishing

one. This is analogous to the vibrations of an elastic rod. If such a rod is clamped at one end, then if the free end is pulled to one side and released, vibrations will continue for some time. This, however, will be the case only when the rod meets with little resistance as in air. If the rod is immersed in some viscous fluid, when released from its strained condition it will slowly return to a position of rest without vibration.

Lord Kelvin in 1853 published a mathematical consideration of this subject and showed that the kind of discharge which will occur depends on the resistance R, the inductance L, and the capacity C.

If

$$R < 2\sqrt{\frac{L}{C}}$$

the discharge will be oscillatory and the frequency n is given by

$$n = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}} \tag{147}$$

If the second quantity under the radical is so small in comparison with the first that it may be neglected, the equation becomes

$$n = \frac{1}{2\pi}\sqrt{\frac{1}{LC}} \tag{148}$$

and the period P, which is the reciprocal of the number of vibrations per second, is

$$P = 2\pi\sqrt{LC} \tag{149}$$

Equation (149) is often called the fundamental equation of wireless telegraphy for L and C are very important factors in its operation. If L is given in henrys and C in farads, P is the period in seconds.

A wave-length λ is equal to the product of period by the velocity v with which the wave travels. Hence

$$\lambda = Pv = 2\pi v\sqrt{LC} \tag{150}$$

126. Electric Resonance.—When an electric circuit is such that oscillations in it will produce waves of a certain length, then oscillations may be produced in this circuit by waves of the same

length from another oscillator. This is the principle of electric resonance and is analogous to a similar phenomenon in sound where one tuning fork will respond to another of the same pitch.

Illustrating this principle an experiment was devised by Lodge, the apparatus for which is shown in Fig. 129. Two Leyden jars of about equal capacity are used. One, A, is continuously charged by means of an induction coil or influence machine so that sparks will pass across the short air gap at c. The other jar is provided with conducting rods connecting with the inner and outer coatings and extending out as shown. If the two jars be placed a short distance apart with the conducting frames parallel, whenever a spark passes at c there will be a corresponding surging of electricity in the neighboring conductor provided it is "tuned" for resonance. This tuning may be effected by moving the slider s until the proper length of circuit is secured. To make the resonance apparent, a wire clip n is placed over the edge of the jar so as to touch the inner coating but leave a short spark gap between it and the outer coating. Some of the electricity will then leap across this short gap and will be seen as a bright spark whenever a spark occurs at c.

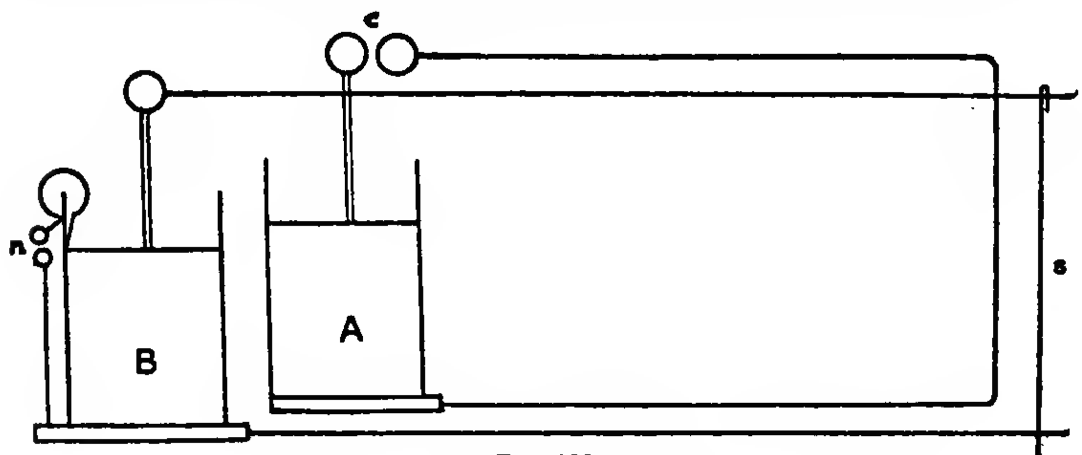

FIG. 129.

127. Experiments of Hertz.—As early as 1862 Maxwell showed from theoretical considerations that an oscillatory discharge of a condenser causes ether waves which travel out with finite velocity and that this velocity may be expressed by

$$v=\frac{1}{\sqrt{k\mu}}$$

as already shown in § 123.

About 25 years later Hertz, a German physicist, undertook a series of experiments which conclusively proved the claims of Maxwell. For the oscillator Hertz used a system of small capacity, as shown in Fig. 130, the discharge taking place between two small, polished, metallic knobs. The object of this is to secure waves of short wave-length, for as shown by equation (150) the waves will be of great length if any considerable capacity is used. Hertz was able to reduce the wave-length to less than one metre. A loop of wire *D* with a short air gap between its terminal knobs was used as a detector or resonator. The length of this loop was adjusted so that it was in tune with the oscillations at *s*. When *D* is held in front of *s* so that the plane of *D* is perpendicular to a line from *s* and so that the air gaps are parallel, then a spark will appear at *n* whenever one occurs at *s*. With such a resonator Hertz was able to detect the presence of electromagnetic waves in a field and to trace out their paths. He proved that the laws of reflection, refraction, and polarization held for these waves as in the case of light. By reflecting waves back on their path he was able to produce stationary waves in which the antinodes could be located by the fact that the resonator would at that point show a spark while at a node no spark would be observed. The distance between two nodes is one-half a wave-length. Thus the wave-length could be determined. The frequency of the oscillator could be computed and then the velocity v of the waves would be

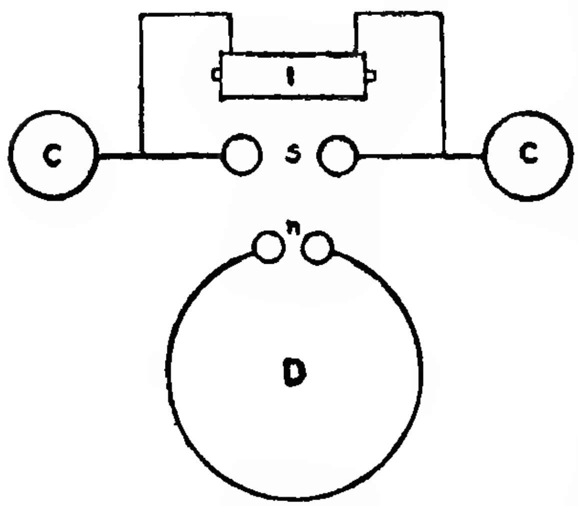

FIG. 130.

$$v = \lambda n$$

where λ is wave-length and n is the number of waves per second. His results show that the velocity of these waves is the same as that for light.

Hertz's experiments, therefore, proved the physical existence of electromagnetic waves and that such waves all travel with the velocity of light, the only difference being wave-length.

128. Electric Nodes and Internodes.—An interesting experiment has been devised by Seibt to show interference of electric waves. A long coil of wire *AB*, Fig. 131, is wrapped on a wooden core and the lower end, as shown, is attached to the helix *S*. The particular coil here described consists of a wooden cylinder six feet long and about one inch in diameter. This is closely wrapped from end to end with No. 30 silk-covered copper wire in a single layer.

A single fine wire *cd* is stretched parallel to *AB* and grounded at *G*. This wire is insulated from the coil.

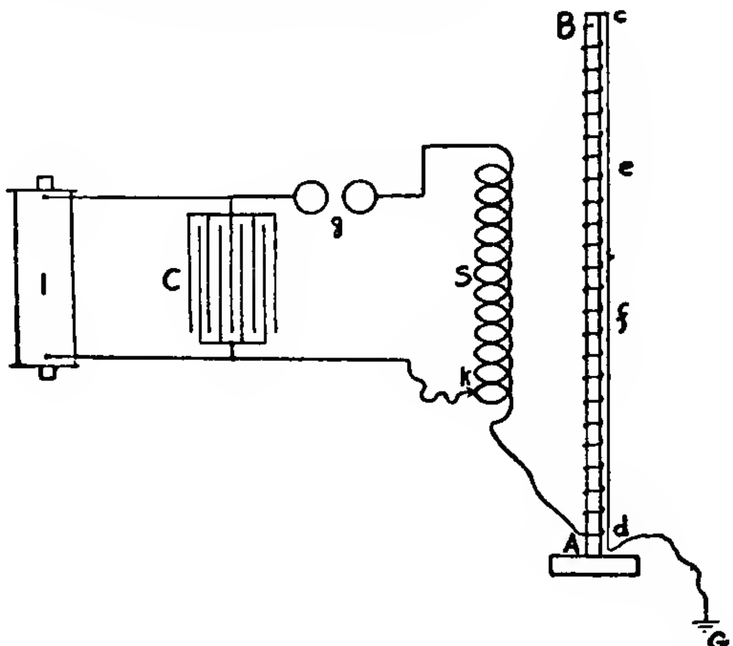

FIG. 131.

A condenser *C* is continuously charged by a high-tension transformer *I*. An oscillatory discharge of the condenser takes place through the helix *S* and the gap *g*. The length of the electric wave will depend on the capacity of *C* and the inductance of *S*. By using a single Leyden jar, about a quart size, as condenser and moving *k* to a certain position on the helix, a glow of light will be seen at several points, as *c*, *e*, and *f*, between *AB* and *cd*.

These indicate the position of the nodes, for the advancing and reflected electric pulses collide at these points, producing a high potential which results in a discharge to the wire *cd*.

By changing the capacity of *C* or the point of contact *k*, the period of the electric impulses is changed and also the position of the nodes. Resonance will first occur when the length of the

electric wave is four times the length of the wire on AB. When this occurs, a brush discharge will appear at the top of the coil.

129. Wireless Telegraphy.—After Hertz had shown the existence of electromagnetic waves and the possibility of detecting them it was evident that if such waves could be detected a long distance from a point of ether disturbance we would have a valuable means of communication without wires. This method has come into general use for communication between ships at sea, between ships and land, or between any two points even when separated 1000 miles or more. Ether waves of this kind can be detected at a greater distance at night than in daytime and are less obstructed on water than on land. Theoretically there is no limit to the distance through which ether waves may be sent, for ether pervades all space, but in practice the success of the method depends on the amount of energy which may be radiated out into space and the delicacy of the apparatus used to detect the waves.

The wireless outfit, therefore, consists of two parts: (1) the *sender*, where oscillations are made to generate waves; and (2) the *receiver* or *detector*.

The sender may be any device that will produce a rapidly alternating current. Usually a high-tension transformer or an induction coil is used to charge a condenser, and the oscillating discharge of the condenser causes the waves. The length of the wave may be regulated by the capacity of the condenser and the inductance of the circuit.

If the circuit of the sender is closed, as in Fig. 129, not much energy is radiated into space, so one side of the spark gap is connected to the earth and the other, called the aerial, is raised high in the air. The wave-length of the system may be found by tuning a standard oscillating circuit to resonance with it. This standard, called a wave metre, may be set to a great number of different wave-lengths.*

A variety of devices have been employed for the detection of waves at the receiving station. In fact the passage of such a wave disturbs the electric condition of almost any object in its path.

* By a law which went into effect December 13, 1912, wave-lengths from 600 to 1600 metres are reserved for government use in the United States. The general regulation of *radio communication* is under the direction of the Secretary of Commerce and Labor.

The first detector perfected by Marconi for long distance work is the *coherer*. This consists essentially of a few clean filings of nickel and silver, nickel and iron, or filings of other metals, placed at *c*, Fig. 132, in a glass tube between two metal plugs. Wires lead out from these plugs, one to the aerial *A* and the other to the ground *G*. The resistance of the filings is so great that practically no current flows although they are in circuit with battery *B′*. When, however, an electric wave passes, their resistance is greatly decreased and *B′* will operate the relay *R*, and the relay will then close the circuit of *B* through the sounder *S*. Thus any code of signals made at the sending station will be indicated by the sounder at the receiving station. When the filings are once made to *cohere*, as it is called, they retain their conducting power until they are mechanically shaken apart. This may be done automatically by an electric vibrator, such as an electric bell, by connecting it in series with the battery *B* and placing it so that the tapper will strike the glass tube.

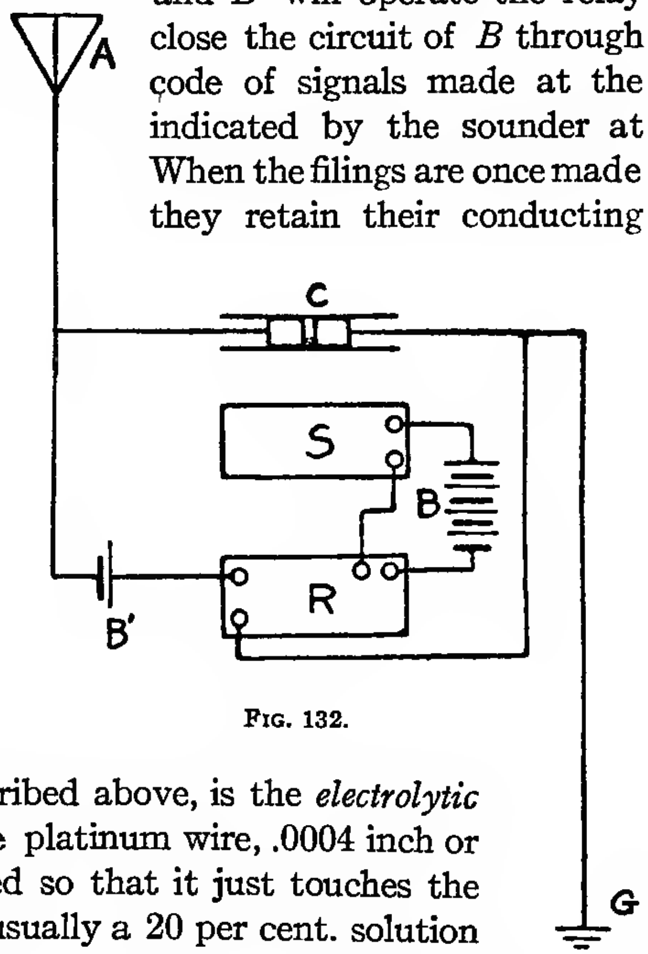

FIG. 132.

Another form of receiver, much more sensitive than the coherer described above, is the *electrolytic detector*. Here a very fine platinum wire, .0004 inch or less in diameter, is adjusted so that it just touches the surface of an electrolyte, usually a 20 per cent. solution of nitric acid. This, as shown at *D*, Fig. 133, is put in circuit with an adjustable noninductive resistance *R*, a battery *B*, and a telephone receiver *T*.

The strength of the battery is kept just below that necessary to cause a current to flow against the *e.m.f.* of polarization in *D*. (See § 66.) Then when a current resulting from the passage of electric waves raises the potential at *D* above the critical *e.m.f.* of polarization, the battery current will flow and produce a sound in the telephone. As soon as the waves cease *D* will again polarize and stop the current.

This detector is therefore self-restoring, and though very sensitive it is easily burned out and requires frequent adjustment.

Another form is called the *magnetic detector* and is based on the principle that when iron is being magnetized and the induction (see § 39) is lagging behind the magnetizing force, any slight change in this force will cause a large change in the induction.

The operation of this detector is shown in diagram, Fig. 134. An iron wire *w* kept in continuous motion by clock-work passes through a glass tube *m* around which is a coil of wire in series with the aerial and ground line. Concentric with this is another coil *n* to the terminals of which is attached a telephone receiver. While the wire moves through the field of the magnets, any current on the aerial will cause a large change in the magnetic field within *n*, and this will induce a current which flows through the telephone. No battery is needed.

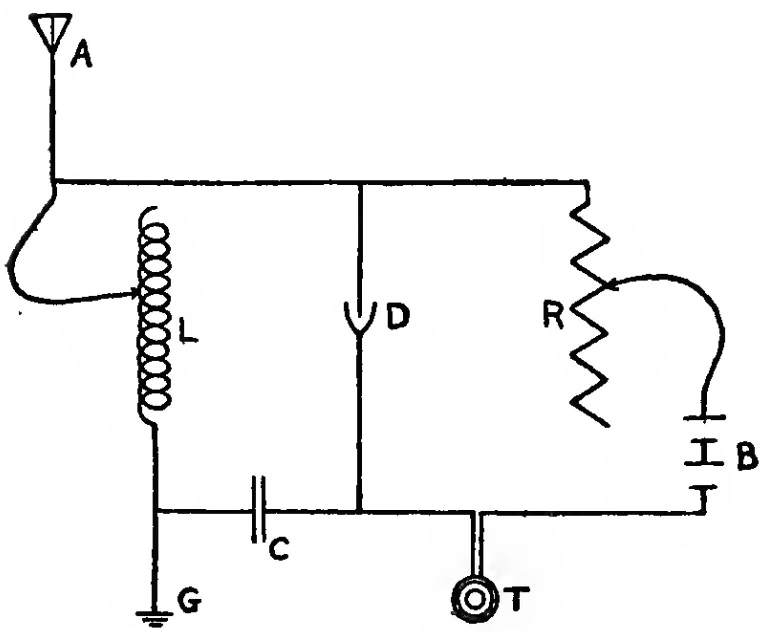

Fig. 133.

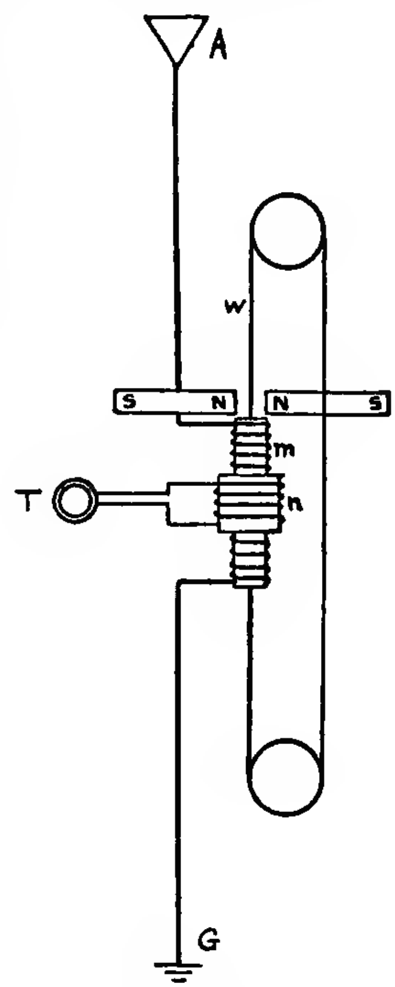

Fig. 134.

It is also found that certain crystals such as galena, silicon, and carborundum may be effectively used to indicate the passage of electric waves. Such are called *crystal detectors*. A rapidly alternating current will pass through such crystals in one direction but not in the opposite direction. Hence, if a telephone receiver be shunted to the opposite sides of the crystal, then a passing

train of ether waves will cause oscillations in the line and a series of unidirectional pulses through the telephone.

In Fig. 135 is shown, in a general way, the plan of a transmitting and receiving outfit. The key K is pressed in such a manner as to produce dots and dashes as in ordinary telegraphy but the operation is slower. The induction coil charges the condenser C which discharges through the gap p, producing oscillations. The inductance of L resists the passage of this current (see Fig. 124) and, when the switch S is closed on the side of the transmitter, the energy is radiated from the aerial in form of waves. To receive a message the switch is thrown to the other side and sounds are produced by the telephone receiver in the manner explained above.*

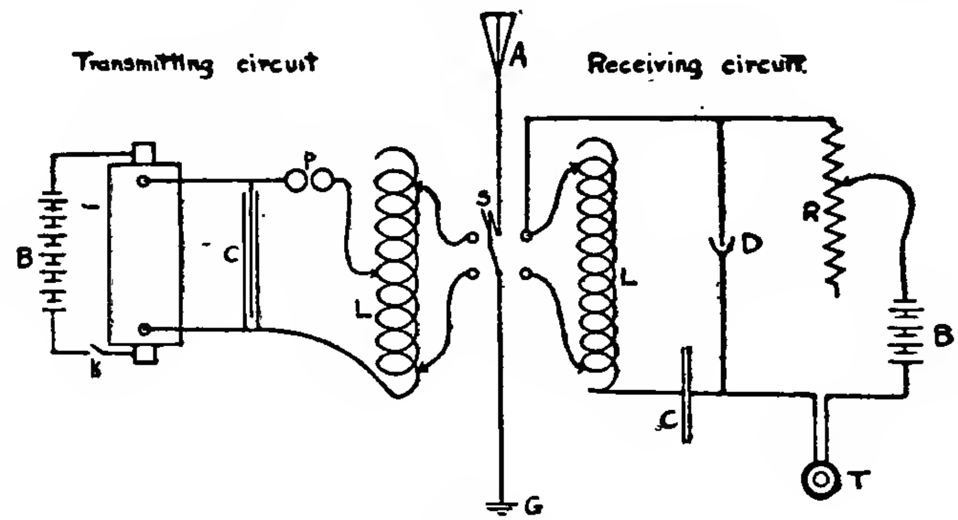

FIG. 135.

130. Light.—When electromagnetic waves are very short they produce the phenomena which we call light. Waves in ether, as already shown, may be very long, 1000 miles or more, and may vary in length down to exceedingly short distances. The shortest wave of this kind, produced by experimental means, is about 4 mm. Even this short length is more than 5300 times as great as that of the longest visible wave, the longest wave-length of red light being $.75\mu$, *i.e.*, .75 micron or .00075 mm.

We have seen how electric oscillations, as in the discharge of a Leyden jar, cause waves in ether the length of which depend

* The signals used are similar to the Morse code used in ordinary telegraphy, but unfortunately the modifications of this code for wireless communication are not uniform and so we have the American, the Continental, and other codes. The international signal of distress is . . . – – – . . . , S O S.

13

on the period of oscillation: the shorter the period the shorter the wave, for the velocity of transmission of waves is the same whatever their length. It, therefore, becomes a subject of interest as to the source of oscillations of such rapidity as to produce light waves, the longest of which is only $.75\mu$ and the shortest $.38\mu$.

131. Source of Light Waves.—There is strong evidence in favor of the theory that light waves originate in the rapid vibratory motion of electrons. All electrons are charges of electricity or at least carry such charges. If we regard them as moving in a circular orbit within the atom, their motion along any diameter of the orbit as axis would be a simple harmonic motion. An electric charge thus oscillates back and forth and produces waves of the same kind as those produced by the oscillations in the discharge of a Leyden jar, but very much shorter. The velocity of electrons must then be very great to produce such short waves. There is experimental evidence of such velocity, as is shown in the next paragraph.

132. Radio Activity.—If, as suggested above, there is a very rapid vibratory or orbital motion of electrons, and if the forces which cause stability of motion should cease, electrons would fly off in a straight line just as the planets of the solar system would move out through space in a straight line if the force of gravitation should cease.

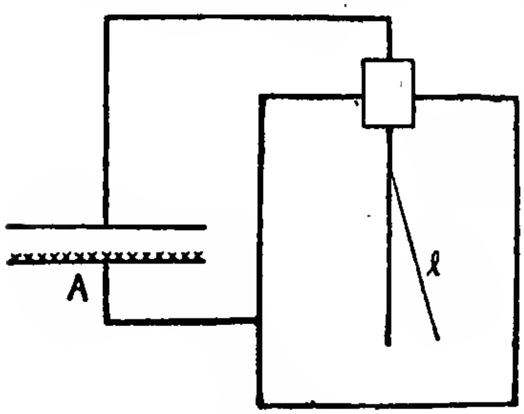

FIG. 136.

Now, Becquerel in 1896 discovered that uranium is a mineral undergoing this very process of change. Its atoms are breaking up and the very small particles of which they were composed then unite in new combinations, forming a different substance. This process is a result of changes which are taking place within the mineral itself and is not caused by outside influences. Substances of this kind are said to be *radio active*.

It was soon found that many other minerals were also more or less radio active. One method of detecting the radio activity of substances is by noting their ionizing effect on air between metal plates connected to an electroscope. If the substance be spread

on plate A, Fig. 136, its activity is indicated by the rate at which the charged gold leaf l will fall.

Another method of detecting radio activity is by photographic effect. Fig. 137 shows theresult of placing 6 different active minerals on an ordinary photographic plate and setting aside for 48 hours. The plate was then developed in the usual manner. The strongest effect was produced by 1, which is *pitchblende.* The others are: 2, *carnotite;* 3, *fergusonite;* 4, *monazite;* 5, *samarskite,* and 6, *thorite.*

FIG. 137.

It was this intensity of radiation from pitchblende that led M. and Mme. Curie to attempt to separate from this mineral the active material which it was supposed to contain. This resulted in the discovery of two new active substances: one called *polonium,* named after Mme. Curie's native country, is much more active than uranium but not so constant in its activity; the other called *radium* is nearly a million times more active than uranium. Only a small fraction of a gram of radium can be obtained from a ton of pitchblende and the chemical process used in the separation is a laborious one. As ordinarily used, radium is in combination with some other element, forming a bromide or chloride.

It has been found that three distinct types of radiation may be given out by radio-active substances. These have been called the *alpha*, the *beta*, and the *gamma* rays. If a small quantity of radium is placed at R, Fig. 138, at the bottom of a hole drilled in a mass of lead, rays will pass in straight lines out into the air, but if a strong electric or magnetic field is produced in the path of these rays it may be shown by the photographic effect or otherwise that some are turned aside to β (the beta rays), others are deflected in an opposite direction to α (the alpha rays), while others are not deflected but pass straight out to γ (the gamma rays).

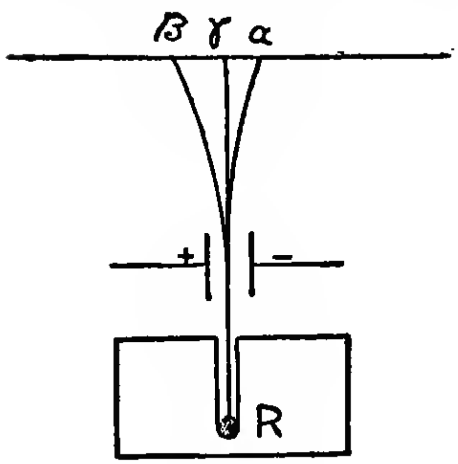

FIG. 138.

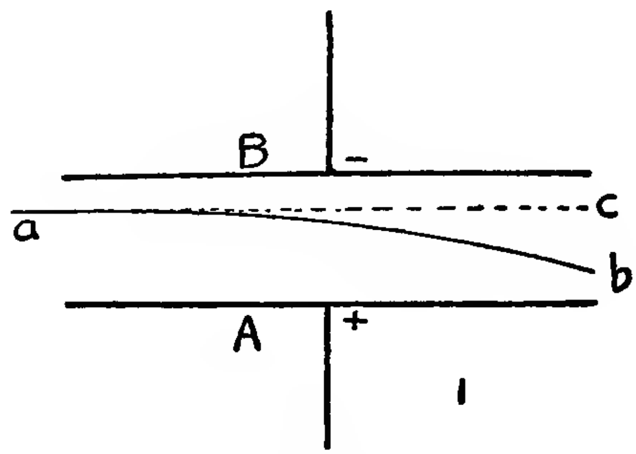

FIG. 139.

Further investigation shows that the β rays consist of a stream of electrons identical with those found in cathode rays, *i.e.*, they are negative corpuscles and so are drawn to the positive side of an electrical field as shown in the figure. The α rays are found to consist of positive corpuscles and so are bent toward the negative side of the field. The γ rays do not appear to be composed of particles of any kind but to be very short ether waves resulting from sudden impacts of β particles.

It is probable that the velocity of the α and β particles is the same as they had in orbital motion before they flew off. Hence, for this reason among others, it becomes an interesting experiment to determine the velocity.

133. Velocity of β Particles.—Let A and B, Fig. 139, be two parallel plates which may be joined to the poles of a high voltage battery. A strong electric field will thus be produced in the space between the plates. Let the strength of this field be denoted by E. Now suppose an electron to be moving in the direction ac.

Let its electric charge be e and its mass m. Then the force F_1 acting on it at right angles to ac is

$$F_1 = Ee \tag{151}$$

which deflects the electron to b.

Let a strong electromagnet now be so placed as to produce a magnetic field between plates A and B, the direction of this field in the particular case shown in the diagram being toward the reader and at right angles to the electric field. By the motor rule we see that this would cause a deflection of the electron to some point above c, Fig. 139. Let the strength of the magnetic field be denoted by H and let e and m have the same meaning as above. Then the force F_2, acting at right angles to the path of the electron in the magnetic field, is

$$F_2 = Hev \tag{152}$$

This equation will be clear from a comparison with equation (116), for a current is regarded as a stream of moving electrons and the strength, i, of a current is the number of electrons per unit of time, t, hence e/t is the same as i and e/t times l is ev where v is velocity, for l/t is v.

If now both fields are used at the same time and their relative strength be so adjusted that the electrons will move straight through to c, *i.e.*, without any deflection toward either b or d, then F_1 must be equal to F_2 and

$$Ee = Hev \tag{153}$$

$$\therefore v = \frac{E}{H} \tag{154}$$

The velocity of the electron is then equal to the ratio of the strength of the electric to the strength of the magnetic fields when both are measured in the same kind of units.

Experiments based on this principle have been made not only for β rays but also for α and cathode rays. The velocity of β rays appears to vary from about $6(10)^9$ to $2.85(10)^{10}$ cm./sec. The α rays have a velocity of about $2(10)^9$ cm./sec. and cathode rays about $3(10)^9$ cm./sec. though this last value varies somewhat with the difference of potential at the terminals of the vacuum tube.

In this connection it is also of great interest to determine the ratio of the charge, e, to the mass m of a particle and also the magnitude of the charge, as shown in the next paragraph.

134. The Ratio of e to m and the Value of e.—Referring to Fig. 139, let the electrical field only be employed. A charged particle moving at right angles to this field is acted on by a constant force parallel to the field, hence while the particle is moving a distance ac it also moves with uniformly accelerated motion over a distance cb. The conditions and results here are similar to the horizontal projection of a mass near the surface of the earth. The horizontal motion is uniform and the vertical motion is uniformly accelerated.

If t is the time required for the body to move from a to c, this will also be the time of the accelerated motion through a distance cb. Hence, if v is the uniform velocity along ac,

$$ac = vt \tag{155}$$

Also, if a is the acceleration in the direction cb,

$$cb = \frac{1}{2}at^2 \tag{156}$$

Since acceleration is equal to the force divided by the mass and the force here is Ee (see equation 151),

$$a = \frac{Ee}{m} \tag{157}$$

Eliminating t from (155) and (156) and substituting the value of a from (157),

$$\frac{e}{m} = \frac{2v^2}{E} \cdot \frac{\overline{cb}}{\overline{ac}^2} \tag{158}$$

The value of v is found from (154), cb and ac are measured, and E is known, hence the ratio of the charge e to the mass m may be found. A magnetic field may also be used in finding this ratio. Many determinations of this quantity have been made, the results always being around $1.8(10)^7$ for cathode rays and β rays.

Now the value of e/m for univalent ions, as hydrogen, in electrolysis is about 10^4, *i.e.*, the value of e/m for β and cathode rays is about 1800 times as great as in a hydrogen ion.

There is good reason for believing that the charge on a hydrogen ion is of the same magnitude as that on an electron. Hence the mass of an electron must be about $\frac{1}{1800}$ the mass of a hydrogen atom.

135. Optics.—A study of the phenomena which occur in the propagation of very short electromagnetic waves is called *optics.* There are two different ways of viewing these phenomena. One is called *geometrical optics*, which is based on certain laws of reflection, refraction, and the assumption that light travels in straight lines through isotropic media. The straight line, called a ray, is an imaginary line indicating the direction in which a wave-front is moving. The other method of treating this subject is called *physical optics*, which attempts to explain the origin and nature of light waves in accordance with physical principles and suggests what may be expected as a result of certain lines of experimental investigation.

Both methods are valuable and in this chapter can best be used together when either or both will assist in understanding the subject.

136. Theories of Light.—During the seventeenth and eighteenth centuries there were two rival theories in regard to the nature and transmission of light. One is called the corpuscular theory, which assumed that a luminous body throws off very small particles or corpuscles and these move with great speed through space. Vision was explained as a result of the impact of corpuscles on the retina of the eye.

The other was known as the undulatory theory, which claimed that light is transmitted as a wave motion on a medium which pervades all space. This medium was called the luminiferous ether. We now know other waves, some shorter—X-rays for example—and many much longer than light waves, such as wireless waves, all of which travel on ether.

The chief advocate of the corpuscular theory was Sir Isaac Newton (1642–1727). His chief objection to the wave theory was that he could not, by this theory, explain the rectilinear propagation of light as observed when light passes the edge of an opaque object. In case of sound waves, water waves, etc., there is, as ordinarily observed, no clearly defined region back of an obstruction where the medium is quiet. Waves of this kind bend around

edges. Light, however, does not appear to bend around an obstruction, but there is a distinct straight line separating the region of shadow from the illuminated region.

This distinction is only an apparent one, resulting from the fact that sound and water waves which we ordinarily observe are comparatively long, while light waves are exceedingly short. When short waves—ripples—are produced on the surface of water an obstruction will produce a region of no disturbance bounded on its sides by straight lines. Very short sound waves will, as shown later, produce back of an obstruction a definite sound shadow similar to a light shadow.

It may be shown experimentally that in any kind of wave propagation if the wave-length is short as compared to the size of the obstruction, or the size of the opening through which the waves pass, the phenomena of rectilinear propagation will be observed.

The cause of the straight line between a shadow and the lighted region is not that light will not bend around an opaque body, but that light waves in the region of the shadow interfere with and destroy each other.

The wave theory of light was established chiefly by Huygens (1629–1695), Young (1773–1829), and Fresnel (1788–1827). Huygens formulated the theory. Young and Fresnel about a century later modified some of the assumptions of Huygens and explained in a convincing manner the destructive interference of light waves.

Many phenomena of light can be explained by either theory, but others cannot be satisfactorily explained by the corpuscular theory. Refraction, for example, was explained by this latter theory as a result of the attraction of a body of matter, such as glass or water, for an approaching corpuscle of light. The corpuscle would therefore be accelerated in its motion and would move faster in a denser medium. Here, then, is an opportunity to test the theories, for by the wave theory the velocity of light should be less in a denser medium. This experiment has been made (§ 140) and it is found that the velocity is always less in a denser medium, though the experiment came too late to have a part in establishing the wave theory.

The phenomena of diffraction—*i.e.*, the deflection of light in passing the edge of an opaque object or through a narrow aper-

ture—has been satisfactorily explained only on the assumptions of the wave theory.

Whatever the light phenomena may be, the wave theory appears to be competent to give a satisfactory explanation, and while it is not known just what the nature of the ether disturbance is, there is strong evidence for the theory that light is a periodic disturbance caused by the oscillations of an electric charge and propagated in form of waves.

137. Wave-Front and Huygens' Principle.—If O, Fig. 140, is a luminous point in an isotropic medium, then a wave disturbance will move out in all directions with equal speed. Such a wave is spherical and is the locus of all points in the same phase of vibration. The figure represents a section through the centre of such a sphere, and the inner circle represents the wave-front at a given instant.

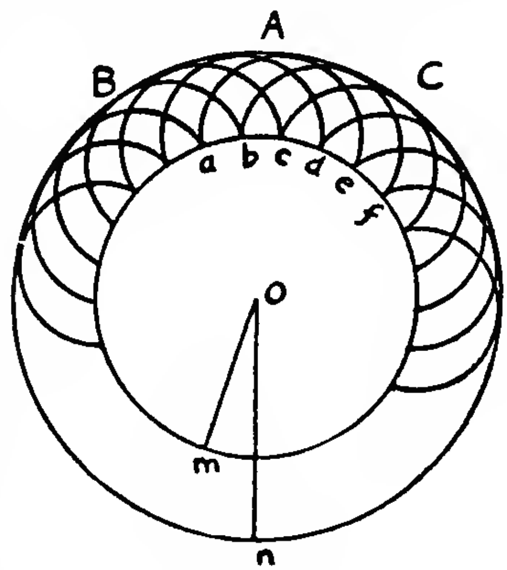

FIG. 140.

According to the principle of Huygens, more fully worked out by Fresnel, each point in a wave-front is regarded as a centre of disturbance from which secondary waves arise. Thus points a, b, c, etc., in the figure are regarded as point sources of light from which arise the wavelets shown above these points.

The disturbance at any point, as A, will then be the resultant of all the secondary waves that reach that point. The larger circle thus becomes the new wave-front, and each point in it then becomes the centre of secondary waves, and so on.

The surface BAC is called the envelope of the secondary waves. It is in this envelope that the disturbance of the secondary waves is greatest, their effect at other points being destroyed by interference.

A line perpendicular to a wave-front is called a *ray*. The rays of a spherical wave as Om or On are diverging and are simply the radii of the sphere. If O is at an infinite distance the wave-front is plane and the rays are parallel. A number of such rays is called

a *beam*. When a wave-front is made to move toward a point, as may be done by use of a lens or mirror, the rays are convergent and a number of them are together called a *pencil*.

138. Interference of Waves.—Let *AB*, Fig. 141, represent a plane wave-front approaching the screen *CD*. Small holes *a* and *b* permit a minute portion of the wave-front to pass through. According to the Huygens principle these points become centres of disturbance and send out waves to the right of the screen. A succession of waves such as *AB* will cause a corresponding series of waves on the opposite side of the screen. The whole lines represent one phase of vibration and the broken lines an opposite phase. It will be noted that at points along *nm* the two sets of waves are in the same phase of vibration, hence the disturbance is the sum of the disturbance of each. If these are light waves and are allowed to fall upon a screen, *m* will be a point of maximum brightness. At points along the lines *cd* and *tu* the waves meet in opposite phase, hence at *d* and *u* are points of minimum disturbance or darkness. Along *ef* and

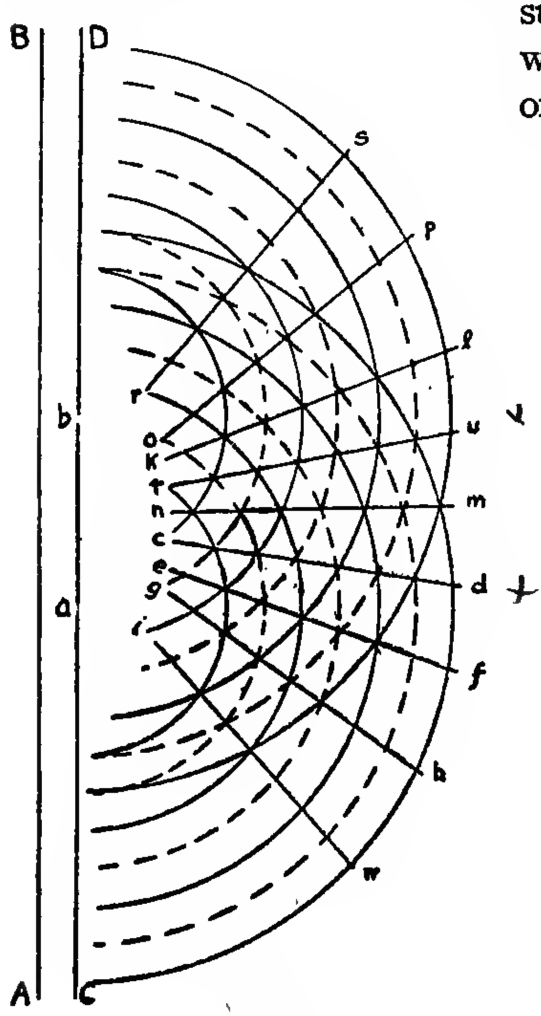

Fig. 141.

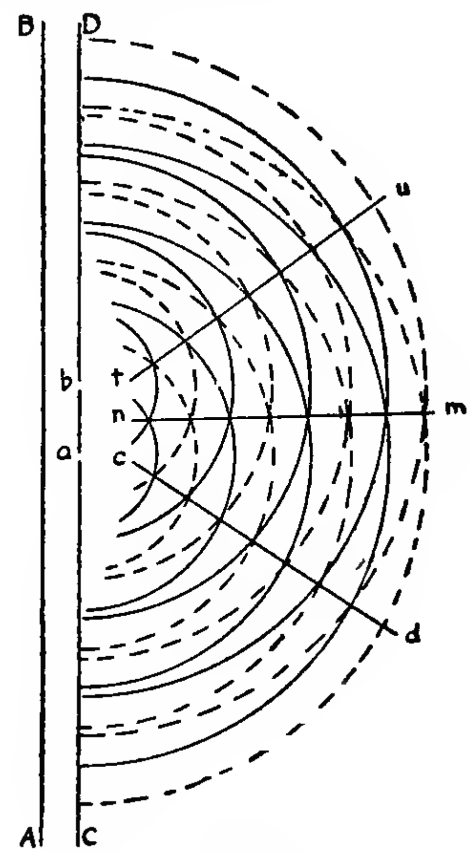

Fig. 142.

kl they are again in the same phase, hence *f* and *l* are points of maximum brightness.

All points on *nm* are equally distant from *a* and *b*, but each point on *cd* is a half wave-length farther from *b* than from *a*, and each point on *tu* is a half wave-length farther from *a* than from *b*. The points along *ef* are a whole wave-length farther from *b* than from *a* and so are again in the same phase.

It will be noted also that since *a* and *b* are much farther apart than the length of a wave, the lines *nm*, *cd*, *tu*, etc., are close to one another. If *a* and *b* were further separated, these lines would be still closer.

In Fig. 142 the same wave-length is used as in the preceding figure, but the distance from *a* to *b* is only slightly greater than a wave-length. It will be noted that *nm*, *cd*, *tu*, etc., are widely separated. If the distance from *a* to *b* were just one wave-length, *ef* and *kl* would coincide with *CD*. If *ab* were one-half wave-length, the lines *cd* and *tu* would coincide with *CD* and all points to the right of the screen would be in a state of disturbance, the maximum disturbance being along *nm*.

It appears, therefore, that light would, under certain conditions, spread out into the region back of the screen, and the condition is that the opening through which the wave disturbance passes be commensurate with the length of the wave.

139. Rectilinear Propagation.—In our ordinary experience the opening through which light is admitted to any chamber or room is large as compared to the wave-length of light. As a consequence the light appears to move in a straight line past the edge of the opening and does not spread out in the room as sound waves ordinarily do. To show that this is what would be expected according to the wave theory, let *ab*, Fig. 143, be a large opening in the screen *CD*. Let a series of plane waves represented by *AB* fall upon *CD*. At the opening *ab* they will pass through and proceed in straight lines to *cd*. Why do the light waves not spread out into the regions above *bd* and below *ac*, as might be inferred from Fig. 141?

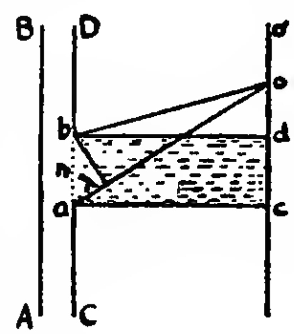

FIG. 143.

According to Huygens' principle (§ 137) each point between *a* and *b* will, when they are in a wave-front, become independent

centres of disturbance and will send out secondary waves which will an instant later form the wave-front beyond them. In Fig. 141 we considered the effect when there were points of disturbance only at *a* and *b*, while here a similar disturbance arises from all the points between *a* and *b*.

Now, let *o* be a point one wave-length farther from *a* than from *b*. Then *n*, midway between *a* and *b*, will be one-half wave-length farther from *o* than *b* is, and nearer to *o* than *a* is. Hence waves from *b* and *n* will be in opposite phase at *o* and will destroy each other. Likewise for waves from *n* and *a*. Also for any point below *b* there is a point below *n* which is sending out waves that are opposite in phase. Hence *o* is a point of quiescence, or darkness.

Another point, as *o′*, would be two wave-lengths farther from *a* than from *b*, and here again there would be complete interference, for one-half of the points between *b* and *n* will produce waves opposite in phase to those produced by the other half and so likewise for waves from points between *n* and *a*. In the same manner it may be shown that any point above *d* or below *c* which differs in distance from *a* and *b* by an even number of wave-lengths will be dark as a result of destructive interference.

Between *o* and *o′*, however, there must be a point which differs in distance from *a* and *b* by one and one-half wave-lengths. If we regard the points between *a* and *b* as divided into three equal groups, the waves from one group will destroy those from the next, for each wave from one group is one-half wave-length behind a corresponding wave from the other, but one-third of the waves are not destroyed and so will produce disturbance—light—between *o* and *o′* one-third as intense as if none of the waves had interfered. Likewise there must be a point above *o′* which differs in distance from *a* and *b* by two and one-half wave-lengths, and by a reasoning similar to that above we see that here four-fifths of the waves are destroyed and only one-fifth of them produce light. Thus the farther we go above *d* or below *c* the less intense the disturbance becomes.

As shown in Fig. 141, the lines *cd*, *tu*, etc., come closer and closer to *mn* the greater the distance between *a* and *b* is in comparison with the wave-length. Hence, since waves of light are exceedingly short compared to the distance here assumed between *a* and *b*, the points *o*, *o′*, and the points of partial light between them

will be practically coincident with *d*, and corresponding points below *c* will likewise coincide with that point. Thus the wave theory gives a satisfactory explanation of why light does not bend around the edges of apertures in cases which are ordinarily observed. If, however, the opening is very small, such as a hole made by a fine needle in a thin sheet of metal, a portion of a plane wave-front will pass through the hole, spread out, and illumine an area on a screen much greater than the area of the hole.

Sound waves as ordinarily used are several feet in length and so are commensurate with openings, such as windows or doors, through which sounds are admitted. Hence sound waves may pass through a door, for example, and spread to all points of a room beyond, as partially illustrated in Fig. 142.

If very short sound waves are used, the same phenomena will be produced as is ordinarily observed in case of light. An interesting experiment illustrating this may be shown by use of a sensitive flame. This, as shown in Fig. 144, is an ordinary pin-hole gas burner which produces a tall, slender flame. When the gas pressure reaches a certain point the flame will flare—*i.e.*, will drop in height, spread out laterally, and give out a roaring sound.

When such a flame is just on the point of flaring, it is very sensitive to short sound waves such as are produced by a shrill whistle. Even the rattle of a bunch of keys at a distance of 50 feet or more will cause the flame to flare.

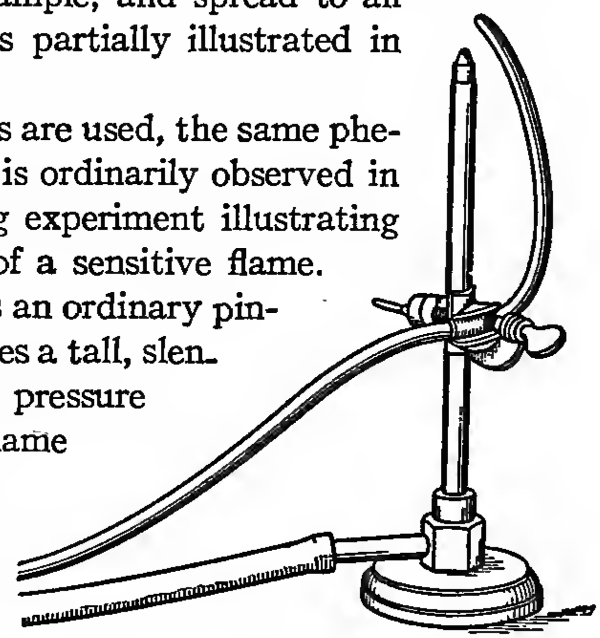

FIG. 144.

By use of the attachment shown in the figure it is easy to produce such a flame at ordinary gas pressure. This is simply a tube drawn to a fine point where a small gas jet is kept burning a short distance above the pin-hole burner. This small jet may be called the igniter, and there will be a flame only from this point upward. By this arrangement a very sensitive flame may be easily prepared by first lighting the igniter and then turning on gas through the pin-hole until the proper pressure is reached.

Sound waves so short that they produce no audible sound will be detected by the sensitive flame. Such a sound may be produced by a very short whistle. Galton's whistle is very good for this purpose.

If now a body, such as a strip of card-board, be interposed between the whistle and the flame, the flaring ceases, showing that the card caused a sound-shadow, and by moving the flame from side to side it will be observed that the edges of the shadow are quite distinct, thus showing that waves of sound when very short are also propagated in straight lines past the edge of an obstacle.

140. Velocity of Light.—Since light travels with a velocity of about 300,000 km. or 186,000 miles per second it is evident that an exact determination of its speed would be a matter of considerable difficulty. A wave of light travels from the sun to the earth in about 8.5 minutes and would pass around the earth 7.5 times in one second. Hence any experiment to determine this velocity would require that the distances between two points of observation be very great or that means of recording very short intervals of time be available.

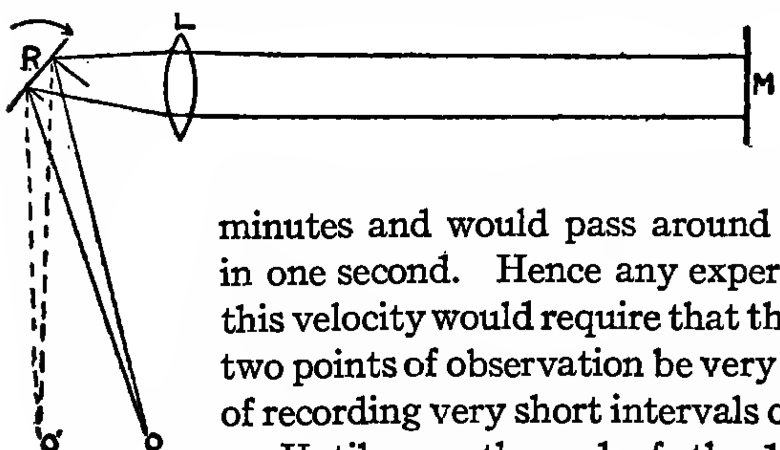

FIG. 145.

Until near the end of the 17th century it was believed by many that the speed of light was infinite, *i.e.*, that no time was required for light to pass from one point to another. We know now both from theory and experiment that an electromagnetic wave, whether long or short, moves through space with a velocity of about $3(10)^{10}$ cm. per second.

A number of ingenious experiments have been devised to determine as exactly as possible the velocity of light. We will here describe only the method devised by Foucault in 1862 and used in a modified form by Michelson and Newcomb working independently in 1882.

The method employed will be understood from Fig. 145, where *o* is a source of light from which a diverging beam falls on a rotating mirror *R* and is reflected through a lens *L* which makes the rays parallel. The waves then pass on through a long distance to a plane mirror *M*, from which they are reflected back in the same

path and, passing through the lens, will be again reflected from R and come together at o, the point from which they started. This is on the assumption that R remains stationary while waves passed from it to M and back to R. If, however, R is rotating in the direction of the arrow and has turned through a small angle while light passed from R to M and back, then the focus of the returning beam will no longer be at o but at some point o'. In one of Michelson's experiments the distance from R to M was 625 m., and by using a lens of long focus the distance from o to R was 9 m. Then the displacement from o to o' was 13.3 cm. when the mirror made 257 rotations per second.

It is plain that if we know the distance from R to M and the time required for light to pass over twice this distance—*i.e.*, out and back—the velocity can easily be calculated.

Let θ be the angle through which the mirror turns while light goes from R to M and returns. Then the angle through which the reflected beam Ro' has been deflected is 2θ, for a reflected beam always turns through an angle twice as great as that through which a mirror turns. Let D be the distance from R to M and t the time required for light to pass over a distance $2D$, then the velocity V is

$$V = \frac{2D}{t} \tag{159}$$

Let n be the number of revolutions made by the mirror in one second, then $2\pi n$ is the total number of radians per second and the time required for θ radians is

$$t = \frac{\theta}{2\pi n} \tag{160}$$

Substituting this value of t in (159),

$$V = \frac{4\pi n D}{\theta} \tag{161}$$

Let s be the distance from o to o', and d the distance from R to o, then

$$s = 2\theta d$$

$$\text{or} \qquad \theta = \frac{s}{2d} \tag{162}$$

$$\text{and} \qquad V = \frac{8\pi n d D}{s} \tag{163}$$

The quantities represented in this equation may be measured and by this method Michelson found

$$V = 299{,}850 \text{ km. per second}$$

Newcomb, using the same method but with different distances, found

$$V = 299{,}860 \text{ km. per second}$$

These figures are probably accurate to within .01 of one per cent.

The original purpose of Foucault's experiment was not to determine the velocity of light, but whether or not the velocity is less in a denser medium. He showed that when light is passed through water its velocity is retarded. This, as shown in § 136, was a test of the two theories of light, and from that time the corpuscular theory was admitted to be untenable.

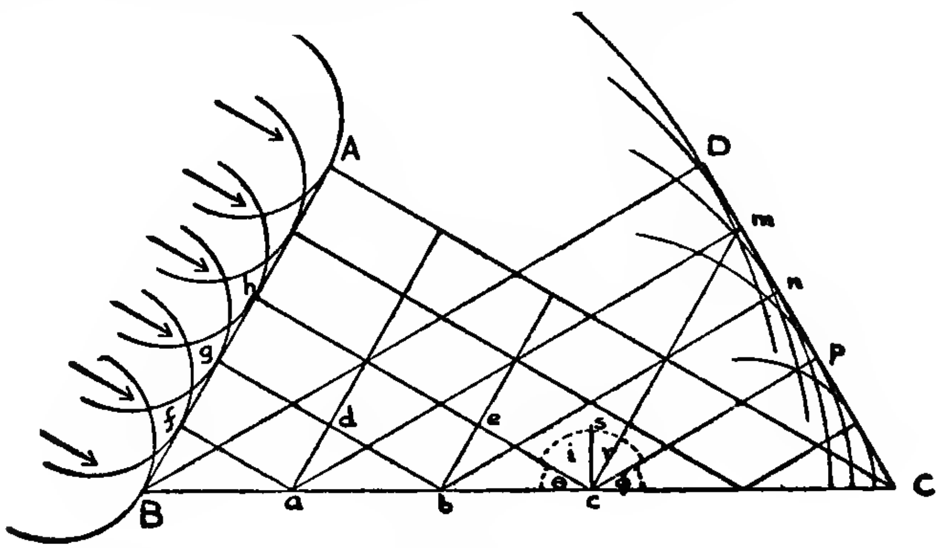

Fig. 146.

141. Reflection from a Plane Surface.—We will first consider the reflection of a plane wave from a plane surface. Let BC, Fig. 146, be a mirror or any reflecting surface and let AB be a plane wave-front. For simplicity only a few of the secondary waves are represented. The front AB moves in a direction indicated by arrows. From points f, g, h, etc., draw perpendiculars to the wave-front. Thus fa, gb, hc, etc., are lines which are defined as rays.

The point B on the mirror will be reached first by the incident wave-front. This point will then be a centre of disturbance and will send out a wave in all directions above the mirror. At the end of a very short time the radius of this spherical wave will be

BD, the line BD being here taken equal to AC. BD may be any length. With B as centre and BD as radius, describe an arc at D, or a whole semicircle above the mirror.

When the wave started out from the point B, the point f on the wave-front had not yet reached the mirror by a distance fa. When a then becomes a centre of disturbance as a result of the incidence of f, the spherical wave sent out will be behind that from B by the distance fa. Hence with a as a centre and a radius equal to $BD-fa$ describe an arc at m. Likewise with b as centre and a radius equal to $am-db$ describe an arc at n. So also for other points on BC. It will then be seen that a line DC, tangent to all these arcs, will mark the position of the new wave-front after reflection, and since the reflected wave from B had the same time in which to reach D as the incident wave had to move from A to C, then the lines AC and BD are equal; consequently the angle ABC is equal to DCB or the angle of incidence is equal to the angle of reflection.

Angles of incidence and reflection are, however, usually defined as the angles included by a normal to the mirror and the incident and reflected rays. Let sc be a normal to the mirror, then i is the angle of incidence and r the angle of reflection. Since the right triangles cpC and chB are similar; the angle θ is equal to the angle ϕ and consequently i is equal to r.

142. Curvature.—The curvature at any point is the rate of departure of a curved line from a tangent to the curve at that point.

Curvature is conveniently expressed in terms of the angle formed by the tangents drawn through the extremities of unit length of arc, as θ', Fig. 147. It is seen that θ' is equal to θ, the angle at the centre of the circle. If s is length of unit arc, R the radius of the circle, and θ is measured in radians, then

$$s = R\theta$$

or

$$\frac{\theta}{s} = \frac{1}{R} \tag{164}$$

The first term of this equation is, by definition, the curvature. Hence if C stands for curvature,

$$C = \frac{1}{R} \tag{165}$$

Curvature may therefore be expressed as the reciprocal of the radius.

If an angle larger than θ be taken, the arc s will be increased in the same proportion and θ/s will still equal $1/R$, *i.e.*, the curvature does not change on this account. If R is made longer, say oc, twice oa, then the arc s' is twice as long as s, hence the curvature of s' is one-half as great as that of s. In general, then, curvature varies inversely as the radius and from our definition is equal to $1/R$.

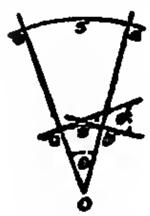

FIG. 147.

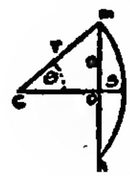

FIG. 148.

In the particular case shown in the figure where s is assumed to be of unit length, say 1 cm., equation (164) may be written

$$\theta = \frac{1}{R}$$

but whatever the angle or length of arc, it is seen from the reasoning above that

$$\frac{\text{angle}}{\text{arc}} = \frac{1}{R}$$

The sagitta of an arc may also be taken as a measure of curvature. In Fig. 148 the sagitta is the line s drawn from the middle of the chord mn to the vertex of the arc.

It is readily seen from the construction of the figure that

$$r^2 = a^2 + (r - s)^2$$

$$\therefore s = \frac{a^2}{2r - s}$$

where a is one-half the length of the chord.

If θ is small, as it should be in the following discussions (see Fig. 158), s in the denominator may be neglected in comparison with $2r$, hence

$$s = \frac{a^2}{2r}$$

The sagitta is therefore inversely proportional to the radius of curvature, *i.e.*, directly proportional to curvature.

143. Reflection of a Spherical Wave from a Plane Surface.—Let o, Fig. 149, be a point from which light waves emanate, and let mm' be a reflecting surface. When a wave-front reaches the mirror, as it will first at d, this point becomes a centre of disturb-

ance and sends out a spherical wave in the opposite direction. So also for points in the wave to either side of d when they are incident on the mirror.

Let a and b be two elementary areas on an advancing wave. The total surface of the spherical wave may be regarded as composed of such areas, each of which is a plane. Now when a or b or any other of the infinite number of these small planes reaches the mirror there will be reflection in the manner described for Fig. 146 where it is shown that when a, for example, is reflected it will retain the same inclination to the mirror as it had before incidence but will be oppositely directed—*i.e.*, the end of a which first reaches the mirror will after reflection be farthest from it. Consequently when all these small plane waves have been reflected, the spherical wave-front will be reversed and the curve sds', which would have advanced to c if the mirror had not been there, now becomes ses'.

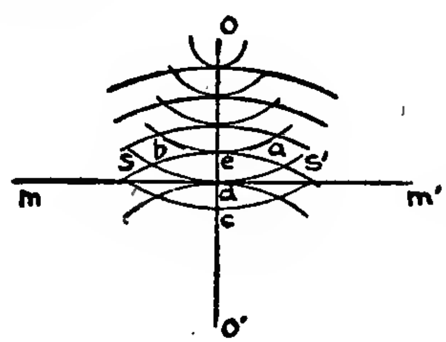

FIG. 149.

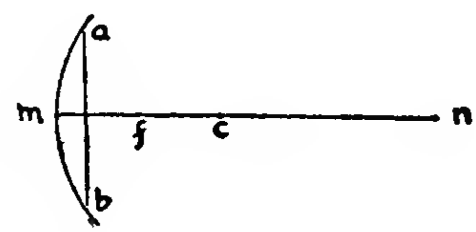

FIG. 150.

The reflected wave-front will therefore have the same curvature as the incident wave, and an observer looking into the mirror will receive spherical waves as if emanating from o', for this is the centre of the reflected curves.

What is seen at o' is called a *virtual* image of o, for the waves appear to have originated from o' rather than from o.

It is also evident, from the fact that the curvatures of the incident and reflected waves are the same, that the image is as far back of a plane mirror as an object is in front of it.

144. Reflection from a Curved Surface.—Consider first a concave reflector and let it be a spherical mirror with radius R—the distance mc in Fig. 150. The curvature of the mirror is then $1/R$. The point c is the centre of curvature, and the line mn drawn through c to the mirror is called the principal axis.

Let the line *ab* represent a plane wave perpendicular to this axis and approaching the mirror. The ends *a* and *b* will first reach the mirror and will be reflected while the middle portion of the wave-front will be reflected last. This will give to the wave-front a curved form having its centre, say, at *f*. This point *f* is called the principal focus of the mirror. We may therefore define a principal focus as a point on the principal axis to which a plane wave, perpendicular to this axis, will converge after reflection, or, since rays are perpendicular to a wave-front, as the point at which rays parallel to the principal axis meet after reflection.

Let the distance from *m* to *f* be denoted by F. Then at the instant the whole of *ab* has been reflected and is ready to start back, the curvature of the wave-front is $1/F$. The concave mirror will therefore change the wave-front from one whose curvature is zero to one whose curvature is $1/F$. This is the change in curvature which the mirror is capable of impressing on a plane wave-front and is sometimes called the focal power of the mirror.

Let the incident wave *ab*, Fig. 151, be spherical with a centre at *o*. It is plain that when *a* and *b* reach the mirror the central part of the wave is nearer *m* than it would be if the wave were plane, consequently the curvature produced by reflection will be less than $1/F$ by the amount of curvature which the wave possessed before incidence. If O represents the distance from *m* to *o*, then the curvature of the incident wave is $1/O$, and if after reflection the centre of the wave is at *i* and we denote by I the distance from *m* to *i*, the curvature of the reflected wave is $1/I$. Hence

$$\frac{1}{F}-\frac{1}{O}=\frac{1}{I}$$

or

$$\frac{1}{O}+\frac{1}{I}=\frac{1}{F} \tag{166}$$

If the source, *o*, of light waves is at any point between *c* and an infinite distance from the concave mirror, a real image *i* will be formed between *f* and *c*, for when *o* is at an infinite distance *i* will be at *f*, as shown in Fig. 151; and when *o* is at *c* the curvature of the incident wave is the same as that of the mirror, and so *i* will also be at *c*.

If o is at any point between f and c, the curvature of the incident waves will be greater than that of the mirror, and the middle portion of the wave will be incident first. The distance of the ends a and b, however, from the mirror is not so great as it would be if f were the source of the waves, and the curvature impressed by the mirror is $1/F$ less the curvature which the incident wave had. Hence equation (166) can, as before, be used to find the position of the image when that of the object is known, or the position of the object when the distance of the image from the mirror is known, the focal distance being given in both cases.

When the object, o, is between f and c, Fig. 152, a real image, i, will be at some point beyond c, for when o is at c the image will be there also, and when o is at f the wave will be made plane by reflection and so i will be at an infinite distance.

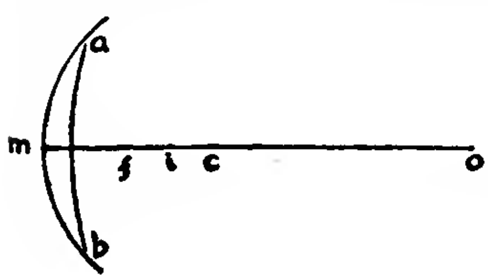

Fig. 151.

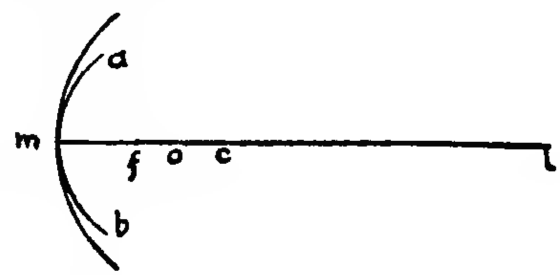

Fig. 152.

If o is between f and m, the curvature of the incident wave is greater than that from f, consequently the ends a and b must move so far before they are reflected that the point m in the wave gets ahead of other portions of the reflected wave-front. The change in curvature is sufficient not only to make the wave plane but to actually reverse it, so that the image i is virtual and appears on the opposite side of the mirror. Equation (166) will under these conditions give a negative answer as it should. Thus, suppose the focal length of the mirror is 12 cm. and the source of light waves is 3 cm. from the concave side of the mirror, then

$$\frac{1}{3}+\frac{1}{I}=\frac{1}{12}$$

$$\therefore I=-4$$

This means that the image of the point o is virtual and is located 4 cm. back of the mirror.

It may be shown by equation (166) that the principal focus f is half way between the mirror and its centre of curvature c. Let R be the distance from m to c—*i.e.*, the radius of the sphere of which the mirror is a part—then $1/R$ is the curvature of the mirror. Now when the source of light is placed at c, both O and I are equal to R. Hence for this condition

$$\frac{1}{R}+\frac{1}{R}=\frac{1}{F}$$

$$\therefore F=\frac{R}{2}$$

Another way of stating the same fact is that since

$$\frac{1}{F}=\frac{2}{R}$$

the curvature impressed by a concave mirror on a plane wave is twice the curvature of the mirror.

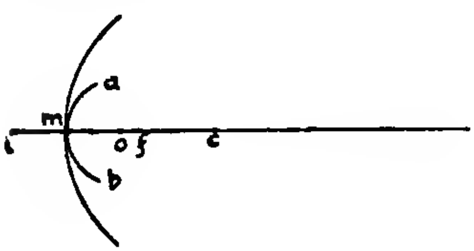

FIG. 153.

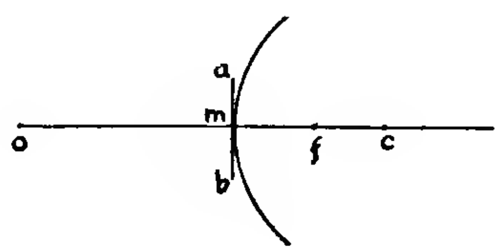

FIG. 154.

When a plane wave is incident on a convex mirror, the middle portion of the wave is reflected first and the ends a and b, Fig. 154, are then at a distance from the mirror equal to the sagitta of the arc in case the wave were incident on the concave side, consequently the change in curvature will be the same as if the plane wave were incident on the concave side, *i.e.*, twice the curvature of the mirror. Hence the image will be at f as before, *i.e.*, the principal focus, f, is again midway between c and m.

If, now, a spherical wave originating in o is incident at m on a convex mirror, Fig. 155, points a and b will have to move forward not only to the position of the plane wave, but, in addition, whatever movement the plane wave went through before all portions

of it were reflected. The change of curvature in a plane wave is from zero to $1/F$, hence, using for distances the same letters as above,

$$\frac{1}{F}+\frac{1}{O}=\frac{1}{I}$$

or
$$\frac{1}{I}-\frac{1}{O}=\frac{1}{F} \tag{167}$$

The image in a convex mirror will therefore always be virtual and will be located between f and m.

In cases described above where the image is real, o and i are called conjugate points or conjugate foci, for if the source of light is at either one the image will be at the other.

To make a drawing which will show the position, size, and nature of an image in a spherical mirror it is more convenient to apply the principles of geometrical optics and use rays instead

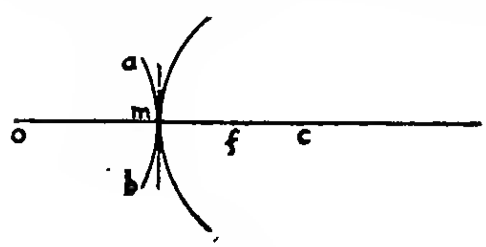

FIG. 155.

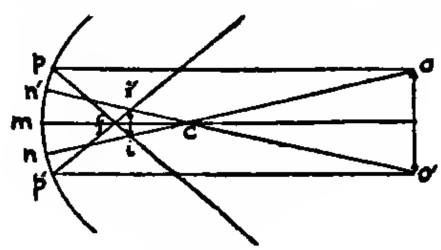

FIG. 156.

of wave-fronts. Thus in Fig. 156 let oo' be a luminous object, every point of which is a source of light waves. It will be sufficient in this case to locate the image of the two ends of oo'. To locate the image of o, draw two rays, op parallel to the principal axis and on perpendicular to the mirror. After reflection op will pass through f and on will return on its incident path. The two will meet at i, hence i is the image of o. In the same manner draw two rays from o'. These will meet at i'. Hence i' is the image of o'. The image of any other point of oo' may be located in a similar manner. All will be found to be at some point between i and i'.

The image in this particular case is thus found to be real, inverted, and smaller than the object, and since points midway on oo' are farther from the mirror than the extremities, the line ii' will be slightly convex toward the mirror.

In the same manner images may be found when oo' is between c and f, f and m, or on the convex side of the mirror.

145. Spherical Aberration. Caustics.—In previous discussions it was assumed that a spherical wave-front would still be spherical after reflection from a spherical mirror. This, however, is not the case, for the farther the point of incidence, p, Fig. 156, is from the vertex, m of the mirror, the more nearly will f approach m. Let a ray parallel to the principal axis be incident at p, Fig. 157. The reflected ray will cross the principal axis at some point f. Let cp, the radius of curvature of the mirror, be designated by R, cf by a, and fp by b.

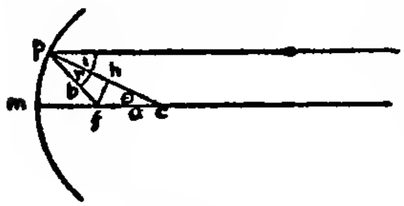

FIG. 157.

$$\text{Angle } \theta = \text{angle } r = \text{angle } i$$
$$\therefore a = b$$
$$hc = a \cos \theta$$
$$hp = b \cos r$$
$$\therefore R = 2a \cos \theta$$

and
$$a = \frac{R}{2 \cos \theta} \tag{168}$$

Equation (168) shows that as θ becomes greater its cosine becomes less and therefore a will be greater, *i.e.*, f will be farther from the centre c. But if θ is so small that its cosine may be regarded as unity,

$$a = \frac{R}{2}$$

For practical purposes, therefore, we say that all the rays from a point are focused at a common point, provided only those rays which are close to the principal axis are admitted to the mirror. Hence only about 15 or 20 degrees of the aperture of such a mirror should be used.

If a large number of rays be drawn to a spherical mirror of large aperture, the reflected rays will intersect as shown in Fig. 158, and a curve drawn from f tangent to all these rays is called a caustic curve, foa and $fo'b$, the cusp of which is at the principal focus f. This effect is greatest when the incident wave is plane,

as in Fig. 157, and decreases as the source of light approaches the mirror. When c is the source, the reflected wave-front is spherical.

A caustic curve may be plainly observed by letting sunlight or lamplight fall on the concave surface of a cup or glass partly filled with milk or turbid water.

146. Parabolic Mirror.—A concave mirror whose surface is a paraboloid of revolution will bring plane waves to an exact focus—*i.e.*, without spherical aberration—and, conversely, if the source of light is at the focus, the reflected wave will be plane.

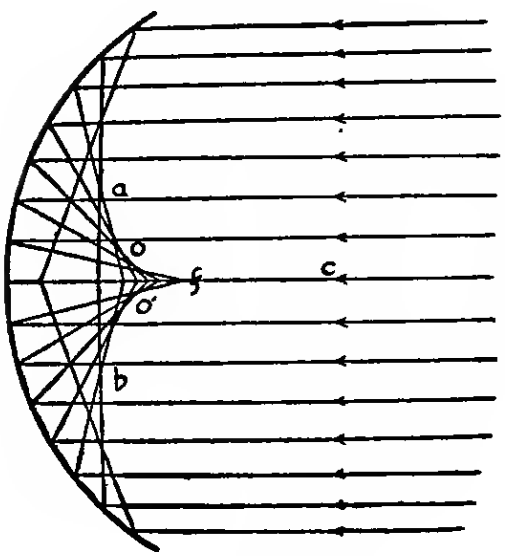

FIG. 158.

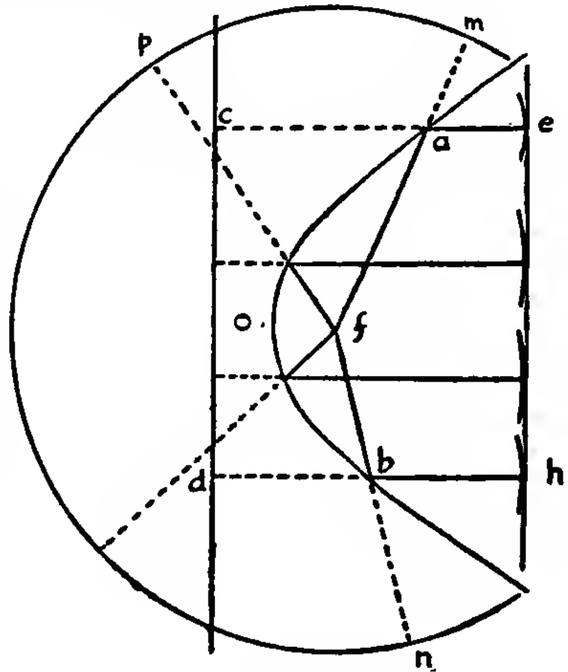

FIG. 159.

Let the curve aob, Fig. 159, be a section of the paraboloid of which f is the focus and cd the directrix. Any point on a parabola is equally distant from the focus and the directrix, as $af=ac$, $bf=bd$, etc. Let a point of light be placed at f. If the mirror had not been there a spherical wave-front would at a certain instant have reached mpn, but the ray fa was reflected to ae, fb to bh, and so for other rays. At the instant the wave would have reached m it will now reach e, ae being made equal to am. Likewise bh is made equal to bn and so for other rays. Hence

$$ce=fm$$

and $$dh=fn$$

But $$fm=fn$$

$$\therefore ce=dh$$

The line *eh*, which is the position of the wave-front after reflection, is therefore parallel to *cd*, and so the reflected wave-front is plane.

It is evident, then, that when a plane wave is incident on such a mirror it will be made to converge to *f* without aberration.

Mirrors of this kind are extensively used as reflectors of head-lights and search-lights, for the light of an arc lamp placed at the focus will be projected as a powerful beam.

A valuable use of paraboloid reflectors is found in objectives of reflecting telescopes. Such mirrors may be made of polished speculum metal composed of an alloy of copper and tin, or of silvered glass.

A very perfect mirror of this kind may be formed by rotating a pan of pure mercury. If the rotation can be kept steady and uniform the surface of the mercury will be depressed at the centre and raised at the sides of the pan, thus forming a paraboloid as long as the rotation is continued.

Problems

1. Two plane mirrors are placed at an angle of 60° to one another with an object midway between them. By drawing rays find the position of the images.

2. If a plane wave is focused at a point 10 cm. from a concave mirror, what must be the position of a luminous point that its image may be 12 cm. from the same mirror?

3. Find the position of a pair of conjugate foci of a concave mirror whose radius of curvature is 40 cm.

4. By use of rays make a drawing to show the position and nature of the image of a candle flame in a convex mirror.

5. A ray of light parallel to the principal axis is incident at a point 60° from the vertex of a concave mirror whose radius of curvature is 50 cm. How far from the principal focus will the reflected ray intersect the principal axis?

6. An object 2 cm. long is placed 10 cm. in front of a concave mirror whose radius of curvature is 12 cm. What will be the length of the image?

7. A beam of light is reflected from a plane mirror to a scale 2 m. distant from the mirror. If the mirror is turned through .2 radian, how many centimetres will the spot of light move on the scale?

Ans. 1. Five images.
2. 60 cm.
3. *e.g.*, 25 and 100.
4. Erect and virtual.
5. 25 cm.
6. 3 cm.
7. 80 cm.

147. Refraction.—When waves of light pass from one medium to another of different density, part of the light is, as a rule, reflected and another part enters the second medium where it may be at once absorbed, or, if the medium is transparent, is transmitted with a change in velocity and direction. The change in direction which occurs when light passes from one medium to another is called *refraction*.

Let AB be a plane wave-front incident on a plate of glass G, Fig. 160. The greater part of the light will enter the glass and pass on through it.

Let t be the time required for the wave to pass from A to C and v_1 the velocity in air or other rare medium. When one end of the wave is at A the other end B is just entering the glass. The point B then becomes a centre of disturbance and sends out waves with velocity v_2 in the glass. When A has arrived at C, B will have moved a certain distance BD, less than AC, for the velocity is less in a denser medium. Hence

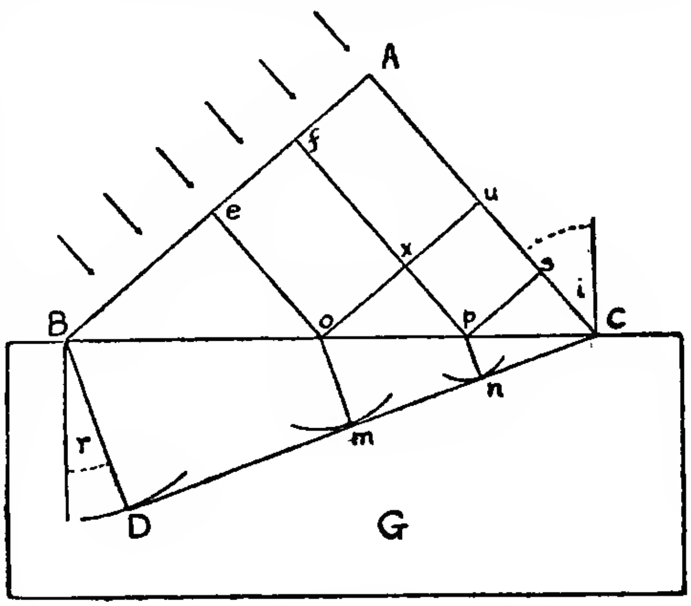

Fig. 160.

$$\frac{AC}{BD}=\frac{v_1t}{v_2t}=\frac{v_1}{v_2}$$

As the successive points in AB reach the glass each becomes a centre of disturbance. When A has arrived at u, for example, e is at o and, as shown above,

$$\frac{uC}{om}=\frac{v_1}{v_2}$$

and $$\frac{sC}{pn}=\frac{v_1}{v_2}$$

Hence

$$BD : om : pn = BC : oC : pC$$

Consequently a line from C to D is tangent to all the spherical waves, and so is the position of the wave-front in the glass.

Let normals to the surface be drawn at C and B. Then i is called the angle of incidence and r the angle of refraction. The angle i is equal to the angle ABC and r to BCD. Hence

$$AC = BC \sin i$$

and

$$BD = BC \sin r$$

Hence

$$\frac{AC}{BD} = \frac{v_1}{v_2} = \frac{\sin i}{\sin r} = \mu \tag{169}$$

where μ is called the index of refraction, defined as the ratio of the sine of the angle of incidence to the sine of the angle of refraction. Refraction, therefore, is caused by a change in the velocity of the wave-front when passing from one medium to another of different optical density.

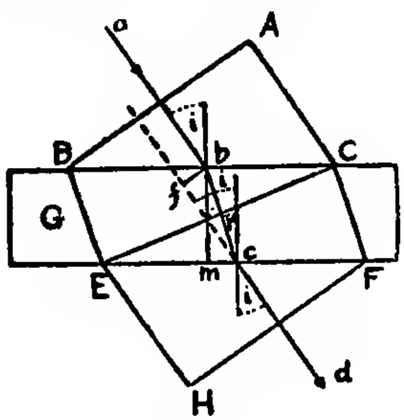

Fig. 161.

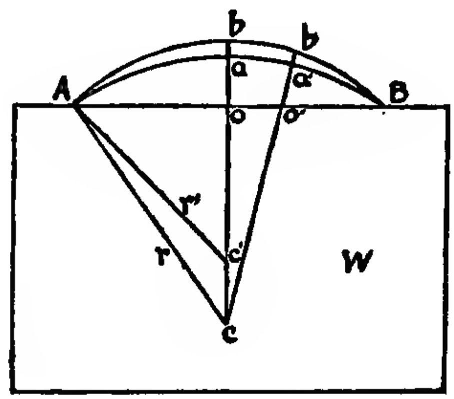

Fig. 162.

If the wave-front is parallel to the interface separating the two media then it is evident from Fig. 159 that, although the velocity is different in the second medium, the direction is not changed.

When the sides of a second medium are parallel, as in case of plate glass G, Fig. 161, the direction of an incident wave will be deflected toward the normal on entering the glass, but, on emerging from the glass to air, will be deflected an equal amount from the normal so that the direction is the same as before incidence. This is apparent from Fig. 161, for the wave traverses a distance BE or CF in glass in the same time as it moves over AC in air. When the wave at E is just emerging into air, the point C must still

traverse a distance CF in glass. Hence EH will be equal in length to AC and HF is parallel to BA. Consequently ab lies in the same direction as cd.

There is, however, a lateral displacement bf, the amount of which depends on the refractive power of the medium and the distance between the two sides. This may be calculated when the angle of incidence and the index of refraction are known, for

$$bf = bc \sin (i - r)$$

and
$$bc = \frac{bm}{\cos r}$$

Let the thickness of the glass, bm, be denoted by d, then

$$bf = d\,\frac{\sin (i - r)}{\cos r}$$

When i and μ are given, r can be found from equation (169).

This phenomenon may be observed by holding a pencil back of a piece of heavy glass. The part seen obliquely through the glass will appear to be out of line with that above the edge of the glass.

148. Refraction of a Spherical Wave at a Plane Surface.—Let a vessel W, Fig. 162, be filled with a transparent liquid and let AB be the surface separating the liquid from the air above. Let c be the origin of spherical light waves.

In a certain time a wave would have moved through the liquid to a with AaB as the wave-front, but at o it began to emerge from the liquid into a less dense medium and hence to move faster, so that in the same time that the wave would have advanced from o to a in water it moved from o to b in air. Let the velocity in air be v_1 and in water v_2, then

$$\frac{ob}{oa} = \frac{v_1}{v_2} = \mu$$

$$\therefore ob = \mu \cdot oa$$

Likewise

$$o'b' = \mu \cdot o'a'$$

where μ is the index of refraction from air to water or other medium denser than air.

The arc AbB is not spherical but when the arc is short, as that used in vision, it may be regarded as spherical. Since the curvature is increased, the centre of the wave will now be at a point c' instead of c, and an observer looking down into the water will see c', an image of c.

Since the sagittæ of the arcs are inversely proportional to the radii of curvature (see § 142),

$$\frac{ob}{oa} = \frac{cA}{c'A} = \frac{v_1}{v_2} = \mu \tag{170}$$

When an observer looks vertically down into a transparent medium both A and B will be very close to o, for the area of cross section of the pencil of light that can enter the pupil of the eye is very small. The arcs and their sagittæ will consequently be very short and may be disregarded in comparison with the radii. Hence under these conditions we may put $cA = co$ and $c'A = c'o$. Then

$$\frac{ob}{oa} = \frac{co}{c'o} = \frac{v_1}{v_2} = \mu \tag{171}$$

Since the index of refraction from air to water is $\frac{4}{3}$,

$$\frac{co}{c'o} = \frac{4}{3}$$

$$\therefore c'o = \frac{3}{4} co$$

i.e., the vertical depth of water appears to be only three-fourths of what it actually is.

These principles may be applied in finding the relative velocity of light in air and other media, *i.e.*, the index of refraction from air to other media. Suppose a microscope is focused on c, Fig. 162, when the vessel is empty. Then let the vessel be filled with a liquid to a height d above c. The microscope must then be raised a distance cc' to bring it into focus again. Hence from equation (171)

$$\frac{co}{c'o} = \frac{d}{d - cc'} = \frac{v_1}{v_2} = \mu$$

149. Relative and Absolute Index of Refraction.—The ratio of the velocity of light in vacuum to the velocity in another sub-

stance is called the *absolute* index of refraction. The ratio of the velocity of light in one substance to that in another is called the *relative* index.

The absolute index for air under standard conditions is 1.00029, *i.e.*, the velocity of light in vacuum is only 1.00029 times as great as in air. Hence the term *index of refraction* of a substance ordinarily refers to the relative index from air to a substance. The absolute index may be found by multiplying this relative index by the above number.

The relative index for any two substances is found by taking the ratio of the relative indices of these substances to a third substance, usually air. For example, it is known that the index for air to glass is $\frac{3}{2}$, and for air to water is $\frac{4}{3}$. Hence, expressing the index as a ratio of velocities,

$$\frac{\text{velocity in air}}{\text{velocity in glass}} = \frac{3}{2}$$

and

$$\frac{\text{velocity in air}}{\text{velocity in water}} = \frac{4}{3}$$

Dividing the former by the latter,

$$\frac{\text{velocity in water}}{\text{velocity in glass}} = \frac{9}{8}$$

Hence we may write

$$\mu_{wg} = \frac{9}{8}$$

or

$$\mu_{gw} = \frac{8}{9}$$

Hence the relative index of any medium in reference to a second medium may be found by dividing the index of the second medium in reference to air or a vacuum by that of the first medium.

150. Critical Angle. Total Reflection.—A critical angle of refraction is an angle of such a value that the refracted ray is parallel to the interface which separates the two media.

Let *o*, Fig. 163, be a point source of light waves in a dense medium such as water or glass. The disturbance propagated along the ray *oa* will emerge into air without bending. Other rays such as *of* will be refracted more and more from the normal

as the angle r becomes greater. At a certain angle C called the *critical angle*, the ray oc is refracted to ch, *i.e.*, the angle of refraction is 90°. (The angle made by a ray with the normal will, as a rule, be called r in the denser medium and i in the rarer medium whatever may be the direction of the ray.) Whenever i is 90°, r is the critical angle.

When a portion of the wave, such as moves along the ray oe, reaches the surface, the deflecting effect resulting from an attempt to pass into a rarer medium turns that part of the wave over and it turns back into the first medium according to the law of reflection. Since none of the light passes into the second medium, this is called *total reflection*.

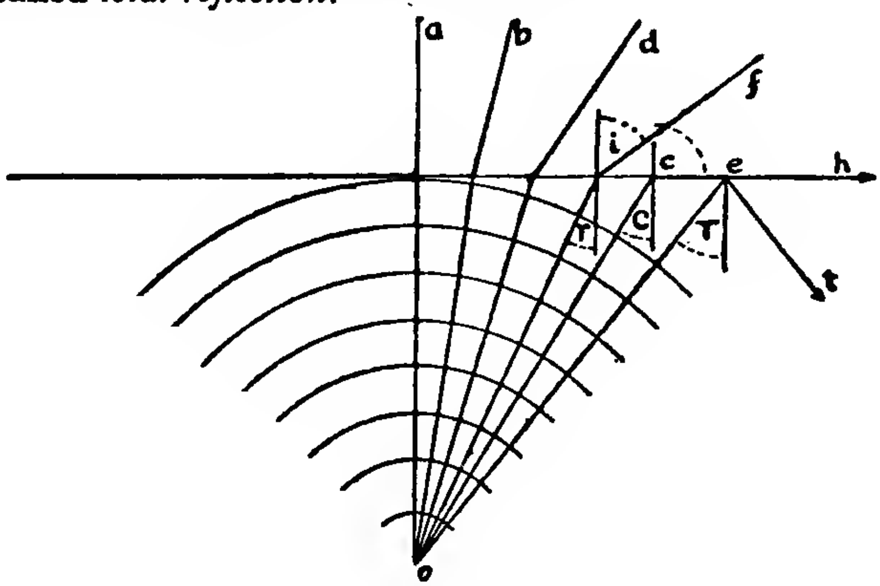

FIG. 163.

The critical angle of any medium in its relation to air may readily be calculated when the index of refraction, μ, is known. Thus,

$$\frac{\sin i}{\sin r} = \mu$$

$$\sin i = \sin 90° = 1$$

$$r = C = \text{critical angle}$$

$$\therefore \sin C = \frac{1}{\mu} \qquad (172)$$

The index for water, for example, is 1.33, hence

$$\sin C = \frac{1}{1.33}$$

$$\text{and} \qquad C = 48° \, 36'$$

The greater the index of refraction of any substance is, the smaller the critical angle; *e.g.*, the index for diamond is about 2.47, hence its critical angle is about 24° 26′. The brilliancy of a diamond is therefore due to the fact that a considerable portion of the light which enters it is totally reflected.

Many of the best optical instruments, which require that light be reflected at any point in its path, are provided with total reflecting prisms instead of mirrors.

Let ABC, Fig. 164, be a cross section of a prism, right angled at A and the side AB equal to AC. A ray of light entering the prism at right angles to AB will pass straight through to c, making an angle of incidence $i = 45°$.

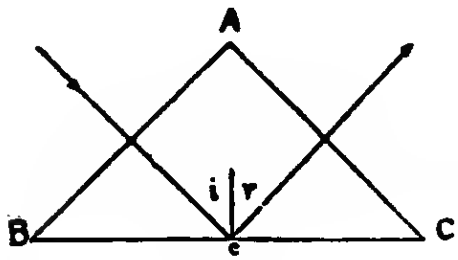

FIG. 164.

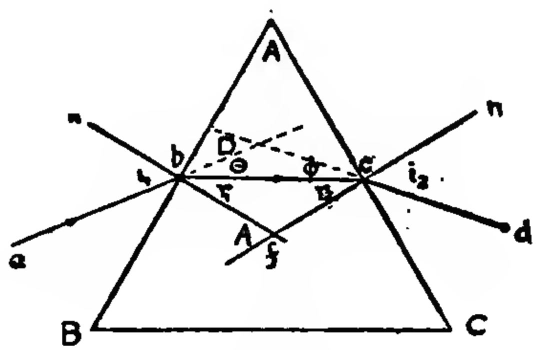

FIG. 165.

If the prism is made of either crown glass or flint glass, whose critical angles are 43° 2′ and 37° 34′ respectively, the light will be totally reflected at c and will pass out at right angles to AC.

151. Refraction by a Prism.—An optical prism is usually made of glass or some substance denser than air. Since the sides are not parallel, a beam of light entering the prism on one side will emerge from the other side in a changed direction, the refraction being toward the base of the prism.

Let a beam *abcd* be passed through the prism ABC, Fig. 165. The emergent beam will deviate from the path of the incident beam by the angle D. The exterior angle D is equal to $\theta + \phi$, but

$$\theta = i_1 - r_1$$

$$\text{and} \qquad \phi = i_2 - r_2$$

$$\therefore D = i_1 - r_1 + i_2 - r_2$$

$$= i_1 + i_2 - (r_1 + r_2)$$

Let the angle at the vertex of the prism be denoted by A.

Then A is the supplement of the angle bfc. r_1+r_2 is supplementary to the same angle. Hence

$$A=r_1+r_2$$

$$\text{and} \qquad D=i_1+i_2-A \tag{173}$$

Both theory and experiment show that the value of D is least when $i_1=i_2$. Under this condition

$$D=2i-A$$

$$\text{or} \qquad i=\frac{D+A}{2} \tag{174}$$

$$\text{Also} \qquad r_1=r_2$$

$$\therefore r=\frac{A}{2} \tag{175}$$

Hence the index of refraction of the material of the prism is

$$\mu=\frac{\sin i}{\sin r}=\frac{\sin\dfrac{D+A}{2}}{\sin\dfrac{A}{2}} \tag{176}$$

By measuring the angle A and the angle D when it is minimum, the value of μ may readily be calculated by equation (176).

152. Spectrometer.—One important use of a spectrometer is the measurement of A and D of equation (176). The instrument consists essentially of a *collimator* C, Fig. 166, a *telescope* T, and a *graduated circle* on which the positions of C and T may be read. The collimator is provided with an adjustable slit s for the admission of waves of light and a lens l which makes the wave-front plane, *i.e.*, makes the rays parallel. An image of the slit is seen in the telescope. A prism-holder h is located in the centre of the circle. The telescope may be turned to any position around the circle. The details of manipulation may be found in laboratory manuals.

One method of finding the angle of a prism is to turn the telescope to a position T_1, Fig. 167, where it is perpendicular to the side AB, and note the scale reading. Then the telescope is turned to a position T_2, perpendicular to the side AC, or the table carrying the prism is turned so as to bring the side AC perpendicular

to the telescope in position T_1. The number of degrees between these two positions is evidently the angle ϕ which is the supplement of A. Hence

$$A = 180^0 - \phi$$

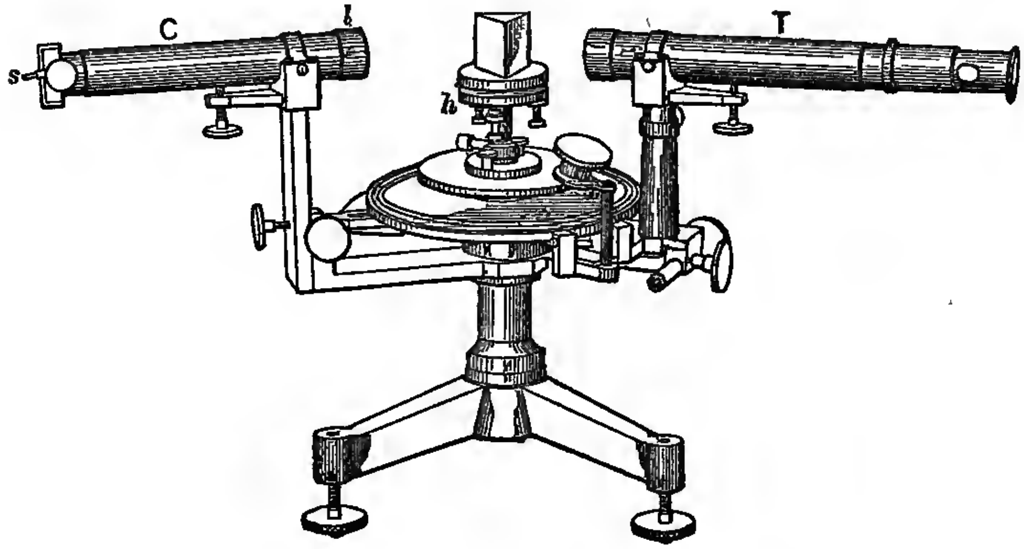

FIG. 166.

In this method the collimator is not used, but to aid in setting the telescope exactly in positions T_1 and T_2 it is provided with a Gauss eye-piece. This, as shown in Figs. 166 and 168, is a short

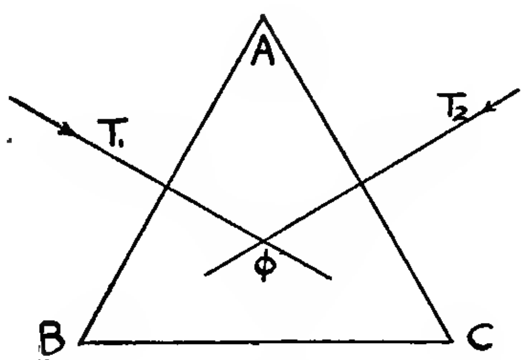

FIG. 167.

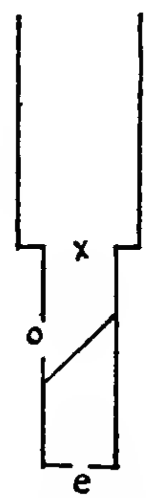

FIG. 168.

tube with an opening o in one side. Light admitted at o falls on a plate of glass set at an angle of 45°. The light is thus directed upon the spider lines x and on through the telescope to the face of the prism where it is reflected. An observer at e may then see

the spider lines directly and also their image. When they exactly coincide, the telescope must be at right angles to the reflecting surface.

In a second method of measuring A, use is made of both collimator and telescope.

Let C be a beam of parallel rays from the collimator falling upon the edge of the prism as shown (Fig. 169). Part of the rays fall on the face AB and are reflected to T_1 while other rays fall on face AD and are reflected to T_2.

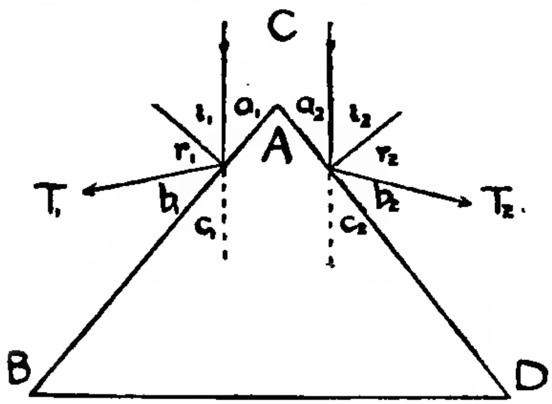

FIG. 169.

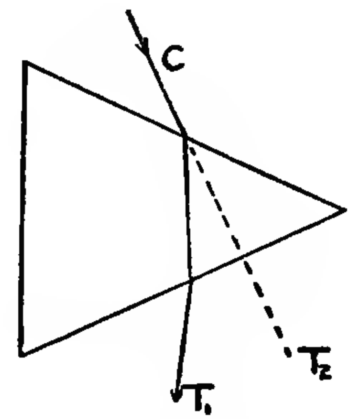

FIG. 170.

If the telescope be turned to position T_1 an image of the slit of the collimator will be seen. The telescope is then turned to the position T_2 where the image of the slit will again be seen. The number of degrees through which the telescope is moved between these two positions is twice the angle A. This will be apparent if we consider that

$$a_1 = b_1$$

$$a_1 = c_1$$

$$c_1 = \frac{A}{2}$$

$$\therefore b_1 + c_1 = A$$

Also
$$b_2 + c_2 = A$$

$$\therefore A = \frac{b_1 + c_1 + b_2 + c_2}{2}$$

The spectrometer is also used in finding D, the angle of minimum deviation.

A source of monochromatic light such as a sodium flame is

placed in front of the slit of the collimator and a beam C, Fig. 170, directed against one side of the prism. Refraction will occur, as shown in Fig. 165, and the beam will emerge from the prism in the direction T_1 where it is received in the telescope. By turning the prism table and following the image of the slit with the telescope, a position will be found where, although the rotation of the prism is continued, the motion of the image will be reversed. It is this position where the image stops that is sought, for then the incident and emergent beams of light make equal angles with their respective faces of the prism, and any increase or decrease of the angle of incidence will cause a greater deviation of the direction of T_1 from the direction of C. Let T_1 be the position of the telescope when the cross hairs are centred on the slit at the moment the image is ready to reverse its direction. If then the prism is removed and the telescope is turned to position T_2, the slit is observed directly in line with the collimator where the deviation is zero. The angular distance between T_1 and T_2 is therefore the angle of minimum deviation.

153. Refraction of Spherical Waves at a Spherical Surface.—The change of curvature which a medium with curved face may impress on a wave-front depends not only on the curvature of the face on which the light is incident but also on the refractive index of the medium.

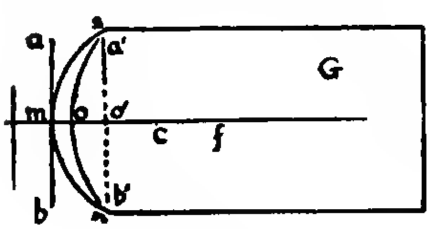

FIG. 171.

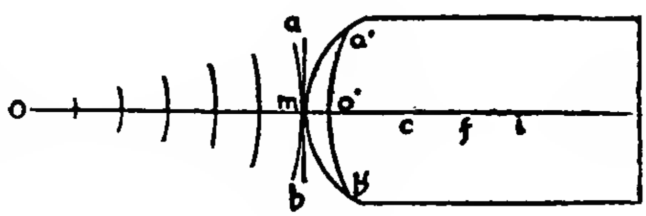

FIG. 172.

Let a body G, Fig. 171, denser than air, say glass, have a curved surface *smn*. Let *ab* be a plane wave-front, entering the glass. The wave would in a certain time in air reach the position of the dotted line $a'b'$, but when any part of *ab* enters the glass it moves more slowly and falls behind other portions which are still in air. In the same time that the wave would move from m to o' in air it moves from m to o in glass, and since index of refraction, μ, is the ratio of the velocities of light in the two media under consideration, $\mu \,.\, mo$ is called the air equivalent of *mo*.

After the wave enters the glass its centre of curvature is f, called the principal focus. Let F be the distance from f to o, then $\frac{1}{F}$ is the curvature-producing power of glass in the form shown in the figure. Any increase in the density or curvature of the glass increases the curvature-producing power.

If a spherical wave is refracted at a convex surface, the incident wave ab, Fig. 172, must first lose its curvature $\frac{1}{O}$, and what then remains of the power of the glass to produce a curvature $\frac{1}{F}$ is used in producing the curve $a'o'b'$ with a curvature $\frac{1}{I}$, where O, F, and I are the respective distances of o, f, and i to their wave-fronts at the surface of the glass, the first before refraction and the others just after.

A convex lens always reverses or tends to reverse a spherical wave-front which emanates from a point, as o. Hence the equation for this kind of a surface is

$$\frac{1}{F} - \frac{1}{O} = \frac{1}{I}$$

or

$$\frac{1}{O} + \frac{1}{I} = \frac{1}{F}$$

If the surface were concave instead of convex it is plain from what is said above that refraction tends to increase the curvature which the incident wave already has. Hence the equation for this condition is

$$\frac{1}{F} + \frac{1}{O} = \frac{1}{I}$$

or

$$\frac{1}{I} - \frac{1}{O} = \frac{1}{F}$$

154. Lenses.—There are two general classes of lenses, convex and concave. The former always tend to produce a converging wave-front and the latter a diverging one.

The double-convex lens X_1, Fig. 173, the plano-convex X_2, and the concavo-convex X_3 are the common forms of convex lenses. The double-concave V_1, the plano-concave V_2, and the

convexo-concave V_3 are the common forms of concave lenses. The form X_3 is both concave and convex, but the convex side has the greater curvature, and so the sum of the effects is that of a converging lens. Likewise the sum of the effects of V_3 is that of a diverging lens.

To deduce a general lens formula involving index of refraction, curvature, and conjugate foci, let a double-convex lens shown in Fig. 174, one face of which has a curvature $\frac{1}{R_1}$ and the other, $\frac{1}{R_2}$, be placed in the path of spherical waves originating at o. After the waves pass through the lens they converge to i, so that o and i are conjugate foci. While the wave ab is passing through the lens from m to n, a portion of the wave at a moves through the air a distance $ap+pa'$. The arc pdp' is described with radius op, and pvp' with radius ip. Hence

$$ap+pa'=md+vn=sm+sd+vs+sn$$

The air equivalent of mn is $mn \cdot \mu$, *i.e.*, this quantity is numerically equal to $ap+pa'$. Hence

$$sd+vs+sm+sn=\mu(sn+sm) \quad (177)$$

All these distances are measured from the common chord, pp', of the several arcs and so are the sagittæ of these arcs.

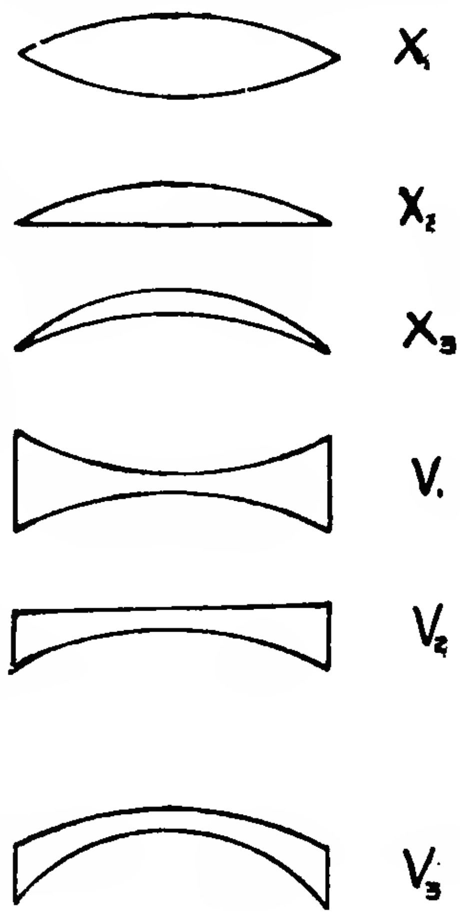

FIG. 173.

Now, putting the reciprocals of the radii in place of the sagittæ as a measure of curvature, and using O, as before, for the object distance and I for the image distance, we have

$$\frac{1}{O}+\frac{1}{I}+\frac{1}{R_2}+\frac{1}{R_1}=\mu\left(\frac{1}{R_2}+\frac{1}{R_1}\right)$$

or

$$\frac{1}{O}+\frac{1}{I}=(\mu-1)\left(\frac{1}{R_2}+\frac{1}{R_1}\right) \quad (178)$$

This is a general equation for a double-convex lens within the limits of error involved in assuming that curvature is proportional to sagittæ of arcs. (See § 142.) For a thin lens the equation may be regarded as correct, for then the arcs are a short portion of the entire circumference of the circle. It is also assumed that the wave-front after refraction is spherical, an assumption which will introduce no sensible error provided only the middle portion of the lens is used.

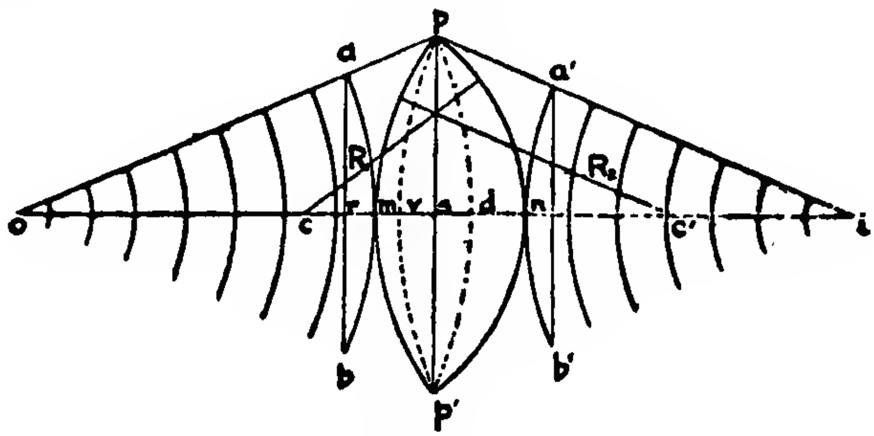

FIG. 174.

If the object o, Fig. 174, is at an infinite distance from the lens, then O, the radius of curvature of the wave ab, is equal to ∞ and the curvature is zero. Under this condition equation (178) becomes

$$\frac{1}{I} = (\mu - 1)\left(\frac{1}{R_2} + \frac{1}{R_1}\right) \tag{179}$$

But the point where an image is formed when the incident rays are parallel to the principal axis is called the principal focus of the lens, and the distance of this point from the lens is called the focal length. Calling this distance F,

$$\frac{1}{F} = \frac{1}{I}$$

and, from equation 179,

$$\frac{1}{F} = (\mu - 1)\left(\frac{1}{R_2} + \frac{1}{R_1}\right) \tag{180}$$

When the radii of curvature of the faces of a lens, and the index of refraction, are known, focal length can be calculated by equation (180).

The general equation for lenses may therefore be written

$$\frac{1}{O}+\frac{1}{I}=\frac{1}{F} \tag{181}$$

where O is the object distance, I the image distance, and F the focal distance.

As the object approaches the lens, the image moves farther away, and so there must be a point where they are at equal distance from the lens. Hence if $O=I$, equation (181) shows that each is $2F$ distant from the lens.

When the object is at the principal focus, equation (181) shows that

$$\frac{1}{F}+\frac{1}{I}=\frac{1}{F}$$

$$\therefore \frac{1}{I}=\text{zero}$$

$$\text{and} \qquad I=\infty$$

i.e., the diverging wave-front will be made plane.

If the object is between the principal focus and the lens, the wave after passing the lens will still be divergent, but less so. Hence I will be negative, *i.e.*, the image will be virtual and will appear on the same side as the object.

If the lens is plano-convex, X_2, Fig. 173, the curvature of one face is zero and (178) becomes

$$\frac{1}{O}+\frac{1}{I}=\frac{\mu-1}{R} \tag{182}$$

If the lens is concavo-convex, X_3, Fig. 173, then (178) becomes

$$\frac{1}{O}+\frac{1}{I}=(\mu-1)\left(\frac{1}{R_2}-\frac{1}{R_1}\right) \tag{183}$$

for each face tends to neutralize the bending effect of the other, their resultant effect being the difference.

In case of concave lenses it is evident that all plane or divergent waves will be made more divergent, so that both the focal and

the image distances are negative, *i.e.*, both the image and the principal focus are virtual. Hence, for this condition, equation (178) must be written

$$\frac{1}{O}-\frac{1}{I}=-(\mu-1)\left(\frac{1}{R_2}+\frac{1}{R_1}\right) \tag{184}$$

or

$$\frac{1}{I}-\frac{1}{O}=\frac{1}{F} \tag{185}$$

155. Optical Centre.—The optical centre of a lens is that point through which, if rays are drawn, they will not be changed in direction.

In double-convex and double-concave lenses having equal curvature on the two sides, the optical centre is at the centre of the lens.

This point, for any spherical lens, may be found by drawing parallel radii as *cm* and *c'm'*, Fig. 175. These are perpendicular to the surface of the lens, and tangents at *m* and *m'* are parallel. Hence a ray of light drawn through *mm'* will not be bent from its course.

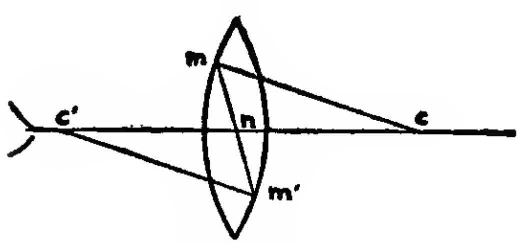

Fig. 175.

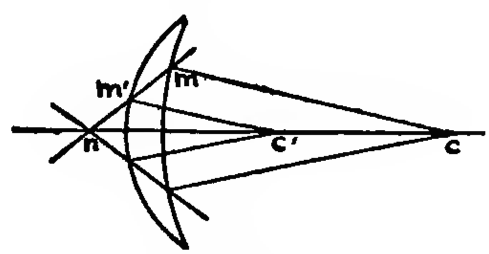

Fig. 176.

Lines joining points corresponding to *m* and *m'* for any pair of parallel radii will all pass through *n*, the optical centre. In a similar manner the point may be found for other lenses. In plano-convex and plano-concave forms the optical centre is at the vertex of the curved side, for there is the only point where a tangent is parallel to the other side. In the concavo-convex and convexo-concave forms the optical centre is outside the lens as at *n*, Fig. 176. Any ray passing through the lens and the point *n* may within certain limits be regarded as a straight line. In all such cases there will be lateral displacement (see Fig. 161), and it is assumed that when a ray enters the lens at, say, *m*, it will be so slightly bent that it will practically pass through a parallel tangent at *m'*.

156. Location of Images by Drawings.—When the focal length and optical centre of a thin lens are known, the position, size, and character of an image may be shown by drawing rays in accordance with the principles given in previous paragraphs.

Each point of an object is a source of spherical waves, and each point of an image is formed by waves which came or appeared to come from a corresponding point of the object. Thus, in Fig. 177, *A*, *oo′* is the object and an infinite number of rays go out from each point, but it will be sufficient to consider only the waves from the ends *o* and *o′* and thus fix the ends of the image.

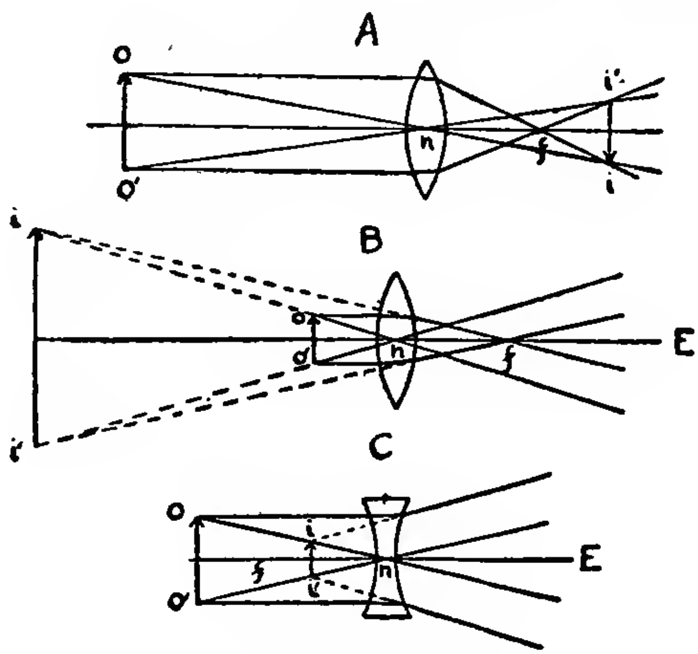

FIG. 177.

It is also a convenience to select two rays from *o* and two from *o′* whose direction we know after they emerge from the lens. Rays from each point drawn parallel to the principal axis will after refraction pass through the principal focus *f*, and rays through the optical centre *n* will not be deflected. Hence the two rays from *o* will meet at *i* and so will all other rays from *o*. This last statement may be regarded as correct only within certain limits. (See § 157.)

Likewise the waves from *o′* meet at *i′* and so the image is at *ii′*. The lines *oo′* and *ii′* may be regarded as the bases of triangles having equal angles at *n*, and therefore the size of object and image bear the same ratio as their distance from the lens.

If the object is nearer to the lens than the principal focal length, Fig. 177, *B*, the rays after passing the lens will still be divergent, though less so than the incident rays. Consequently, to an observer at *E*, the waves will appear to originate at *i*, *i'*, and intermediate points. The image is virtual, erect, and enlarged.

A convex lens used in this manner is commonly known as a magnifying glass or simple microscope.

In concave lenses, Fig. 177, *C*, rays will pass straight through the optical centre, and those parallel to the principal axis will, after refraction, if extended backwards, pass through the virtual focus *f*.

If a lens is thick, then in place of an optical centre there are two points called the principal points, *n* and *n'*, Fig. 178, such that a ray from *o* to *n* will emerge from the lens as though coming from *n'*, and *n'i* will be parallel to *on*. The location of the principal points is at the intersection of the principal planes *ac* and *bd* with the principal axis. If a ray from *o*, parallel to the principal axis, is incident at *a*, it will emerge as if from the opposite point *b* on the other plane and will pass through the principal focus *f*.

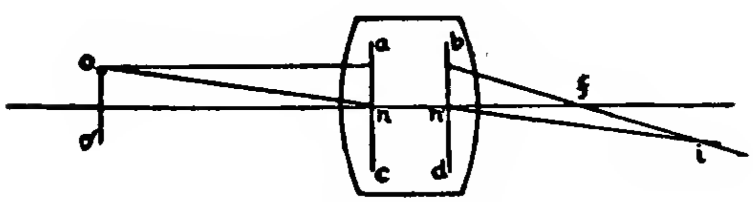

FIG. 178.

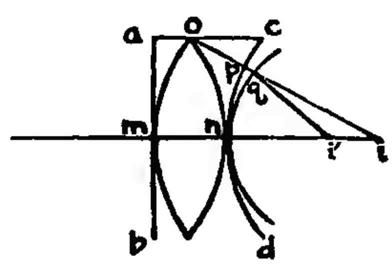

FIG. 179.

The position of these planes is different for lenses of different shape and material. For a double-convex lens of crown glass they are about one-third of the thickness of the lens from each side. Focal distance as well as object and image distances are measured from these planes, and equation (181) will then express the relation of these distances as for thin lenses. In thin lenses it is assumed that the principal planes coincide.

157. Spherical Aberration.—In the previous discussions it has been assumed that a wave-front is spherical after it has passed through a spherical lens. That this is not strictly true is shown as follows:

Let *ab*, Fig. 179, be a plane wave-front entering a double-convex lens. When the wave has just passed through, it has traversed the centre of the glass a distance *mn*, while in air it

would have passed over a distance $ac = mn \cdot \mu$. Then, drawing a circle through c and n, with its centre on the principal axis, this is the form the wave would have if it were spherical. But aoq, the distance the wave travels through the point o of the lens, is also practically all in air and so is equal to ac. Hence q will fall within the circle and the wave will converge to some point as i' instead of i. This effect is slight when only the central portion of the lens is used, and we may then assume that the refracted waves are spherical and meet at a common point.

Problems

1. A block of glass with parallel sides is 3 cm. thick. Its index of refraction is 1.6. A wave of light is incident at 40°. What will be the lateral displacement of the wave when it emerges from the opposite side?
2. What is the focal length of a double-convex lens made of glass having an index of refraction 1.5, the radii of curvature of the faces being 20 cm. and 40 cm.?
3. Where is the principal focus of a double-convex lens, the two sides having equal curvature and the index of refraction being 1.5?
4. What is the critical angle in passing from a certain liquid to air if the index of refraction of the liquid is 1.65?
5. If an observer is at the bottom of a pond of clear water 3 m. deep, what is the radius of the circle at the surface of the water through which he can see the sky?
6. If the angle of a prism is 60° and the index of refraction is 1.5, what is the angle of minimum deviation?
7. A convex lens placed 17 cm. from a candle flame forms an image on a screen. When the lens is moved 65 cm. nearer the screen another image is formed. What is the focal length of the lens?
8. A luminous disc 2 cm. in diameter is placed 15 cm. from a convex lens whose focal length is 10 cm. What is the size of the image?

Ans. 1. 9.18 mm.
2. 26.6 cm.
3. At centre of curvature.
4. 37° 18′.
5. 340 cm.
6. 13° 44′.
7. 14 cm.
8. 4 cm. in diameter.

158. The Spectrum.—*Spectrum* is a name applied to a band of various colors which appear on a screen, or may be seen directly by the eye, when white light or other light of a composite character is so resolved that waves of the same length are grouped together and the groups separated from each other.

There are two important methods by which spectra are usually produced: (1) By use of a diffraction grating, and (2) by passing light through a prism. That produced by the former method is called a diffraction spectrum and the other a prismatic spectrum.

159. Diffraction Grating.—A simple form of grating consists of a plane piece of glass on which parallel lines are ruled with a diamond. The number of lines may be several thousand per centimetre. The space between the lines is clear glass and so will transmit light, but the grooves cut by the diamond are virtually opaque, for the light incident on them is diffused.

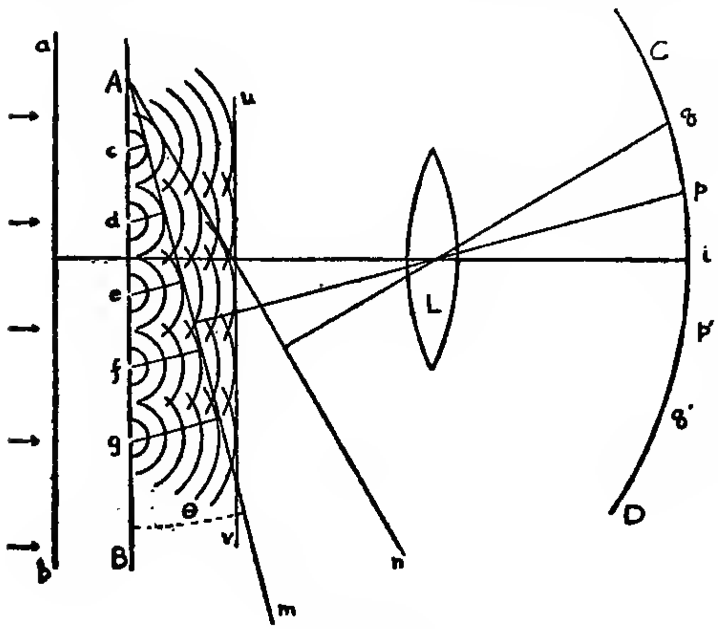

Fig. 180.

The principle of the grating will be understood from the following consideration.

Let AB, Fig. 180, represent a cross section of a grating where all parts are greatly magnified. The short spaces c, d, e, f, and g are the ends of slits through which light passes, but the spaces between the slits are opaque. Let ab be a wave-front of yellow light or any light of one wave-length. When ab is incident on the grating each slit will become a centre of disturbance and the wave-front uv, parallel to ab, will continue to the right of the grating and, passing through the lens L, will be focused at i. The only effect of the grating for this wave-front is a decrease in the quantity of light.

It is to be noted, however, that there are other lines, as Am and An, along which waves are in the same phase of vibration. Hence these lines are also wave-fronts. When Am passes through the lens it is focused at p, and An at q. Similar focal points, or images of the slits, will be found at p' and q' and still others both above q and below q'. So now, in addition to the ordinary image at i, we have images of the first, second, etc., orders on either side of i. The reason for this is that the opaque parts of the grating prevent

that destructive interference of waves which would occur at all points except i if the grating were removed. (See Fig. 143.)

Let the angle which the wave-front Am makes with the grating be denoted by θ. Also let the distance between two slits, as A to c, c to d, etc., be denoted by s. This distance s is called the grating space or grating constant.

From c to Am is one wave-length, λ. Hence

$$\lambda = s \sin \theta \tag{186}$$

This is for an image of the first order, *i.e.*, at p.

Let An make an angle ϕ with the grating, then since the distance from c to An is 2λ,

$$2\lambda = s \sin \phi \tag{187}$$

This is for an image of the second order, *i.e.*, at q.

Then if θ is the angle which any wave-front makes with the grating and n is the order of the image on the screen,

$$n\lambda = s \sin \theta \tag{188}$$

This is a general equation and shows that θ varies with λ.

Now white light may be regarded as the resultant of many different wave-lengths. Hence when such light passes through the grating, θ will have a different value for each wave-length, as shown by equation (188). Instead of a line image we will therefore have a band containing in succession all the colors into which white light is capable of being resolved. The different colors are the sensations produced by different wave-lengths, so that for each wave-length the line or plane Am, Fig. 180, will have a different inclination to the grating. The longest light waves are red and the shortest are violet, consequently the red at p will be farthest from i and next to it will be orange, then yellow, green, blue, and finally violet nearest to i. A similar band of colors will appear at p'. These are diffraction spectra. The ones at p and p' are spectra of the first order. Other spectra will appear at q and q' and at other points beyond, but will be less brilliant on account of the overlapping of images there.

In spectra of the first order each color, not only those named above but all intermediate shades, occupies a separate position on the screen. Hence this is called a *pure spectrum*.

As long as θ is small we may, in accordance with equation (188), say that dispersion—*i.e.*, θ—is proportional to wave-length. For this reason a diffraction spectrum is taken as the standard and is called the *normal spectrum.*

160. Wave-length of Light.—Principles discussed in the previous paragraph suggest an excellent method of finding the wave-length of light in any desired part of the spectrum. The method of procedure will be plain if Fig. 181 is compared with Fig. 180. In both, AB is the grating and Am is the wave-front producing a spectrum of the first order. But, in Fig. 181, L is the lens of the eye and CD is the retina. The light whose wave-length is sought is placed at X and admitted through a narrow slit close to the light so that the image may be more exactly located. When

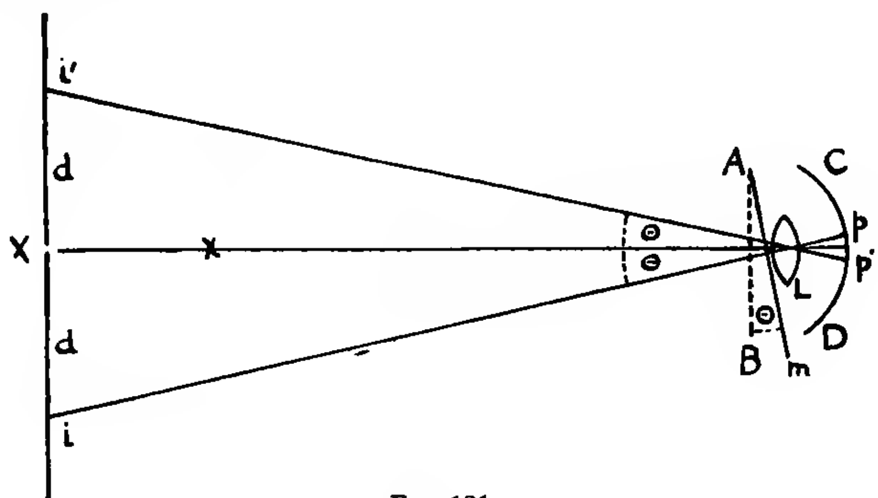

FIG. 181.

this light passes through the grating an image is formed on the retina at p and p', and the eye locates the images at i and i'. The angle θ between AB and Am is equal to the angle made by pi or $p'i'$ with the axis x. The distances from X to the lens and from X to i are measured. Call these distances x and d, respectively. Then

$$d = x \tan \theta$$

From this θ is found and, by a simple substitution of its value in equation (186) with the value of the grating space s which is given by the makers, the value of λ is found.

In a similar manner any image in spectra of the second or third order may be located and the value of λ computed.

Another simple method of measuring θ is to mount the grating on the spectrometer (Fig. 166) and note the angle through which

the telescope must be turned from a position in line with the collimator to a position where a color of the first or other order may be seen.

161. Dispersive and Resolving Power of Gratings.—By dispersive power is meant the difference in the value of θ for the various colors of the spectra. The greater the dispersive power the longer the spectrum will be.

It may be readily inferred from Fig. 180 and also from equation (188) that since

$$\sin \theta = \frac{n\lambda}{s}$$

any decrease in s, the grating space, will increase $\sin \theta$. If θ is small it may be used in place of its sine, and then the dispersion θ is inversely proportional to the grating space, s.

The closer the lines of a grating are to each other, *i.e.*, the greater the number of lines per centimetre, the greater the dispersion.

The equation also shows that dispersion varies directly with the order of the spectrum. In the second order the spectrum is twice as long as in the first.

The resolving power of a grating is its ability to cause images having different wave-lengths to stand out sharply defined. This power is proportional, not to the number of lines per centimetre, but to the total number of lines on the grating.

It has been shown that when an image of any particular color is formed at p or q, Fig. 180, the space between p and q or p and i is dark because of destructive interference. The closer this region of interference approaches an image the sharper the image will be.

If a wave-front is such as to make an angle with the grating only very slightly greater or less than Am makes, an image would be formed just above or below p, *i.e.*, the image at p would not be distinct. But if there are a sufficient number of grating spaces there will be one, many spaces from c, whose distance from the wave-front differs by one-half wave-length from the distance of c to the same wave-front. Hence there will be complete interference of the waves from these two grating spaces. If, then, there are as many grating spaces below the one whose waves are

a half wave-length behind as there are from it up to c, all the waves from one-half of the grating that would form an image on either side of p are destroyed by waves from the other half, for each grating space in one-half has a corresponding one in the other, differing by a half wave-length in their distance from the wave-front.

A grating made by Professor Michelson is nine inches long and is ruled with 114,300 lines.

The gratings described above are called transmission gratings, for light passes through them. Other gratings having lines ruled on polished metal are called reflection gratings. The principles involved in the use of the latter are the same as for the former.

162. Prismatic Spectra.—The deviation of a beam of light when passed through a prism depends on the angle of the prism, the index of refraction, the wave-length, and the angle of incidence. The shorter the wave-length the greater as a rule will be the deviation. Hence, when light of a composite character, such as white light, is passed through a prism, it is resolved into groups of similar wave-lengths, and if these are made to fall on a screen there will appear, for white light, all shades of color from red to violet, the deviation being least for red and most for violet.

The angle between rays of violet and red waves is the dispersion for these two wave-lengths.

It appears, then, that since the shorter waves are most refracted, they are the ones whose velocity is most retarded in passing through a denser medium.

When light from the sun or other source of light is admitted to a darkened chamber and passed through a prism, a spectrum is formed consisting of a succession of images in different colors. These images, however, overlap, and there is not a distinct separation of the various wave-lengths. Much better results are obtained by first passing the light through a narrow slit and then, after dispersion by a prism, interpose a lens which will form on the screen a succession of images of the slit in the various colors into which the prism has resolved the light.

It is obvious that index of refraction for any given substance should be given in terms of some specified wave-length or for some designated position or line in the spectrum, for refraction is different for different colors. The D line in the yellow part of

the spectrum is usually employed in determining what is called the mean index of refraction.

It is found, however, that prismatic dispersion is not proportional to refraction or to wave-length. Long waves as a rule are but little dispersed, and short waves are abnormally spread out by a prism. Also, for prisms of different material, the relative dispersion in various parts of their spectra do not agree. Dispersion of this kind is called *irrational* as contrasted with *normal* dispersion produced by a grating.

163. Kinds of Spectra.—Spectra are usually classified as: (1) *bright-line*, (2) *continuous*, and (3) *absorption* or *reversed* spectra.

An incandescent gas will ordinarily give a bright-line spectra. Atoms of a gas are widely separated from each other and move in long, free paths without collision. The electrons which are regarded as the source of light waves may then produce a long train of waves of a definite wave-length. These will cause a single bright line of a certain color without the other colors of the ordinary spectrum. Within the same atom electrons may have different periods of vibration, and so there may be several trains of waves of different wave-length each producing a bright line in the spectrum. Light from sodium vapor, for example, as when sodium chloride is burned in a Bunsen flame, will produce a bright yellow line without any other colors. Cadmium vapor gives red, green, and blue lines. Vapor of mercury gives lines of yellow, green, blue, and violet. Other spectra of this kind are obtained by heating salts of various substances to a state of incandescent vapor.

If a vapor or gas is put under great pressure the lines become broader and tend to unite in continuous spectra.

Incandescent solids and liquids will produce continuous spectra, *i.e.*, there is no break in the band of color, however pure the spectra may be. White light from such a source is therefore resolved into every possible wave-length from red to violet.

A third form of spectrum of very great importance and utility is the absorption spectra. When sunlight is admitted through a narrow slit and is dispersed by a prism or grating, numerous dark lines parallel to the slit are observed at intervals throughout the spectrum. Fraunhofer in about the year 1815 counted more than 700 of these lines and definitely mapped out about 350 of them,

and hence they are known as the Fraunhofer lines. He designated the more important lines by letters, as shown in Fig. 182, where *A* lies in the extreme red, *B* and *C* in the lighter red, *D* in the yellow, *E* in green, *F* and *G* in the blue, and *H* at the limit of violet.

These lines are of great importance for purpose of reference as in designating wave-length for any particular point of the spectrum. Thus we say the wave-length for *F* is 4861 Ångström units. An Ångström unit is 10^{-10} metres, *i.e.*, 10^{-7} mm. or 10^{-4} microns. The *D* line is in fact two fine lines very close together called D_1 and D_2, whose wave-lengths are 5896 and 5890 Ångström units.

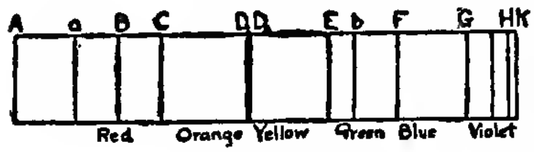

FIG. 182.

The index of refraction is different for each change of wave-length and these values are given for the various lines of the spectrum.

By use of Fraunhofer lines it is possible to determine a number of elements in the sun, for the dark lines are the region where waves that would fall on that part of the spectrum have been absorbed by some substance through which the light passed.

FIG. 183.

The sun is surrounded by an envelope consisting of gases and the vapors of many substances. Light from the hotter portions of the sun must pass out through these vapors, and those waves are absorbed which the vapors themselves would give out if they were highly incandescent. The bright line spectrum of the vapor of iron, for example, is shown in part in the middle band of the photograph, Fig. 183. On either side is a spectrum of sunlight. The exact position of the numerous lines of iron in line with dark lines of the solar spectrum leaves little doubt that there is iron in the sun. In a similar manner it is shown that many other substances are constituents of the sun.

The sun is rotating on its axis, and if a spectrum is formed of light coming from that limb which is moving toward the earth, all the lines except A will be shifted toward the violet. This shifting is what would be expected according to Doppler's principle that when a body from which a train of waves emanates is in motion, wave-length will be shorter in the direction of that motion. This is often observed in case of sounding bodies in motion and is just as true of luminous bodies in motion. Hence the shifting of spectral lines to a slightly higher position than they would have if the source of light were stationary. Light from the opposite limb of the sun causes a shift in the opposite direction.

Since the A line is not thus affected it is concluded that the earth's atmosphere absorbs those waves that would produce a bright line at that point.

There appears to be no limit to the number of Fraunhofer lines, for every improvement in the methods of dispersing light beings more lines in view.

The ratio of the difference in deviation of rays that fall on the C and F lines to the deviation of those that fall on the D line is called the dispersive power and may be written

$$\frac{D_F - D_C}{D_D}$$

where D stands for deviation and subscripts F, C, and D are lines of the spectrum.

This ratio may be expressed in terms of refractive indices as follows:

From equation (173)

$$D = i_1 + i_2 - A$$

We also have

$$\mu = \frac{\sin i}{\sin r} = \frac{i_1}{r_1} = \frac{i_2}{r_2}$$

within limits where the ratio of angles may be regarded as equal to the ratio of their sines. Hence

$$i_1 = \mu r_1$$

and $$i_2 = \mu r_2$$

Substituting these values of i_1 and i_2 in the equation above and keeping in mind that $r_1+r_2=A$, we have for the deviation of the three rays under consideration

$$D_F=(\mu_F-1)A$$
$$D_C=(\mu_C-1)A$$
$$D_D=(\mu_D-1)A$$

Substituting these values in the expression for dispersive power P

$$P=\frac{(\mu_F-1)A-(\mu_C-1)A}{(\mu_D-1)A}$$

or $$P=\frac{\mu_F-\mu_C}{\mu_D-1} \tag{189}$$

164. Limits of the Spectrum.—That portion of dispersed waves which appears in various colors is called the visible spectrum. There are, however, in the radiations from such a source as the sun or arc lamp, many waves which are longer than the longest red waves. This is known as the *infra red* region of the spectrum. The length of wave in the A line is $.7604\mu$. The wave-length of the longest infra red wave yet found is 300μ. Designating this interval in octaves as in music, the infra red region extends about $8\frac{2}{3}$ octaves below the A line. These long waves may be detected by the heat effects, as when a thermopile or bolometer is moved along the region of the spectrum. (See p. 237, " Mechanics and Heat.") The wave-length can then be computed, for in a normal spectrum wave-length is directly proportional to deviation.

The visible spectrum is only a little more than one octave in length—*i.e.*, from about A to H—these lines being near the extreme limits of red and violet respectively. This interval in wave-lengths is $.7604\mu$ to $.3968\mu$.

Above the violet is a region called *ultra violet*. The shortest wave-length that has been measured in this part of the spectrum is $.1\mu$. Ultra violet waves, therefore, extend nearly two octaves above the H line.

Short waves may be detected by their actinic effect—*i.e.*, by the chemical effect which they produce—as when they fall on a photographic plate.

Such a plate is affected in a similar manner by Röntgen rays, or X-rays, and these are believed to be very short ether waves

produced by the impact of electrons on a metal target placed in the path of cathode rays. (See Fig. 1.) X-rays, however, cannot be reflected, refracted, or diffracted, and hence no way is known by which their wave-length can be determined. Both X-rays and ultra violet waves will cause fluorescence in a substance such as barium platino-cyanide.

165. The Spectroscope.—A spectroscope is an instrument by which spectra may be conveniently and accurately studied. One common form is shown in Fig. 184. The principle of construction is the same as in the spectrometer (Fig. 166), with the addition

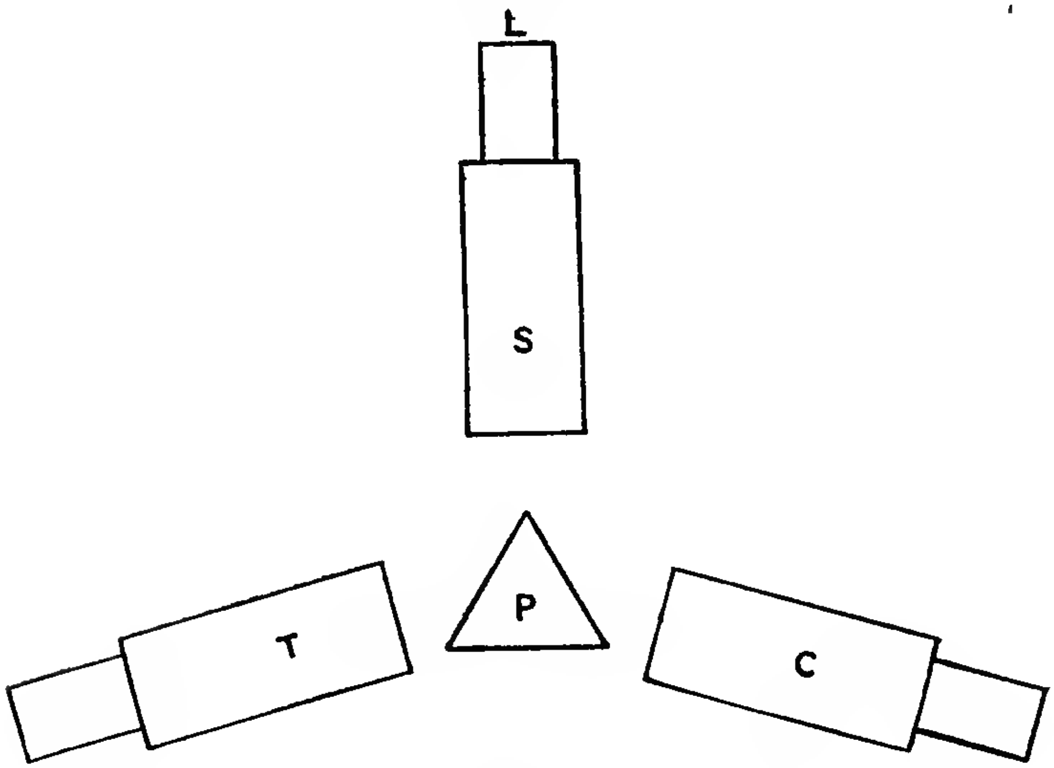

Fig. 184.

of a third tube *S*, which contains a finely divided transparent scale. Light from *L* passes through the scale and is reflected from a side of the prism into the telescope. Thus an observer may locate the various lines of the spectrum on the scale.

A substance in a state of incandescent vapor will give a bright-line spectrum characteristic of that substance. When these lines and their position have been learned, it is often possible to make a chemical analysis of an unknown substance by vaporizing some of it in a Bunsen flame placed in front of the slit of the collimator. An observer looking through the telescope identifies characteristic

spectra of known substances, or light from the unknown vapor may be admitted directly through one-half of the slit and that from a known substance may be admitted at the same time by use of a total reflecting prism placed over the other half. The two spectra are then seen side by side. The known substance is present in the unknown if its lines run straight across both spectra. The third tube is not needed.

By another method a photographic camera is put in place of the telescope. With one-half of the plate covered a photograph is made of the spectrum from one source of light, then after shifting the cover to the other half a photograph of the other source is made. A permanent record is thus secured.

166. Chromatic Aberration.—When white light is refracted by a lens as in Fig. 177 dispersion will occur as in a prism. The violet which is refracted most will be focused at a point nearer the lens, the red at a point farthest away, and the other wave-lengths at intermediate points. The image will therefore be indistinct and surrounded by a fringe of colors. This phenomenon is called chromatic aberration.

This defect of lenses may be corrected by use of achromatic lenses. Such lenses are possible as a result of irrational dispersion already explained.

If two prisms of the same kind of glass and having the same angle are placed in a reversed position as in Fig. 185, then, although white light is dispersed in passing through the first prism, the rays are reunited by the second prism, so that the emergent beam is the same in character and direction as the incident beam. The effect is the same as for a plate of glass with parallel sides. (See Fig. 161.)

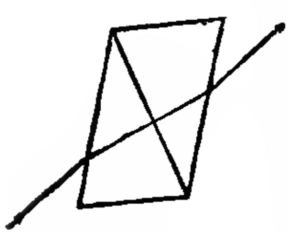

FIG. 185.

If, however, the angles and kind of glass are different in the two prisms it is possible to have deviation without dispersion. Such a combination is *achromatic*. To construct prisms for this purpose which will be achromatic for any desired wave-lengths it is necessary to choose such angles and material of such indices of refraction that dispersion in the region to be achromatized shall be equal. It has been shown in the derivation of equation (189) that deviation is expressed by $A\ (\mu - 1)$. The difference of devia-

tion between, say, the F and H lines, would be the dispersion in that region and would be expressed by

$$A(\mu_H - 1) - A(\mu_F - 1) = A(\mu_H - \mu_F)$$

If then two prisms of different angles, one made of flint glass and the other of crown glass, for example, be selected so that $A(\mu_H - \mu_F)$ of one is equal to that of the other and the prisms are placed as in Fig. 185, the dispersion of one between F and H will be corrected by that of the other and there will still remain a certain amount of deviation.

In the same manner two lenses of different curvature and material may be combined to produce an achromatic lens. A plano-concave flint glass is usually combined with a double-convex crown glass and the curvatures so selected that for optical instruments the lens will be achromatic for the brilliant colors—*i.e.*, in the region of the yellow—but for photography the upper regions of the spectrum are thus corrected.

If as a result of irrational dispersion it is possible to have deviation without dispersion, it should also be possible to have dispersion without deviation. This latter result may be obtained by choosing two prisms of such values for A and μ that the deviation $A(\mu - 1)$ of one is equal to that of the other. This principle is employed in the construction of direct-vision spectroscopes, *i.e.*, spectroscopes through which the observer looks directly toward the source of light.

167. Color.—As already suggested, the different colors which we perceive are *sensations* caused by different wave-lengths of light. Consequently this topic belongs to physiology rather than to physics.

According to the Young-Helmholtz theory of color sensations there are in the retina of the eye three sets of nerve terminals, one of which is particularly sensitive to the waves that produce a sensation called red, another to waves called green, and a third to violet waves.

The sensitiveness of these terminals to the various wave-lengths of the spectrum is shown by the length of the ordinates of the curves in Fig. 186, where R, G, and V are points of maximum sensitiveness. It is found, however, by examination of cases of color-blindness, that each set of terminals is sensitive in some

degree to all wave-lengths, so that each curve extends over nearly the whole spectrum, reaching a maximum at R, G, and V.

A superposition of red, green, and violet of the proper shade and intensity will produce the sensation of white or gray, and by a proper combination of these three primary-color sensations any color of the spectrum may be produced.

It is supposed that the nerve terminals respond or vibrate in sympathy with waves of a certain length, and for any combination of waves the sensation is that of the resultant disturbance, but it is not known with certainty just how light affects the nerve terminals. Another assumption is that the retina may contain substances which are acted on chemically by the light waves.

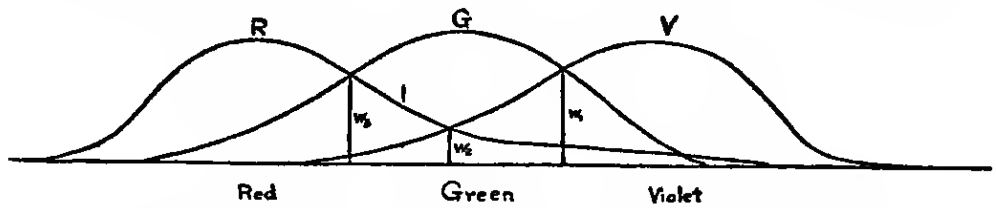

FIG. 186.

A person who is color-blind is defective in one of the three sets of terminals. For one who is red-blind the entire curve R, Fig. 186, is wanting. To him a combination of green and violet will be white in the region of the ordinate W_1. For one who is green-blind the curve G is wanting and a green will be gray, for W_2 is a common ordinate of the R and V curves in that region.

168. Complementary Colors.—Any two colors which, falling upon the retina at the same time, produce the sensation of white are *complementary*. Blue and yellow, for example, or red and blue-green are complementary. A good way to mix colors is by the rotation of discs on which colors are arranged in different proportions. A number of discs of cardboard, each of different color, may be slit along a radial line, and any two or more may then be slipped together so that any desired portion of each is exposed. When these are rapidly rotated each color will, on account of the persistence of the image, be impressed on the retina at the same time. Red and green will give yellow; red and blue, purple. Any desired shade of color may be obtained by mixing the various colors in the proper proportion.

If one looks intently at a blue card held against a white background for a short time, when the card is suddenly removed a

yellow image will appear in the region which was covered by the blue. If the card is red, the after image is green. The color of the image is complementary to that of the card. An explanation offered for this is that after looking at blue for a time those nerve terminals which are particularly sensitive to that color become fatigued. Hence, when only white is in view, that part of the retina which was before covered by blue will be most sensitive to that color, yellow, which with blue would produce white, *i.e.*, after the terminals sensitive to blue are weakened by fatigue the eye is for a short time blue-blind and the red and green terminals will, as the diagram shows, produce a sensation of yellow.

169. Color Resulting from Absorption.—In the previous paragraph we have discussed what may be called addition of colors. The color of transmitted light is a result of the subtraction of colors. The waves which pass through a transparent body are those which have not been absorbed or subtracted by the body. Red glass, so called, is glass that absorbs all waves except red. Blue glass absorbs all waves except blue, and so on. A common experiment illustrating this fact consists in passing a beam of light through a solution of copper sulphate. An examination with the spectroscope will then show that the longer waves of the spectrum, the red and yellow, have been completely absorbed. The light is then passed on through a solution of bichromate of potash and all shorter wave-lengths, blue and violet, will be absorbed. The only waves which pass through both solutions are the ones which produce a sensation of green.

The mixing of colors and the mixing of pigments are, therefore, entirely different processes. By the former process blue and yellow give white; by the latter, green. When white light falls on any mixture of pigments certain waves are as a rule absorbed and the others reflected. A white paint reflects all waves, while a black paint absorbs all. The color of a paint is determined by the waves which are reflected and is not something inherent in the paint. A white body in red light is red and a red body in blue light is black.

170. Polarized Light.—The discussion at the beginning of this chapter would lead us to infer that the disturbance of the ether on the passage of an electromagnetic wave would be in a direction at right angles to the direction of propagation, for when an elec-

tron vibrates it sets up a magnetic field only at right angles to the direction of its motion.

In way of analogy we may think of a cord of indefinite length, one end of which is fastened to the prong of a tuning fork or other vibrating body. Waves pass out on the cord, but any point on the cord moves back and forth in a direction at right angles to the direction the wave is moving.

In each atom of a luminous body are a number of electrons, each of which may be vibrating in a different plane, or any one electron may rapidly change its plane of vibration. As a consequence we find that the ether disturbances along a beam of light take place in every conceivable plane, the plane in all cases including the line of propagation of the wave.

In Fig. 187 is a representation of a wave moving in a direction perpendicular to the paper with the numerous planes of vibration of the ether. These planes may all be resolved in two directions xx' and yy'. If by any means the vibrations in one of these two planes are removed or destroyed, the light is said to be *plane polarized*, for the vibration of the ether is in one plane only. If only a part, less than half, of the planes are removed, the light may be said to be *partially polarized*. Ordinary light, in this sense, is polarized in every direction.

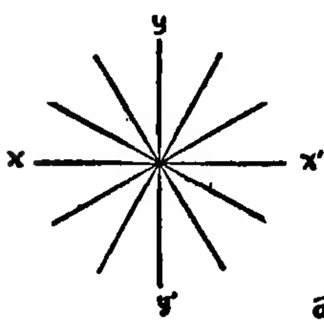

FIG. 187.

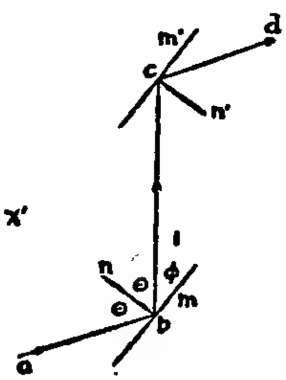

FIG. 188.

Huygens in 1610 discovered that when a beam of light passes through a crystal of Iceland spar it emerges from the crystal as two beams, except when the direction of the light is parallel to the axis of the spar. He also found that if another crystal is held in the path of one of these two beams another division of the light took place when the second crystal was held in one position, but not when either crystal was turned 90° on the line of light as axis.

Because of this experiment Huygens is usually given credit for the discovery of polarization, though he could not explain the phenomenon. We now know that the light was polarized by double refraction, as will be shown later, and the emergent beam

which exhibited peculiar properties contained vibrations in but one plane instead of the symmetrical arrangement shown in Fig. 187.

Early in the 19th century Malus discovered that when light is reflected from glass at a certain angle it exhibits properties like that observed by Huygens in case of double refraction, *i.e.*, by simple reflection at a certain angle the light was polarized.

Experimental evidence of this kind, described in succeeding paragraphs, leads to the belief that the vibrations of the medium through which light travels are not longitudinal as in case of sound but transverse to the line of propagation.

171. Polarization by Reflection.—In Fig. 188 let m be a mirror of plane glass set so that the angle which it makes with the vertical line bc will be about 33°, *i.e.*, $\phi = 33°$. Then let a beam of light ab fall on m, making the angle of incidence, θ, about 57°. The ray bc will then be plane polarized, *i.e.*, will contain vibrations in but one plane. How this has come about may be understood from Fig. 189, where ab is an ordinary beam of light with directions of vibration as shown in Fig. 187, but here resolved into two planes at right angles. The lines and dots on ab represent 189, A, as seen edgewise. Now part of the beam ab will be reflected and part will be refracted and pass on to f. If bc is perpendicular to bf, the former will be completely polarized and the latter partially so, as will be described later. If bc, then, is to contain both planes of vibration, one of them would be in the direction of the propagation of the wave, and that is just what we have claimed cannot be. In the other plane, however, the vibrations continue to be transverse to the direction of the light.

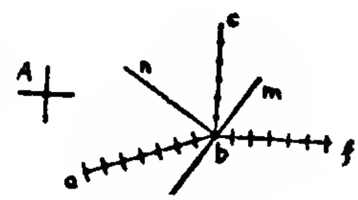

FIG. 189.

Now going back to Fig. 188 the beam bc is made to fall on a mirror m' which is backed with black varnish and is set parallel to m. The eye of an observer at d will receive the beam of polarized light and no difference will be observed between it and ordinary light, but when m' is rotated through 90° on cb as axis, no light will be reflected from m'. In place of the beam seen at d there will now be darkness. Just as m removed the transverse vibrations in one plane, m' now removes those in the other plane at

right angles to the first. A further rotation of m' through another 90° will again bring the light into view, as it should if our theory is correct. At the end of the next 90° there will again be darkness.

In an arrangement of this kind m is called the polarizer and m' the analyzer.

To determine whether light from an unknown source is plane polarized or not, a mirror such as m' is placed in a proper relation to the beam and rotated in the manner just described.

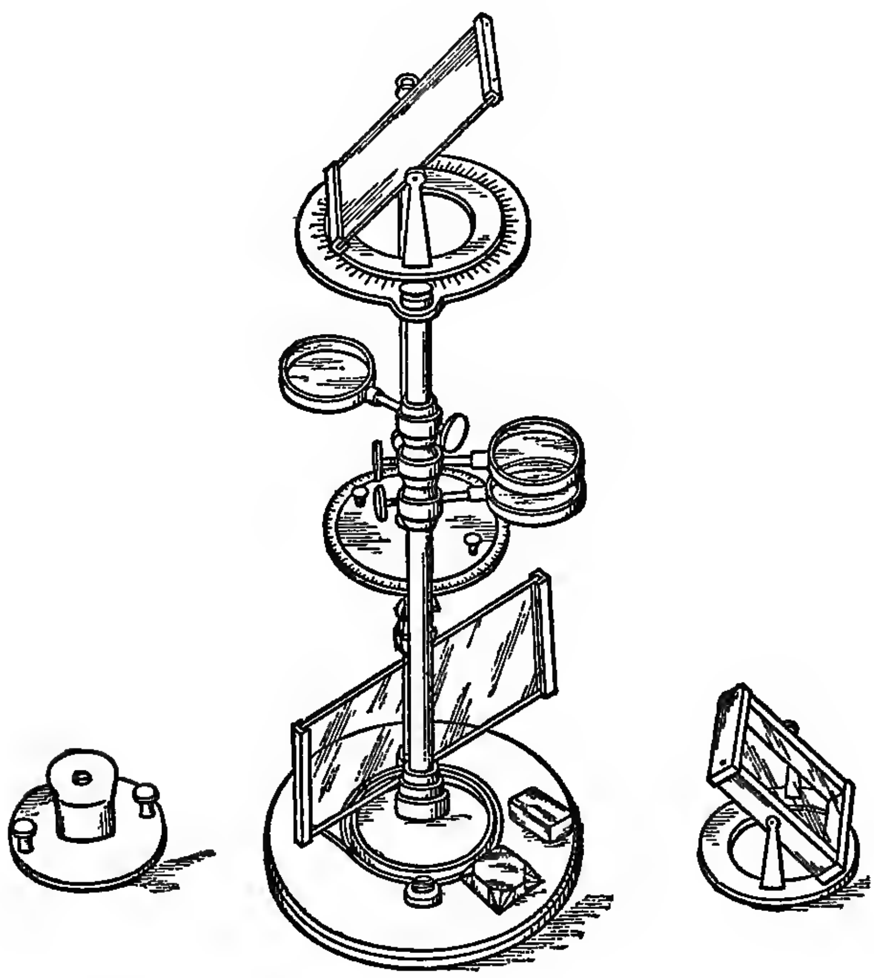

FIG. 190.

A very convenient instrument for experimental work in polarization is the Norrenberg polariscope shown in Fig. 190. The upper mirror is supported on a collar which may be turned on a graduated circle.

172. Brewster's Law.—The angle of incidence at which polarization of reflected light is maximum is called the polarizing angle. A very simple law discovered by Brewster states that when the

angle between the reflected and refracted rays is 90° the reflected ray is completely polarized.

From the construction of Fig. 191 it is seen that

$$r + 90° + f = 180°$$

$$\therefore r = 90° - f = 90 - i$$

$$\frac{\sin i}{\sin r} = \mu = \frac{\sin i}{\sin(90° - i)} = \frac{\sin i}{\cos i} = \tan i \qquad (190)$$

Hence another statement of Brewster's law would be that the index of refraction of a substance is equal to the tangent of the polarizing angle, or

$$i = \tan^{-1} \mu \qquad (191)$$

The polarizing angle will therefore be different for substances with different refractive indices. Ordinary glass has an index of about 1.55, and this is approximately the tangent of 57°. This is the reason the angle of incidence was made 57° in the experiment described above.

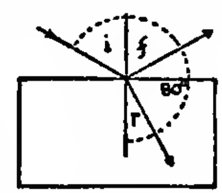

FIG. 191.

Also, since refractive index is different for different colors of the spectrum, if white light is polarized by reflection, not all the colors are shut off by the analyzer, for when the polarizing angle is correct for red, say, it is not quite correct for violet. Hence a slightly colored spot of light will be seen even when the analyzer crosses the polarizer unless monochromatic light is used.

173. Polarization by Refraction.—As shown in Fig. 189 the ray *bc* contains only vibrations at right angles to the plane of polarization—*i.e.*, the plane which includes *ab*, *bn*, and *bc*—but the refracted ray *bf* includes all vibrations of *ab* which were in the plane of polarization and also those transverse vibrations which were not reflected with *bc*. The light in *bf* is therefore only partially polarized. If, now, several plates of glass be placed parallel to *m* in the path of *bf*, each will reflect out a portion of the vibrations like those in *bc*, and so at each reflection *bf* will become more nearly plane polarized. A bundle of about 12 or 15 thin glass plates will completely polarize the refracted beam.

There is no advantage in using a greater number of plates, for, after all the vibrations perpendicular to the plane of incidence

have been reflected, no further reflection will occur and there is a disadvantage resulting from absorption of light by the glass.

If such a bundle of glass, seen to the right in Fig. 190, be used as analyzer in place of the mirror of the Norrenberg polariscope, it will be found that, since the light is now already plane polarized by the lower mirror, when light is transmitted with maximum brightness, as may be observed by looking vertically downward through the bundle of glass, reflected light is a minimum. At a point 90° from this position reflection will be a maximum and transmission a minimum.

174. Polarization by Double Refraction.—Certain crystals, notably crystals of calcium carbonate called calcite or Iceland spar, possess a structure such that when a beam of light is incident normally to one side a part of the beam will pass straight through, just as it would go through glass, while another portion of the beam is deflected to one side. Two separate and parallel beams emerge from the crystal, both of which are plane polarized.

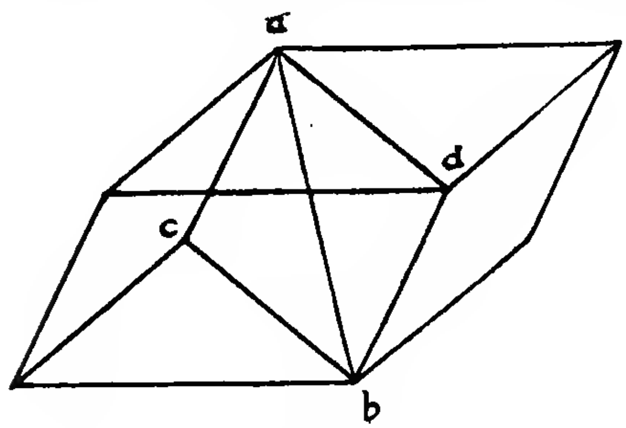

Fig. 192.

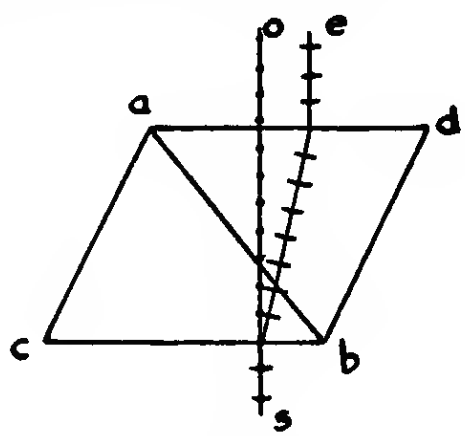

Fig. 193.

The natural shape of a crystal of Iceland spar is shown in Fig. 192. The solid angles at a and b are obtuse, each being included by three obtuse plane angles of 101° 53′. The shape of the crystal is a rhombohedron. A line ab making equal angles with the three faces of the solid obtuse angles at a and b is called the *optic axis*. When the sides of the rhombohedron are equal, as in the figure, ab is a continuous line.

A beam of light passing along the optic axis or any line parallel to this axis does not suffer double refraction. The reason for this is that in all directions at right angles to any point on the optic axis the structure of the material is alike, and so the transverse

vibrations are equally affected in all directions. This is not the case when light is passed through the crystal in other directions.

A plane passed down through *ad*, Fig. 192, so as to include the optic axis, is called the principal plane and is shown in Fig. 193.

Let a beam of light from *s*, Fig. 193, pass through the spar in the direction indicated. It will be divided into two beams, one, *so*, called the *ordinary* ray, has its plane of vibration perpendicular to the principal plane; while the other, *se*, called *extraordinary* ray, has its plane of vibration in the principal plane. Both rays are plane polarized, but in direction at right angles to each other. This can be easily shown by placing a crystal of Iceland spar on the middle support of the polariscope, Fig. 190, over a pin hole through which light is reflected from white paper placed below, to the analyzer above. While the analyzer is turned, the two white images of the pin hole, one caused by the ordinary and the other by the extraordinary ray, will alternately appear and disappear, each reaching maximum brightness twice in a complete revolution. This shows clearly the fact and the direction of the polarization.

If a crystal of this kind is placed over a dot on a sheet of white paper it will be noted that when the crystal is rotated the ordinary ray is stationary and the extraordinary ray turns around it, but the two images and the solid obtuse angle *a* are always in line.

It will also be noted that image of the dot as seen by the ordinary ray is nearer the top of the crystal than is the other image. This shows that the ordinary ray suffers a greater retardation in passing through the crystal (see § 148).

Considerations such as those described above lead to the belief that when light falls on a crystal, as shown in Fig. 193, the point of incidence becomes a source of ether waves which attempt to spread out in all directions, but the velocity of propagation is greater for those transverse vibrations which are parallel to the optic axis. Hence the direction of the ray containing vibrations of this kind only will be that of the resultant of velocities in all directions, the greatest component being in a direction at right angles to the optic axis. The extraordinary ray will, therefore, be deflected toward that path in which waves travel with the greatest facility.

In case of the ordinary ray the transverse vibrations are perpendicular to the principal plane and hence to the optic axis.

The retardation will then be the same in all directions. The waves arising from the point of incidence will be spherical, and the medium is virtually isotropic for this ray.

Many other crystals such as quartz, tourmaline, mica, and sugar possess the property of double refraction, but Iceland spar can be most easily found in large transparent crystals.

175. The Nicol Prism.—The most effective way of obtaining plane polarized light is by use of a doubly refracting crystal as explained above. Both of the rays emerging from such a crystal are completely polarized, but they are close together unless a very thick crystal is used, and their planes of vibration are at right angles. Hence it is desirable to get rid of one of the rays and transmit the other.

Such an arrangement was devised by Nicol. As shown in Fig. 194 he cut a crystal of Iceland spar along a plane *mn*, this plane being parallel to the upper edge of the crystal. The two faces were then polished and again cemented together with Canada balsam.

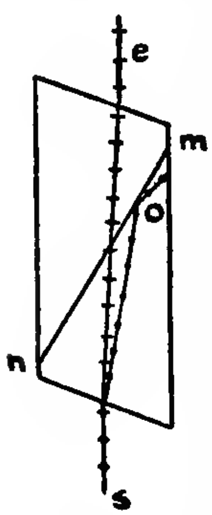

Fig. 194.

The ray of light is incident obliquely and the ordinary, being most retarded, will be refracted most. The index of refraction of balsam is less than that of calcite for the ordinary ray but greater for the extraordinary. The plane *mn* is therefore so chosen that at *o* the ordinary ray is totally reflected to the black coating on the crystal where it is absorbed. The extraordinary ray passes on through the crystal and is plane polarized.

176. Circular and Elliptic Polarization.—In plane polarization, as we have seen, the motion of ether particles as a wave passes is back and forth in a straight line transverse to the line of propagation of the wave. The motion of the ether particle may, however, be circular or elliptical, and the light is then said to be circularly or elliptically polarized.

As has already been indicated, when light has been doubly refracted two images are seen, and if these are viewed by another crystal in the proper position four images will appear. Each of the two rays of plane polarized light has been resolved into two, one ray in each set being an ordinary and the other an extraordinary ray. This kind of resolution will occur and will give

maximum brightness to the images when the plane vibrations of the rays from the first crystal are at an angle of 45° to the optic axis of the second crystal.

If, then, a thin piece of mica is placed on the middle shelf of the Norrenberg polariscope and a plane polarized ray passed up through it, the ray will, when the mica is turned to the proper position, be doubly refracted, for mica possesses the same property as calcite in that respect. The single plane of vibration of light incident on the mica is resolved into two rectangular components.

The ordinary and extraordinary will not, since the mica is very thin, be appreciably separated on emergence, and their fields of disturbance in the ether will overlap. Also, since the ordinary ray is more retarded than the other in passing through the mica, it will emerge with a change of phase unless the retardation happens to be one complete wave-length.

Let the mica plate be of such thickness that the ordinary ray lags a quarter wave-length, $\lambda/4$, or some odd multiple of $\lambda/4$. Then if A, Fig. 195, represents a plane vibration before entering the mica, B will represent the condition after resolution and retardation. But if the amplitudes of o and e are equal and o differs $\lambda/4$ in phase from e, then, just as in compounding any two simple harmonic motions, the resultant motion is a circle. This kind of disturbance in ether is called circular polarization. If the difference of phase is not $\lambda/4$ or the amplitudes are unequal, the resultant motion will be in the orbit of an ellipse, producing elliptic polarization.

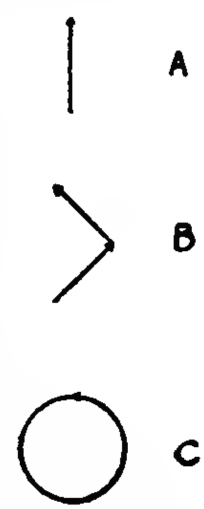

Fig. 195.

A plate of mica or other substance which will cause the ordinary ray to lag $\lambda/4$ is called a quarter-wave plate. Mica is commonly used because it can be split into very thin plates. If the plate is thick the two rays will be so far separated that there will be no overlapping of fields of disturbance on emergence.

Double refraction is the most common method of producing polarization of this kind, but there are other methods of producing similar results. Plane polarized light reflected from a polished pole of a magnet is found to be elliptically polarized. This is known as the Hall effect.

One method of detecting circularly polarized light is by passing the light through a quarter-wave plate. This will change the circular back to plane polarization which can then be readily detected with an analyzer. Elliptical polarization can usually be detected by the fact that in that case the image seen in the analyzer will be alternately bright and dim as the analyzer is rotated. The same effect, however, is produced by partially polarized light and when in doubt the quarter-wave plate may be used as above.

177. Color of Thin Plates.—When plane polarized light is passed through a thin plate of mica or other substance having similar optic properties a brilliant play of colors will be seen when the plate is viewed through an analyzer. This can be very readily observed by use of the Norrenberg polariscope or simply by holding the mica in light reflected from the surface of a polished table top and viewing it through a pile of glass plates. (See § 173.) A rotation of either the mica or the analyzer will cause colors to appear and disappear. If in one position there are bright colors a rotation of 45° will cause them to disappear. The next 45° will again show colors which are the complements of those first seen. In the next 45° the colors again disappear, and at 180° from the first position the first colors will again be in view.

In explanation of these effects suppose the thickness of the mica is such that the ordinary ray lags $\lambda/2$ behind the extraordinary for red light. When the two rays emerge from the mica one will have been changed in phase by 180°. The plane of vibration will therefore be turned through 90°. But with the same thickness of plate the lag for violet is a whole wave-length or 360°, and the plane of vibration of emergent violet is the same as it was on entering the mica. The two planes of vibration are then at right angles. Hence, when the Nicol is placed so as to extinguish red, violet will be transmitted, and when the Nicol is turned through 90°, red will be seen and violet will be cut out.

For other colors of the spectrum the mica plate will, as has been shown, produce elliptic polarization. For colors near the red, as orange and yellow, the long axis of the ellipse will be nearly parallel to the plane of vibration of the red, and similarly for colors near the violet. Hence those colors which have wave-lengths approximating red will be mainly transmitted through

the Nicol with red and other colors with the violet. As the two sets of waves together produce white, any color seen in one position of the Nicol will be replaced by its complementary color when the Nicol is turned through 90°.

178. Rotation of the Plane of Vibration.—If the Nicol on the Norrenberg polariscope is turned so that a beam of plane polarized light sent up through it is extinguished, and a crystal of quartz having its upper and lower faces normal to the optic axis is interposed in the path of light below the Nicol, the beam of light will reappear. The Nicol must then be turned through a certain angle to again extinguish the light, *i.e.*, the quartz turned the plane of vibration through a certain angle though the light remained plane polarized.

The experiment indicated above should be made with monochromatic light such as a sodium flame. The amount of rotation is different for different wave-lengths, being about three times as great for violet as for red. It also varies with the thickness of the plate. For a plate of quartz 1 mm. in thickness the rotation for red light is about 18°. If white light is used the field of view will always be colored, for in any given position of the Nicol but one color is cut out.

In respect to this property it is found that quartz crystals are of two kinds: one rotates the plane of vibration to the right and is called right-handed or dextrogyrate quartz, while the other rotates the plane to the left and is called left-handed or levogyrate quartz.

A number of crystals and solutions possess the property of rotary polarization. The most important direct application of polarized light is in the use of a saccharimeter to determine the character and strength of sugar by the direction and number of degrees through which the plane of vibration of plane polarized light is turned while passing through a fixed length of solution. Sugar, like quartz, because of a difference in crystalline structure, is found to be left-handed and right-handed, the former being called dextrose and the latter levulose.

179. The Zeeman Effect.—It was long believed that an effect of some kind should be observed in a spectrum when the source of light is placed in a strong magnetic field. After the establishment of the electron theory it appeared evident that if the electron is an electric charge and its vibration is the cause of those

electromagnetic waves which we call light, some changes would be noticed in the spectral lines when the electrons are made to revolve or vibrate in a magnetic field.

Many experiments were tried and the problem seemed to be one of making sufficiently powerful instruments to make the effect observable.

In 1896 Dr. Zeeman, of the University of Amsterdam, was able to obtain results which strongly confirm the electron theory. Some of his results show great complexity and are not easy to explain, but we will describe a simple typical case as, for example, when blue-green cadmium light is used.

Zeeman found that when the light is viewed in the direction of the lines of force, through a hole in the pole pieces, this cadmium line was resolved into two lines, a doublet, one on each side of its ordinary position in the spectrum. This shows that there has been an increase in vibration frequency for that line which has moved toward the blue, and a decrease for the other line. He also observed that both lines were circularly polarized, but in opposite directions.

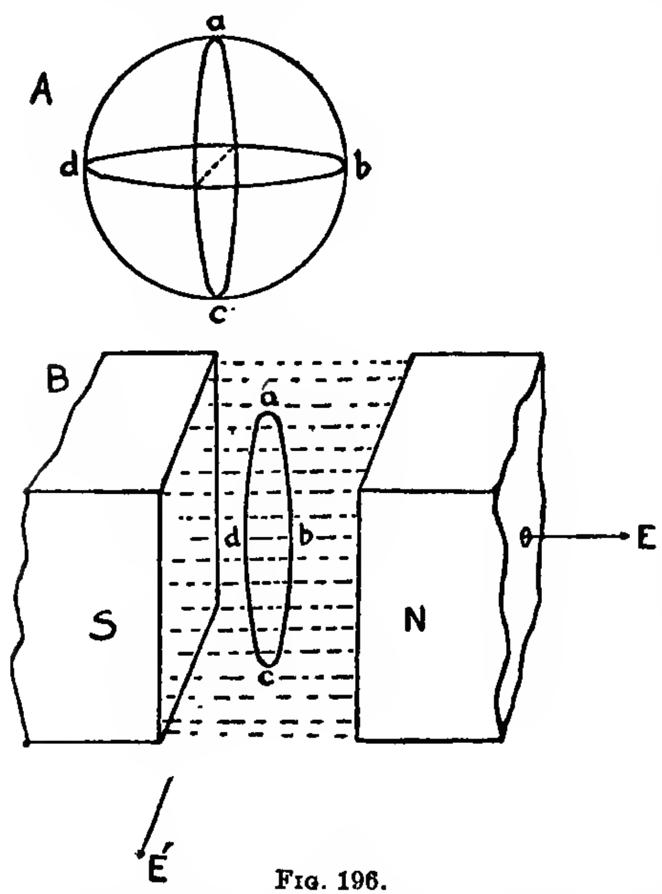

Fig. 196.

To explain this we assume that electrons are moving in orbits in every conceivable direction. All these orbits may be resolved in three planes as shown in Fig. 196, *A*. Now let the electrons, moving in these orbits, be placed in a strong magnetic field. Consider first only the orbit *abcd* placed at right angles to the field and viewed from *E*, Fig. 196, *B*.

Electrons are moving in both directions around the orbit. One moving in the direction *abcd* will, as may be shown by the

motor rule, be subject to a force tending to drive it from the centre. (See Fig. 61.) It will therefore move in a larger orbit and will make fewer revolutions per second. The waves sent out by this motion of the electron are therefore longer, and the resulting line as seen in the spectroscope is shifted toward the red end of the spectrum.

Waves are sent out only in a direction at right angles to the motion of the electron, and since the plane of the orbit is perpendicular to the line of observation, the light received at E will be circularly polarized. This accounts for the lower line of the doublet.

An electron moving around the orbit in the opposite direction, *adcb*, will be acted on by a force tending to draw it to the centre. Hence its motion will be accelerated and it will make a greater number of revolutions per second. The waves sent to E will then be shorter, and the line seen in the spectroscope will be shifted toward the region of the violet and the light will be circularly polarized in a direction opposite to that of the other line, for the orbits are in an opposite direction.

In applying the conventional left-hand rule or motor rule it appears that the lower line as seen from E should be circularly polarized clockwise and the upper line counter clockwise. An examination, however, shows that the opposite is true. Hence the electric current is negative, *i.e.*, electrons are negatively charged.

Now let the spectroscope be placed at E' and the orbits viewed in a direction at right angles to the magnetic field. No waves are sent out in the direction of the motion of an electron, hence only those components of the orbits at right angles to the line of vision will send waves to E'. The effect will be the same as the vibration of an electron between a and c on a diameter of the orbit. The larger orbit will therefore produce plane polarized light shifted toward the red in the spectrum. The smaller orbit will also produce plane polarized light, shifted toward the blue. The plane of vibration in both cases is perpendicular to the magnetic field.

What is actually observed when the light is viewed at right angles to the field is a triplet—*i.e.*, three lines—the middle one being in its ordinary position and plane polarized at right angles to the other two. The two outside lines have the position and

polarization they should have according to the explanation just given. The middle line of the triplet may be explained by considering the effect of the other two orbits shown in Fig. 196, *A*, which for sake of simplicity were left out of the lower figure.

By the amount of separation of the lines in the spectrum Dr. Zeeman was able to calculate the ratio of the charge to the mass, e/m, of an electron and obtained results comparable with those for beta rays and cathode rays. (See § 134.)

180. Artificial Light.—Any light produced by the devices of man is termed artificial as contrasted with natural light such as that from the sun.

Most methods of producing artificial light are very inefficient. Optical efficiency is the ratio of light energy emitted to the quantity of energy expended in producing that light and is usually designated in watts per candle-power (W.P.C.). Thus a carbon lamp at a temperature of about 1850° C. requires 3.5 W.P.C.; a tantalum lamp at 2000° C., 2 W.P.C.; a tungsten lamp at 2100° C., 1.25 W.P.C.; and at 2300° C., 1 W.P.C.

The best of these lamps, however, have a low efficiency, about 2 per cent.

Methods of producing artificial light consist chiefly in heating bodies to such a temperature that the waves sent out will come within the limits of the visible spectrum. Such waves, as shown in § 164, cover less than one-tenth of the entire spectrum, visible and invisible. Hence, even at the highest temperatures, a great deal of the radiant energy consists of waves too long to produce light. As long as we rely on a process of heating to produce light we will have optical inefficiency. It is found, however, that as temperature is raised optical efficiency increases, for the point of maximum energy radiation is thus shifted to the shorter wavelengths. (See Displacement Law, p. 240, " Mechanics and Heat.") The value of the tungsten, gem, tantalum, Nernst, and other lamps consists in the fact that a high temperature can be used without fusion or disintegration. But even with the best lights of this kind, efficiency is low.

The ordinary carbon arc lamp gives out light from the incandescent carbon terminals. The arc itself is not luminous. Hence this also is a heat lamp; although the temperature is high, the efficiency is not over about 10 per cent.

A great improvement has been effected by making the arc luminous. This may be done by impregnating the carbons with certain calcium salts which are vaporized at the hot terminals and render the carbon arc highly luminous. Such are called *flame arcs*.

In the so-called *luminous arcs* carbon is not used. One electrode is simply a good conductor which does not burn away, but the other is a mixture or compound of oxides of iron and titanium. The arc is highly luminous and the luminous particles are at the same time the carriers of the electric current.

In both lamps just described efficiency is high, between 30 and 40 per cent., and the effect is not due to high temperature.

181. Candle-power.—The unit of light giving power of any source is called a *candle*, from the former use of the candle for that purpose. By the efforts of the Bureau of Standards at Washington the principal nations of the world have agreed to adopt as a unit of candle-power *one international candle*. The unit is fixed by an agreement to definite relations which shall exist between it and the various concrete standards in practical use by different nations. The standard commonly used in Germany, for example, is the Hefner lamp, which burns amyl acetate and when the flame is turned to a height of 4 cm. gives one *Hefner unit*. The Carcel lamp burns colza oil and gives one *Carcel unit* when regulated according to certain specifications. This lamp is in common use in France. Another standard is the Vernon-Harcourt pentane lamp in which pentane is burned in a specified manner.

The relation of these standards to the international unit is

$$\begin{aligned} 1 \text{ international unit} &= 1 \text{ American candle.} \\ &= 1 \text{ pentane candle.} \\ &= 1.11 \text{ Hefner units.} \\ &= .104 \text{ Carcel unit.} \end{aligned}$$

The Hefner lamp is extensively used as a flame standard and, as shown above, the value of the Hefner unit is nine-tenths of the international unit.

It is difficult to maintain flame standards, for they vary with humidity, barometric pressure, etc. Probably the best practical standard is an incandescent lamp the filament of which has been

well seasoned by passing a current through it. Such a lamp when standardized and used without over-voltage will remain constant for a long time.

182. Intensity of Illumination.—The intensity of illumination of a surface upon which light falls is the quantity of light per unit area of that surface. Let L, Fig. 197, be a source of light of one candle-power in the centre of a sphere of radius r. Light will pass out in form of cones having their vertices at L. Let A be the area of the base of such a cone. The solid angle θ is then

$$\theta = \frac{A}{r^2} \text{ (see Appendix 2)} \tag{192}$$

and if θ is unity

$$A = r^2 \tag{193}$$

i.e., the area of the base of the cone as measured on the surface of the sphere is equal to the square of the radius, and since the surface of the sphere is $4\pi r^2$ there are 4π unit cones of light.

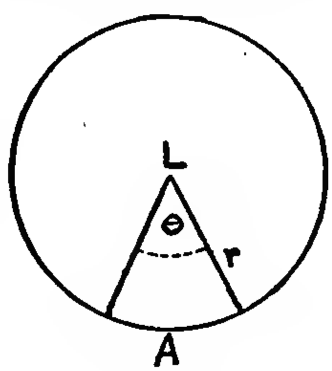

Fig. 197.

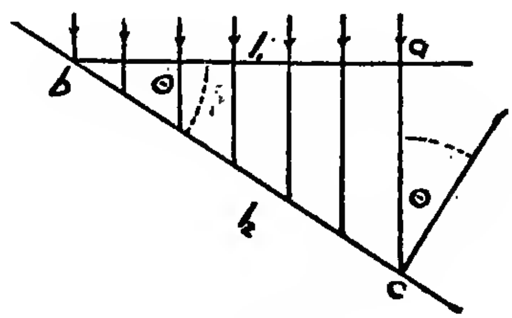

Fig. 198.

The quantity of light included in one unit cone is a *lumen* when the source of light is one candle-power. It is evident that the quantity of light will be the same for any value of r.

The *intensity* of illumination on A, however, will decrease when A increases, for the same number of lumens is spread over a larger area. By equation (193) the area, A, varies directly as r^2, hence intensity varies inversely as r^2. This is expressed by a common law called the law of inverse squares, *viz.*: *The intensity of light varies inversely as the square of the distance from the source.*

If r in Fig. 197 is one foot, the intensity of illumination on A is called one *foot-candle;* if r is one meter, one *meter-candle* or one *lux*. For example, if the source of light is 16 c.p., the intensity

at a distance of 1 ft. is 16 foot-candles; but, at a distance of 2 ft., 4 foot-candles, at 4 ft., 1 foot-candle.

If light falls on a surface obliquely, as in Fig. 198, all the light which would fall on *ab*, normal to the rays, now falls on *bc*. Since intensity, I, is inversely as area,

$$\frac{I_1}{I_2} = \frac{bc}{ab}$$

But

$$ab = bc \cos \theta$$

$$\therefore I_2 = I_1 \cos \theta \tag{194}$$

where *ab* and *bc* represent areas and I_2 is the illumination on *bc*. Hence intensity of illumination is directly proportional to the cosine of the angle made by the rays of light with the normal to the illuminated surface.

183. Photometry.—A photometer is an instrument for measuring the candle-power of lamps by comparison with a standard lamp. From what has been said above it is evident that if illumination varies inversely as the square of the distance from a source of light, then the candle-power of two sources, each of which produces the same illumination of a screen, will be to each other as the squares of their distances from the screen. If c_1 and d_1 represent candle-power and distance of one source of light and c_2 and d_2 of the other, then

$$\frac{c_1}{c_2} = \frac{d_1^2}{d_2^2} \tag{195}$$

If then it were possible to determine accurately when a screen is equally illuminated by the two sources of light, d_1 and d_2 can be measured, and if c_2 is a standard the value of c_1 is readily found.

It is not possible by the sensation of sight to determine with any degree of accuracy the intensity of illumination of any one surface, but shades of difference on two screens close together can be readily detected provided the lights are the same color.

Many different styles of instruments have been devised for this purpose. One form in common use is called the Bunsen photometer. A screen *ab*, Fig. 199, made of paper and having a translucent spot in the centre, is mounted in a box and may be moved along a carriage *cd*. At S_1 and S_2 are the sources of light, one of which

is a standard lamp. When *ab* is equally illuminated on both sides the central spot will be nearly invisible, *i.e.*, will appear like other parts of the screen. Otherwise the spot will appear dark on the side which is more intensely illuminated.

Mirrors *m* and *n* reflect an image of their respective sides of the screen to an observer at E, so that both sides can be seen at the same time.

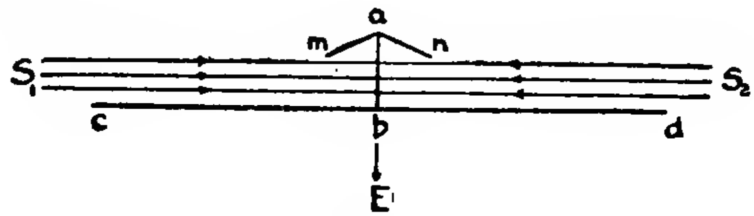

FIG. 199.

The Lummer-Brodhun photometer is provided with a greatly improved sight-box for comparing intensity of illumination on the screen.

Light from two sources S_1 and S_2, Fig. 200, falls on a white screen A, and images of the two sides are reflected by mirrors *m* and *n* to right-angled prisms P. The hypothenuse surface of

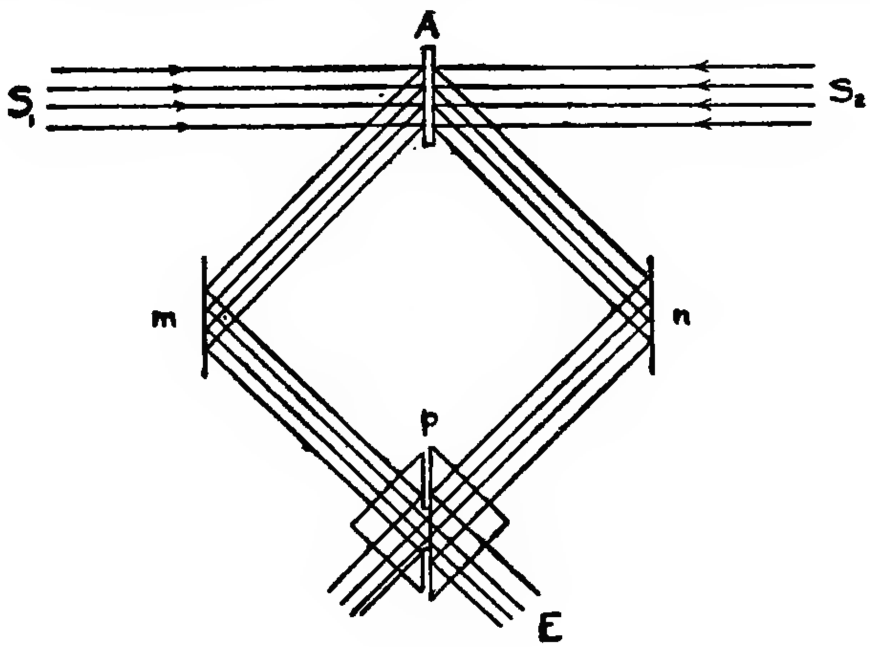

FIG. 200.

one prism is plane, that of the other is partly cut away, the central part of both surfaces being closely joined together so that, within the region of contact, rays from *m* will pass straight through both prisms and enter a telescope at E. Other rays are totally reflected.

Rays from n, outside of the region of contact, will be totally reflected to E, while those in the central region will pass through both prisms.

Thus an observer at E can see, in close contact, a portion of each side of A. The distances of the light from the screen are then adjusted until no difference of illumination can be detected and calculation of candle-power is made as described above.

Problems

1. An observer 449.58 cm. from a yellow flame looks through a grating ruled with 14,500 lines to the inch, and sees a yellow line in a spectrum of the first order 160.02 cm. to the left or right of the flame. What is the wave-length of the light used?

2. A crown glass prism whose angle is 4° 30′ has an index of refraction 1.513 for the *C* line and 1.521 for the *F* line. What must be the angle of a flint prism whose index for *C* is 1.60; and for *F*, 1.62, that the combination may be achromatic between *C* and *F*?

3. Calculate the polarizing angle in case of reflection from water.

4. What must be the relative distances of a Hefner lamp and a light of 1 international candle to produce equal illumination on the screen of a photometer?

Ans. 1. .587 micron.
2. 1° 48′.
3. 53° 10′.
4. 3 : 3.15.

CHAPTER XIII

SOUND

184. Media of Communication.—The two great media of communication are the ether and air. The former transmits electro-magnetic waves which, when of a certain length, are called light. The latter transmits waves of condensation and rarefaction which we call sound.

Man and all higher animals are constantly surrounded by these media and consequently there are two important organs, the eye and the ear, for perceiving disturbances transmitted on the ether and air. Without these organs our knowledge would be very limited.

185. Nature of Sound Waves.—All sound originates in a vibrating body which is immersed in air or other medium capable of transmitting sound waves. To transmit sound waves a medium must be elastic, *i.e.*, any deformation produced by a passing wave must produce a restoring strain. When the prong of a tuning fork, for example, is in vibration, it crowds or condenses the air on the side toward which it moves. This portion of air, in attempting to regain its original state, crowds the air in front of it, this the next, and so on. Thus a pulse or condensation moves out from the fork through the surrounding air. Each particle of air will, while the pulse is passing, make a slight forward movement in the same direction but will return to its original position as soon as the pulse has passed. Now when the prong of the fork moves in the opposite direction a body of air on the side from which the prong is moving will be rarefied. Adjacent air will then fall back into this rarefied region. Thus a pulse of rarefaction moves out from the fork and each particle of air, as the pulse passes, makes a short excursion in a direction opposite to that in which the pulse is moving and returns to its position of rest.

An elastic vibrating body executes simple harmonic motions and sends out a succession of alternate condensation and rarefaction which follow one another through the air, forming a train of waves as represented in Fig. 201. A complete wave, therefore,

is the effect produced in the medium by a complete vibration of a body which causes the wave. Each particle of the medium will then execute simple harmonic motions, and the length of the excursion during the passage of one wave is twice that for either pulse.

The amplitude of vibration is the distance a particle moves to either side of its position of rest, *i.e.*, forward or backward from its position of rest, in line with the direction of propagation of the waves.

A wave-length is the distance from any point in a wave to the next point in the same phase of vibration, as *ac*, *bd*, or the distance between any other two successive particles having the same displacement in the same direction.

Sound waves are therefore *longitudinal*, *i.e.*, the direction of displacement of the medium is in line with the direction of wave propagation, with the wave in case of a condensation and in the opposite direction in case of a rarefaction.

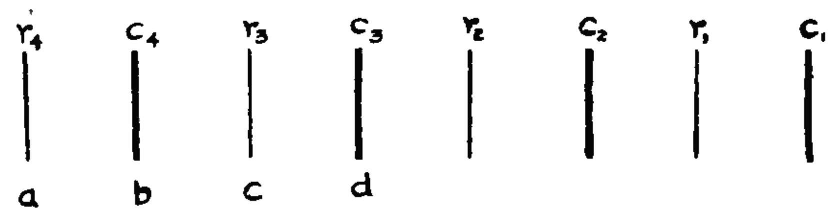

Fig. 201.

Light waves also consist of alternating changes of some kind, but the medium is the ether and not air. Also, in order to explain polarized light, it is necessary to assume that the displacement is at right angles to the direction of wave propagation.

The exact nature and mechanism of a sound wave is well known, but for light this is as yet an unsettled question.

Fluids cannot sustain a shearing stress and therefore waves in air cannot be transverse. The assumption of transverse waves in case of light necessitates the additional assumption that the ether possesses properties of an elastic solid, and this was the accepted theory a number of years ago.

While it is now generally admitted that Maxwell's theory of light is correct, yet nothing is known of the nature of the displacement in an electromagnetic wave. The use of transverse lines in figures such as 189 is simply a convenient device for explanation of polarized light.

186. Phenomena of Sound Transmission.—Many phenomena and their explanations which have been given for light also occur in case of sound and are explained in the same manner. The great difference in wave-length, however, causes the phenomena to appear under different conditions and on a different scale. The shortest wave-length of audible sound is about 1 cm. and the longest about 2000 cm. The shortest visible electromagnetic wave is approximately .00004 cm. and the longest about .000076 cm.

Reflection of sound is a matter of common observation and the explanation given in § 140 for reflection of light is just as applicable here. The conditions, however, are very different because of the great difference in wave-length. For the proper reflection of light a surface must be very smooth for the waves are very short, but sound waves may be reflected from the walls of a room, the side of a building, a cluster of trees covered with foliage, the side of a cliff, etc. Any large reflecting surface at right angles to the line of propagation will return the sound as an echo. Repeated reflections to and fro between the walls of a room or public hall is a great annoyance and often very difficult to correct. The chief causes of poor acoustic properties of a room are large and unbroken expanses of smooth walls, ceiling, and floor. The only effective remedy is to break these up in some manner as by beams across the ceiling, offsets, pictures, or other hangings on the walls, carpets and furniture on the floor, and by covering hard walls and curved surfaces with substances that will absorb sound waves instead of reflecting them.

A spherical surface will reflect sound waves and bring them to a focus just as in case of light, but the effect is often obscured by diffraction unless the curved surface is large or the wave-length is short.

Sound waves may also be refracted, and the phenomenon is explained as in § 147 for light. An experimental illustration may be given by passing sound waves through a lens-shaped bag filled with a dense gas such as carbon dioxide, and noting the increased intensity of sound at the focus.

Interference of sound waves may occur in the manner explained for light in Figs. 141 and 142.

It has been shown that, in case of light waves, diffraction or bending around an object is not ordinarily observed because the

waves are very short. In case of sound, on the other hand, the bending of waves around objects is a matter of common observation and it is only when the waves are very short, as shown in § 139, that sound shadows can be readily observed.

187. Velocity of Sound Transmission.—In physiology sound is regarded as a sensation, while in physics the waves which produce the sensation are usually regarded as sound. The velocity of sound, then, is the velocity of compressional waves through a medium—gas, liquid, or solid.

An equation worked out by Newton is

$$V=\sqrt{\frac{E}{\rho}} \tag{196}$$

where V=velocity, E=elasticity, and ρ=density. In words, velocity is directly proportional to the square root of the elasticity of a medium and inversely proportional to the square root of the density.

Equation (196) may be used to calculate the velocity of sound in various media provided the proper coefficient of elasticity is chosen. For example, if a rod of brass is stroked with a piece of rosined leather, longitudinal vibrations are set up, but since the rod is free to expand or contract laterally while it is subjected to compression or extension lengthwise, the elasticity for this condition is Young's modulus Y. Hence for brass

$$V=\sqrt{\frac{Y}{\rho}}=\sqrt{\frac{10.5(10)^{11}}{8.5}}=3514\text{ m.} \tag{197}$$

Since liquids cannot sustain a shearing stress they cannot have a coefficient of rigidity. For them E is the bulk modulus. The velocity of sound in water, for example, is

$$V=\sqrt{\frac{E}{\rho}}=\sqrt{\frac{2.27(10)^{10}}{1}}=1506\text{ m.}$$

Velocity of sound in gases is of the greatest interest and importance. It has been shown that there are two elasticities of a gas (see " Mechanics and Heat "), one isothermal and the other adiabatic. The former is equal to the pressure, P, but the latter is γP.

Newton attempted to apply his equation on the assumption that elasticity for a sound wave passing through a gas is isothermal. His equation would then be

$$V=\sqrt{\frac{P}{\rho}} \tag{198}$$

which for air at 0° C. and under pressure of 76 cm. of mercury, *i.e.*, one atmosphere, gives

$$V=\sqrt{\frac{76\times 13.6\times 980}{.001293}}=280 \text{ m.}$$

This result falls far short of results obtained by direct experiment, as when cannon are mounted at the tops of two distant hills and observers note the time between the flash of light and the arrival of the sound. From such experiments the velocity was approximately 331 m. This large difference indicates that Newton made a wrong assumption in regard to elasticity.

It was afterwards pointed out by Laplace that since air is a poor conductor of heat and the condensations and rarefactions occur very suddenly, the process is adiabatic. Hence the true equation is

$$V=\sqrt{\frac{\gamma P}{\rho}} \tag{199}$$

and this gives for velocity in air at 0° C. when the barometer stands at 76 cm.

$$V=\sqrt{\frac{1.4\times 76\times 13.6\times 980}{.001293}}=331 \text{ m.}$$

It has not been possible to determine the velocity of sound with a great degree of accuracy but, under conditions given above, 331 m. is probably correct to within several centimetres. This corresponds to 1086 $^{\text{ft.}}/_{\text{sec.}}$

188. Effect of Changes in Temperature and Pressure.—It was shown on page 189 of "Mechanics and Heat" that pressure of a gas may be expressed by

$$P=\rho R_{T}$$

and substituting this value in equation (199)

$$V=\sqrt{\gamma R_{T}} \tag{200}$$

where it is seen that velocity of sound varies directly as the square root of the absolute temperature τ. Therefore, to compare velocities for temperatures τ_1 and τ_2, we may use the proportion

$$\frac{V_1}{V_2} = \frac{\sqrt{\tau_1}}{\sqrt{\tau_2}}$$

For example, if V_1 is velocity in air at 0° C., then at 10° C.

$$\frac{331}{V_2} = \frac{\sqrt{273}}{\sqrt{283}}$$

$$\therefore V_2 = 337 \text{ m.}$$

The increase for 10° C. is therefore $337-331=6$ m., or 60 cm. per degree, very nearly 2 ft.

A change of pressure on any given body of gas does not affect the velocity of sound in that gas. This may be inferred directly from equation (200), for P does not occur there—*i.e.*, V is independent of P—or the same may be shown from (199) for, by Boyle's law, any increase of pressure is accompanied by a proportionate increase in density, consequently the ratio of P to ρ remains unchanged.

189. Musical Tones.—When a train of sound waves of uniform frequency and having a wave-length within the limits of hearing are incident on the drum-head of the ear, a sensation is produced which is called a *tone*. A body which vibrates in simple harmonic motion will produce such a train of waves.

When the frequencies are not uniform and the wave train is composed of a jumble of sounds of different pitch the sensation is called a *noise*, as in tearing paper, the fall of a tray of dishes, etc.

A tone may be *pure*, *i.e.*, caused by a wave train which contains but one kind of wave-length. Such a tone is most nearly produced by a tuning fork.

A tone in most cases is *complex*, *i.e.*, composed of several simple tones each having its own definite wave-length. The longest wave-length produces what is commonly called the *fundamental tone*, while the others produce *overtones*. Fundamentals and overtones unite to form a single tone of definite quality. The sensation produced is a resultant of all the tones. If a curve be plotted with time as abscissa, an ordinate at any point would be the sum of the disturbances of all the waves at that point. (See Fig.

39, " Mechanics and Heat.") Such a tone is complex in the sense that several wave trains have united to produce it. The components may be determined by analysis as shown in § 194.

Tones may differ in three respects: (1) *pitch*, (2) *intensity*, and (3) *quality*. These are discussed in the following paragraphs.

190. Pitch.—Pitch is the degree of acuteness of a tone and depends entirely on wave-length. The shorter the wave the higher the pitch. Since wave-length depends on vibration frequency, pitch may also be said to depend on it.

There are various ways of determining frequency for a given pitch. One method is by use of a siren, the principle of which is shown in Fig. 202. This consists of two circular plates which lie very close together, the upper one being free to rotate on a vertical axis. The plates contain circular rows of holes bored obliquely, as shown to the right in the figure. Air is forced up from below, and in passing from *a* to *b* a component of force turns the upper plate as indicated by the arrow. Each time the holes of the upper plate move over those below, a puff of air comes through. A moment later the air is shut off. Then another puff, and so on. This regular succession of disturbances produces a train of waves that in turn produce a tone whenever the speed of the rotating disc becomes sufficiently great.

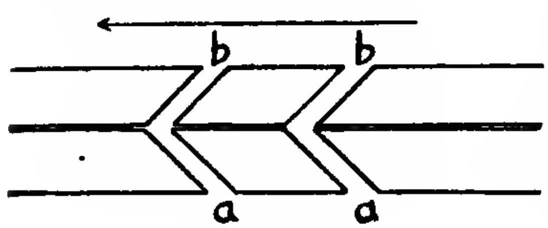

FIG. 202.

If the frequency for a tone of given pitch is desired, the speed of the siren is increased until its tone is in unison with the one to be tested. Then, keeping the speed uniform, a train of wheels is connected to a worm gear at the upper end of the shaft at a noted time. The hands on the dial show the number of revolutions. Knowing the number of holes in the plate, frequency is easily calculated.

191. Intensity.—Intensity of sound as measured by the sensation produced is the degree of loudness of that sound. The test of intensity in this manner is not exact, and the same sound is judged differently by the same person under different conditions. Fairly accurate comparisons, however, can be made of the intensity of two sounds heard at the same time or in immediate succession.

A more accurate method of finding intensity of sound at any point is to determine the rate at which energy of sound waves falls upon unit area of surface at that point.

Intensity depends on the amplitude of the vibrating particles of the medium, and this in turn depends on the amplitude of the vibrating body which is the source of the waves.

Each vibrating particle makes the same number of excursions, for tones of the same pitch, no matter what the amplitude may be. Hence the velocity of the particle is proportional to amplitude. But energy is proportional to the square of velocity as seen in the common equation $E=\frac{1}{2}mv^2$. Consequently the energy or intensity of a sound varies directly as the square of the amplitude of the vibrating particles.

Intensity also varies inversely as the square of the distance for, since sound moves out in all directions from a vibrating body, the energy may be regarded as distributed over the surface of a sphere. Since such a surface varies directly as the square of the radius, the energy per unit area would be inversely as the square of the distance or radius.

192. Resonance.—Resonance is the phenomenon which appears when a body is acted on by a succession of small impulses so timed as to conform to the natural periods of vibration of that body. Thus when a tuning fork, capable of giving out waves of considerable intensity, is sounded in the neighborhood of another fork of the same pitch, the second fork will also vibrate and give a tone of the same pitch. A few impulses from the first fork would produce no perceptible effect, but a large number of them, so timed that each added its effect to that of preceding ones, produce a cumulative effect.

A resonant air column is an important application of this principle, but before describing it we should consider the effects produced when a pulse of condensation or rarefaction passes from one medium to another of different density.

A reference to the model, Fig. 203, will make this plain. Let a, b, c, etc., be small masses supported by wires and connected together by coiled springs.

Now let an impulse be applied to a, moving it to the right. This motion will be communicated to b, then from b to c, and so on through the entire row. Thus a pulse corresponding to a

condensation moves from a to g, the motion of each mass being in the same direction as that of the pulse. When the condensation reaches the solid support B which corresponds to the denser medium, g cannot move to the right and the spring between g and f, being compressed more than any of the preceding ones, recoils and throws f to the left, then e, and so on back to a. *A condensation on meeting a denser medium is returned as a condensation.*

Now let a be pulled to the left. The motion of b, c, etc., will also be to the left while the pulse moves to the right. This is then a pulse of rarefaction. When g, which is fastened to B, is reached, the spring between g and f is stretched more than any preceding ones, for g does not yield. Hence f will be pulled to the right, then e, and so on back to a. Throughout the trip the particles moved in a direction opposite to that of the pulse. *A rarefaction on meeting a denser medium is returned as a rarefaction.*

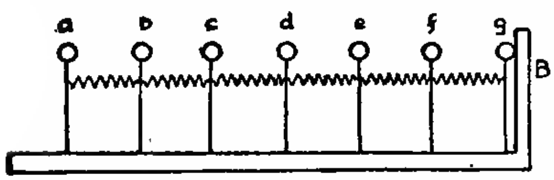

FIG. 203.

Let the body B be now removed. The region to the right of the row of particles may then be regarded as a rarer medium.

Now let a be moved to the right, thus sending a condensation to g. Since g is not supported on the right it will move farther than any of the others and will draw f after it, then e, and so on back to a. While the particles moved with the pulse from a to g, they move opposite to the pulse from g to a. *A condensation on meeting a rarer medium is returned as a rarefaction.*

Let a now be pulled to the left. A rarefaction will move over the row of particles, and since g has no spring or other support to the right of it, it will be pulled farther to the left than any of the others were and so will give f an impulse to the left, then e, and so on back to a. The particles moved opposite to the pulse from a to g but with it from g to a. *A rarefaction on meeting a rarer medium is returned as a condensation.*

Now applying these principles to resonant air columns, let A, Fig. 204, be a tube closed at the end ab but open at the other end to receive a train of waves. Suppose a tuning fork or other vibrating body produces waves the length of which is four times the length of the tube, *i.e.*, the distance from c_1 to c_2 or r_1 to r_2 is four

times the tube length. The condensation c_1 moves down to ab and is reflected as a condensation to the open end. The air outside the tube is in effect rarer than that enclosed in A. Hence when c_1 returns to the open end it sends a pulse of rarefaction back into the tube. This occurs at the instant r_1 has arrived at the mouth of the tube. These two rarefactions are thus united and a pulse of increased amplitude moves to ab, is reflected as a rarefaction, and returns to the open end where it sends back a condensation. At this instant c_2, the second condensation, has arrived at the same point, and the two condensations together constitute a pulse of increased amplitude. This in similar manner is reflected from the denser medium ab and on returning reflects a rarefaction which passes back into the tube with r_2. Thus there is a cumulative increase of the amplitude of particles within the tube. The intensity of the sound grows louder and would increase indefinitely, but for the fact that when a certain intensity is reached the quantity of energy escaping from the tube in form of sound waves is equal to that received.

FIG. 204.

It is seen from the discussion just given that the length of air column must be at least one-fourth of the wave-length of the note to which it resounds.

By a process of reasoning similar to that above it may be shown that there will be resonance, though not so intense, when the tube A, Fig. 204, is $\frac{3}{4}$ of the wave-length, for the condensation c_1 will then pass down and back and start a rarefaction down with r_2. Meantime the rarefaction r_1, one-half wave-length behind c_1, returns in time to send a condensation back with c_3, and so on.

In a similar manner there will be resonance again when the tube is $\frac{5}{4}$, $\frac{7}{4}$, etc., of the length of a wave.

Making use of this method of finding the length of a wave and knowing the frequency of the fork which produces the waves, it is possible to calculate velocity of sound from

$$V = n\lambda \tag{200'}$$

where V = velocity, n = frequency, and λ = wave-length.

A glass tube, Fig. 205, about 3 cm. in diameter and one metre or more in length, is partly filled with water which may be raised or lowered by use of a side tube or pump. A tuning fork making, say, 512 vibrations per second is set in vibration and the water raised to a point, as a, when a loud resonance will be heard. The distance from a to the top of the air column is then $\lambda/4$. Now let the water be lowered until a second resonance occurs, say at b. Then the distance from b to the top of the air column is $3\lambda/4$.

It is found that the effective length of the air column extends a short distance above the mouth of the tube to a point where freedom of the air is such that condensations and rarefactions are reflected back into the tube as if from a rarer medium. Hence a certain quantity, x, must be added to the actual length of the resonant tube to obtain the length of the resonant air column. The value of x is approximately the radius of the tube but varies with the material and size of the tube.

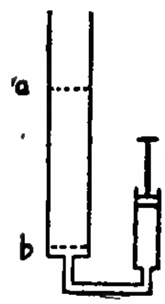

Fig.205.

Since the value of x is uncertain, it is better to eliminate it. Let L_1 be the length of the tube as measured from a to the top, L_2 as measured from b. Then

$$L_1+x=\frac{\lambda}{4}$$

and $$L_2+x=\frac{3\lambda}{4}$$

Subtracting. $$L_2-L_1=\frac{\lambda}{2}$$

and $$\lambda=2(L_2-L_1) \tag{201}$$

The value of λ and n may then be substituted in equation (200′)

193. Resonance in a Tube Open at Both Ends.—By application of the principles explained in § 192 it is seen that when the condensation c_1, Fig. 206, arrives at the open end e of the tube it causes a rarefaction to return to o. This rarefaction on emerging from the tube at o starts a condensation back into the tube and if the length of the tube is one-half the length of the wave, condensation c_2 will join it, thus producing an increased amplitude of vibration. Likewise r_1 will enter A, return a condensation at e,

and on emerging from o will send back a rarefaction which is joined by r_2. Thus the first resonant length for a tube open at both ends is $\lambda/2$. If the length of the tube is made $2\lambda/2$, c_1 will return to o in time to send a condensation back with c_3 and r_1 with r_3. If the length of the tube is $3\lambda/2$, c_1 will return in time to unite with c_4 and r_1 with r_4.

Thus, while for any given train of waves the lengths of resonant air columns in a tube closed at one end are $\lambda/4$, $3\lambda/4$, $5\lambda/4$, $7\lambda/4$, etc., the lengths in tubes open at both ends are $\lambda/2$, $2\lambda/2$, $3\lambda/2$, $4\lambda/2$, etc.

If the length of the air column is kept constant and the length of the sound waves changed, it is evident there will be a series of different wave-lengths for which there will be resonance, for the ratio of wave-length to tube length will be the same, for example, whether the wave-length is kept constant and the tube made twice as long or the tube kept constant and the wave-length made

FIG. 206.

one-half its former length. Hence the series of wave-lengths for which there will be resonance in a closed tube will be in the ratio $1, \frac{1}{3}, \frac{1}{5}, \frac{1}{7}$, etc., and the corresponding ratio of the number of waves is 1, 3, 5, 7, etc.

The first note, *i.e.*, the one having the longest wave-length, is called the fundamental note. The others are overtones.

Likewise for an open tube of constant length there will be resonance when wave-lengths bear the ratio $1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}$, etc., or, in terms of frequency, 1, 2, 3, 4, 5, etc.

Thus in closed tubes the frequencies for overtones are odd multiples of the frequency for the fundamental, while in open tubes the multiples are both even and odd.

194. Quality.—Of the three principal properties of musical tones, pitch and intensity have already been discussed. *Quality* has been deferred until after a discussion of resonance.

According to the investigations of Helmholtz, *quality depends on the combination of fundamentals and overtones and the relative phase and intensity of the various waves which enter into this combination.*

To analyze a complex tone such as that produced by striking the key of a piano, Helmholtz used a series of resonators like those shown in Fig. 207. Each of these is tuned to resonance for a certain vibration frequency.

By placing these in succession to the ear while a tone is being sounded, the fundamentals and overtones may be detected by the resonance which they produce.

Instead of placing the resonators to the ear they may be connected in succession to a recording phonograph, and a permanent record thus made of the fundamental and overtones in any given tone.

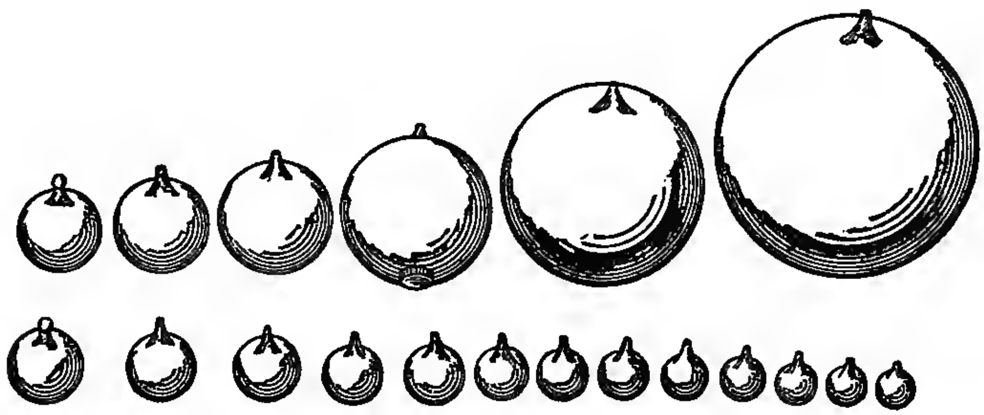

Fig. 207.

Helmholtz then prepared a number of tuning forks whose frequencies were the same as those of the components of the tone he had analyzed. When these were sounded together, each with proper amplitude, the original tone was reproduced.

Thus tones may be synthesized as well as analyzed, and *quality*, or *timber* as it is sometimes called, is that property of a tone by which it is possible to distinguish the tone produced by one instrument from that produced by another, although the pitch and intensity are the same in both.

195. Nodes in Resonant Air Columns.—A node is a point in a medium where particles in the path of a wave are quiescent.

When the condensation c_1 arrives at the end e of an open pipe whose length is one-half the length of a wave, the rarefaction has arrived at o. c_1 will therefore send back a rarefaction from e to meet r_1 at the centre of the pipe. The particles in the line n will thus be subjected to two equal and opposite forces and will have no amplitude. Each rarefaction will return to their respective

ends of the tube and then return to n as condensations. Here, again, the particles at n will not be moved.

An open pipe when giving its fundamental note will always have a node at its centre and an antinode at each end.

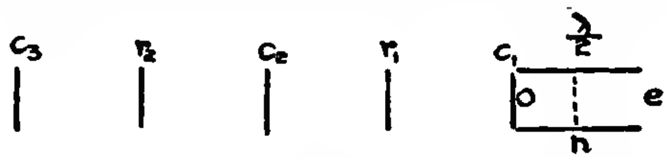

FIG. 208.

For the first overtone of an open pipe, the waves are one-half as long as for the fundamental. Hence the distance oe, Fig. 209, is now the length of one wave. When the first condensation c_1 arrives at e, r_1 will be half-way down the pipe. c_1 sends back a rarefaction which collides with r_1 at a point one-fourth the length of the pipe from e. Here, then, is a node n_2. But r_1 is reflected

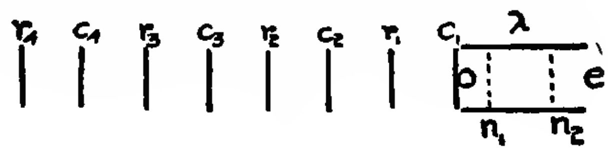

FIG. 209.

back from n_2 as a rarefaction and collides with c_2 at the middle of the pipe. Since both pulses displace particles in the same direction, there will be maximum disturbance at this point. c_2 now starts a rarefaction from the middle of the pipe toward o but collides with r_2 at n_1, one-fourth of the length of the pipe from o.

FIG. 210.

In a similar manner the three nodes of the second overtone or the four nodes of the third may be traced out.

A closed pipe is one-fourth the wave-length of the fundamental tone, hence c_1, Fig. 210, passes down to e and returns as a condensation to o by the time r_1 reaches that point. Hence there will be maximum disturbance at o and the only node in the pipe is at the stopped end e.

For the first overtone of closed pipes the waves must be one-third as long as for the fundamental, hence the length of the pipe is $3\lambda/4$. When c_1, Fig. 211, has moved to e, r_1 will have moved into the pipe to a position one-third of the distance from o to e. Then c_1 will return as a condensation and collide with r_1 at a point one-fourth of a wave-length from e. A condensation will return from this point and move toward o, meeting c_2 at a point one-fourth wave-length from o. Here will be a node n_1, the other being at the closed end.

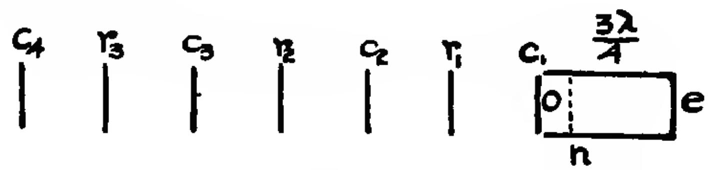

FIG. 211.

In a similar manner nodes may be traced out for the second, third or other overtones.

196. Organ Pipes.—Organ pipes are tubes of wood or metal, open or closed, within which sound waves travel to and fro in the manner just described, thus producing resonance. Instead, however, of a train of waves coming in from an outside source as in the figures of the preceding paragraph, resonance is produced by forcing a stream of air into one end of the pipe.

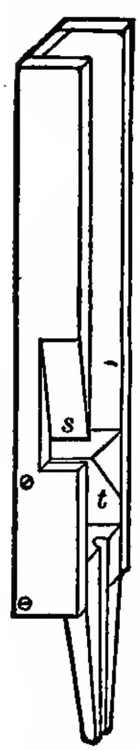

FIG. 212.

As shown in Fig. 212, air forced into the throat t issues from a narrow slit and strikes just within the sharp lip s. This starts a condensation up the pipe and also, because of the pressure produced, pushes the stream of air outside the lip. The condensation goes to the top of the tube and, if the top is open, is reflected as a rarefaction which on returning to s will draw the air inside. Thus the stream of air is made to vibrate from one side to the other of the lip s.

When the stream is forced outward, a rarefaction passes up the pipe and, if the pipe is closed, returns as a rarefaction, thus pulling the stream of air within.

In both closed and open pipes it is the period of the pulses up and down the pipe that determines the rate of vibrations of the current of air. Consequently the tone depends on the length

of the pipe. For a short pipe the round trip of a pulse is shorter, the number of vibrations greater, and the pitch higher.

It is also apparent that the pitch of the fundamental of an open pipe should be one octave higher than for a closed pipe of the same length, for the length of the former is $\lambda/2$, but, for the latter, $\lambda/4$. This may be illustrated by use of a pipe shown in Fig. 213. In the position shown, the slide forms a solid partition across the middle of the pipe. A closed pipe is thus formed which is one-half the total length of the tube. If the slide is pushed in, an open pipe is formed whose length is the total length of the tube. The pitch of the fundamental tones is the same in both cases.

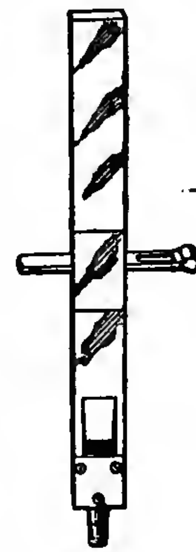

FIG. 213.

197. Velocity of Sound by Kundt's Method.—By a method illustrated in Fig. 214 it is possible to find the relative velocity of sound in gases or solids. *oe* is a rod of brass, steel, glass, or other elastic material about one metre long, rigidly clamped at the middle. *mn* is a glass tube one metre or more in length, tightly closed at *n*. To the end *o* of the rod is attached a light disc which may be inserted into the tube without touching the sides.

The rod *oe* may be made to vibrate longitudinally by stroking it with a piece of leather or cloth covered with rosin. Compressional waves will thus be made to pass back and forth from *c* to each end of the rod.

A condensation starting in the rod at *o* passes to *c*, where the rod is clamped. This point acts as a denser medium and reflects the condensation to *o*, where it is communicated to the air in the

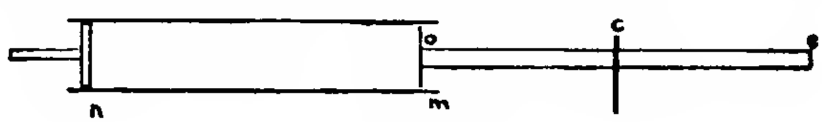

FIG. 214.

tube, but in passing from the rod to the rarer medium—air—a rarefaction is reflected back to *c* and returns as such to *o*, where a rarefaction is imparted to the air. While one complete wave is thus started out on the air, a pulse has traversed the space *oc* four times. The wave-length in the rod is therefore twice the length of the rod *oe*.

The wave-lengths in air within the tube depend on the velocity of sound in air. Since velocity in a gas is much less than in a solid, the waves are much shorter. To determine the length of these some fine powder, such as ground cork or lycopodium powder, is run along the lower side of the tube and the length of the air column adjusted to resonance with the vibrating rod. If the rod is then made to give a clear tone, the powder will be thrown into ridges at the antinodes but will be undisturbed at the nodes.

The distance between two adjacent nodes, as shown in Figs. 209 and 211, is one-half of a wave-length.

Since the vibration frequency is the same for both rod and air, then, from equation (200′),

$$n = \frac{V_r}{\lambda_r} = \frac{V_a}{\lambda_a} \qquad (201')$$

where the subscripts r and a refer to the rod and air respectively.

Now λ_r is twice the length of the rod, λ_a is twice the nodal distance, and if the velocity, V_a, in air is known, the velocity of sound in the rod is easily calculated. In this manner the velocity of sound in various solids may be found.

If V_r is known, the velocity in air or other gas with which the tube may be filled can be found.

Velocity in various gases may be found in this manner without knowing the velocity in the rod, for if the same rod is used for two different gases, the velocity in one of which is known, the velocity in the other is found by simply comparing the nodal distances.

198. Vibration of Strings.—If a flexible cord is stretched between two supports and is struck at a point near one end, the particles at that point are displaced in a direction at right angles to the cord. These displacements are communicated to adjacent particles, thus causing a pulse to move along the cord as shown in Fig. 215, where the stroke has been made upward near A and the pulse or crest at P is moving toward B.

The velocity of the pulse depends on the tension T of the cord and the mass m of unit length of cord. Each small portion of aPb, having mass m, must be in equilibrium between two forces, one due to T which tends to move m toward the centre of the arc, and the other is the centripetal force which must be

exerted to keep m in a circular path. Although the mass m does not actually move along the arc, the effect is the same as if aPb were stationary and the cord drawn from B to A with a velocity equal to that of the pulse from A to B.

The inward force due to tension is T/r, r being the radius of curvature. (See p. 170, "Mechanics and Heat.") The centripetal force is mV^2/r. Hence

$$\frac{T}{r}=\frac{mV^2}{r}$$

$$\text{or} \qquad V=\sqrt{\frac{T}{m}} \tag{202}$$

The pulse P in which the displacements of the cord are upward will move to B, where it will be reflected as a pulse in which the displacements are downward. After another reflection at A they

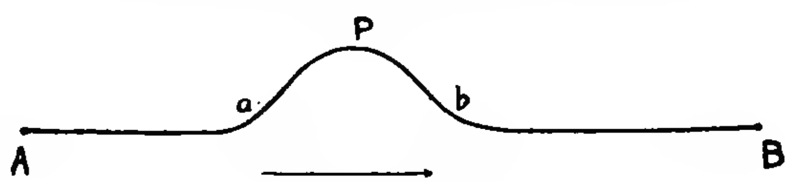

FIG. 215.

are again upward. When the pulse has returned to P, each particle of the cord has made one complete vibration at right angles to the cord, *i.e.*, a pulse has travelled twice the length, l, of the cord during one vibration.

Hence the wave-length, λ, on a cord vibrating as a whole—*i.e.*, not in segments—is $\lambda=2l$.

From equation (200′)

$$V=n\lambda$$

$$\therefore n=\frac{V}{\lambda}=\frac{1}{2l}V$$

Substituting the value of V from (202),

$$n=\frac{1}{2l}\frac{\sqrt{T}}{\sqrt{m}} \tag{203}$$

Stating this equation in words, *the vibration frequency of a flexible string varies inversely as the length, directly as the square root of the tension, and inversely as the square root of the mass of unit length.*

Since the mass of unit length of a string is its volume times its density, ρ, equation (203) may be written:

$$n = \frac{1}{2l}\sqrt{\frac{T}{\pi r^2 \rho}} \tag{204}$$

which shows that vibration frequency varies inversely as the square root of the density, and inversely as the radius of the string.

If the length of a wave is not twice the length of the string, but is $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, etc., of that distance, there will be points where an advancing and a reflected wave would meet in equal and opposite phase of vibration. A node would be produced at this point, as at n_1, n_2, n_3, in Fig. 216.

When a string vibrates in this manner the waves are said to be stationary. The distance between two nodes is one-half a wave-length. The vibrating segments between nodes are called loops. Each loop may be regarded as a string whose length is $\lambda/2$. To find the frequency of vibration, the results obtained from equation (203) must be multiplied by the number of loops.

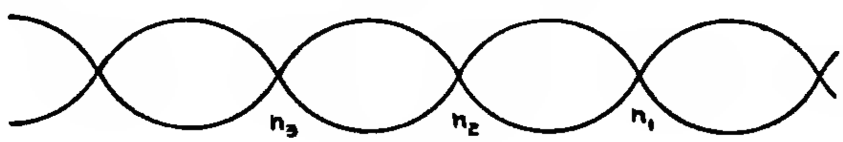

FIG. 216.

When a piano wire is struck the wire will vibrate as a whole, thus producing its fundamental tone, and superimposed on this is a number of shorter loops of various frequencies which produce overtones. The tone which is heard is therefore a complex one and possesses a quality dependent on the resultant form of the wave (§ 194). The point where a wire is struck determines in some measure the kind of overtones, for there cannot be a node at that point. For this reason piano wires are struck at a point about one-eighth of the distance from one end.

199. Diatonic Scale.—A musical scale is a series of tones or notes whose vibration frequencies bear to each other a certain ratio that is pleasing to the ear. The harmony produced by any combination of tones depends on the *ratio* of their vibration frequencies and not on pitch. This may be shown by opening the four stops of the siren, Fig. 202, thus producing what is called a

major chord in which the vibration frequencies of the four tones bear the ratio 4, 5, 6, 8. The siren may be driven at various speeds, thus changing the pitch, but the ratios and the harmony are maintained.

The names and relation of notes in the diatonic scale are as follows.

(1)	C		D		E		F		G		A		B		c
(2)	do		re		mi		fa		sol		la		si		do
(3)	1		$\frac{9}{8}$		$\frac{5}{4}$		$\frac{4}{3}$		$\frac{3}{2}$		$\frac{5}{3}$		$\frac{15}{8}$		2
(4)		$\frac{9}{8}$		$\frac{10}{9}$		$\frac{16}{15}$		$\frac{9}{8}$		$\frac{10}{9}$		$\frac{9}{8}$		$\frac{16}{15}$	

This scale consists of eight notes called the octave, the highest note having twice the vibration frequency of the lowest. According to the notation of Helmholtz and the common practice of musicians, notes are designated by letters as in line (1) above. The plain capitals indicate a certain octave in the bass. The next octave above is indicated by c, d, e, f, etc., and the next by c', d', e', f', etc., another prime being added for each additional octave. c' is called *middle c*. Its frequency for physicists' pitch is 256 complete vibrations per second, or 261 for international pitch. For physicists' pitch, then, $c=128$ and $C=64$. Octaves below C are marked C_1, D_1, E_1, etc., C_2, D_2, E_2, etc.

Each note in an octave is also designated by a name as in line (2).

In line (3) is given the ratio of the frequency of vibration of each note to the frequency for the fundamental or keynote C. C may have any frequency but, whatever it is, $\frac{9}{8}$ of it will give the frequency for D, $\frac{5}{4}$ of it will be the frequency for E, and so on through the scale. If the letters are taken to represent frequencies, $C:D=8:9$, $C:F=3:4$, $C:B=8:15$, and so on for any notes in the scale.

In line (4) is given the ratio of each note to the one preceding. These are called intervals. There are three kinds of intervals in the scale, $\frac{9}{8}$, $\frac{10}{9}$, and $\frac{16}{15}$, the first two being whole tones and the last a half tone.

200. Construction of a Major Diatonic Scale.—When two trains of waves of different wave-length are moving together as shown in Fig. 217 they move with the same velocity, and there are certain regions in the trains where the disturbance will be a maximum or a minimum. c_1 in each train is a condensation,

hence the displacement of air particles there will be maximum, and the same effect is produced at all points where there is coincidence of the c's or the r's in the two trains. At c_6 and r_5, however, the vibrations are opposite in phase, *i.e.*, a condensation and a rarefaction are united, and the result will be silence.

If two continuous trains of waves of this kind enter the ear, the sound will be alternately loud, then soft, then loud, and so on. Intermittent sounds produced in this manner are called *beats*. The number of beats per second produced by any two tones is plainly equal to the difference in their vibration frequencies. When, for example, c' and d' are sounded together, the number of beats is $288-256=32$.

Now the ear is so constructed that when the number of beats is very small or very large, their effect in not unpleasant, but for intermediate numbers, about 30 or 40 in the neighborhood of middle c, the effect is very discordant. All discord is caused by beats.

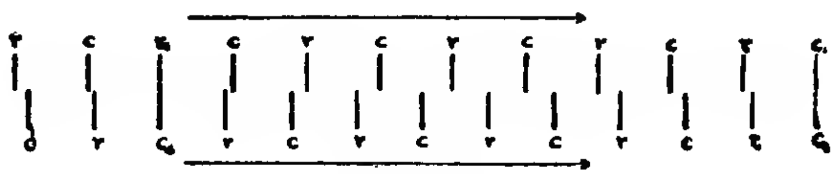

FIG. 217.

It is necessary, therefore, in selecting notes for a musical scale, to avoid, as far as possible, those frequencies which when combined would produce discord as a result of beats.

It is found that the ratio of the vibration frequencies of two tones is most pleasing when it may be expressed by the smallest numbers. Thus $C:c=1:2$, the *octave*, is the most harmonious ratio. The next is $C:G=2:3$, the *fifth*; $C:F=3:4$, the *fourth*; $C:E=4:5$, the *major third*; $E:G=5:6$, the *minor third*.

A combination of three tones whose frequencies are in the ratio $4:5:6$ produces a pleasing effect. The three tones so related form a *major triad*. The diatonic scale may be regarded as made up of three such triads. The first embraces the notes C, E, and G. These must have frequencies in the ratio $4:5:6$. Hence, if $C=4$, $E=\frac{5}{4}$ and $G=\frac{3}{2}$ of that number. Whatever the value of C may be, E is $\frac{5}{4}$ and G is $\frac{3}{2}$ of it. If $C=1$, then $E=\frac{5}{4}$,

and $G=\frac{3}{2}$ as given in the scale on page 289. This is called the triad of the *tonic* because the first note in it is the tonic or key-note of the scale.

The second triad includes the notes G, B, and d. These must also be in the ratio $4:5:6$, or $1:\frac{5}{4}:\frac{3}{2}$. The frequency of these three notes in reference to C is, therefore, $G=\frac{3}{2}$, $B=\frac{5}{4}$ of $\frac{3}{2}=\frac{15}{8}$, and $d=\frac{3}{2}$ of $\frac{3}{2}=\frac{9}{4}$. This is called the triad of the dominant, for G is called the *dominant* in the scale.

In this triad d falls in the octave above. To complete the eight notes of which C is the tonic, $D=\frac{9}{8}$ is used instead of $d=\frac{9}{4}$, the frequency for d being twice that for D.

The third triad includes the remaining notes of the scale, *viz.*, F, A, and c. The ratio of frequencies must again be $4:5:6$ or $1:\frac{5}{4}:\frac{3}{2}$, *i.e.*, $\frac{3F}{2}=c$ and $\frac{5F}{4}=A$. Since c is one octave above C, the ratio of their frequencies is $C:c=1:2$. In reference to C, therefore, $c=2$. $\frac{3F}{2}=c=2$, hence $F=\frac{4}{3}$. $\frac{5F}{4}=A$, hence $\frac{5}{4}$ of $\frac{4}{3}=A=\frac{5}{3}$.

This is called the triad of the subdominant, for F is called the *subdominant* of the scale. This completes the frequency ratios, in reference to the key-note, for the eight notes of the major diatonic scale.

A minor triad is composed of three notes having frequencies in the ratio $10:12:15$. By using three of these a minor scale may be constructed in exactly the same manner as that given above for the major scale.

201. Scale of Even Temperament.—The scale described in the preceding paragraph is what is called a *scale of just temperament.* It serves the purposes of music very well as long as the key-note is C, but it is often desirable to change the key-note for the purpose of producing certain musical effects or to accommodate the pitch to the range of singers' voices. For the latter purpose the change in pitch might be a whole octave and then C of that octave could still be the key-note, but such a large change is seldom desired.

For the human voice and instruments, such as the violin, that can be adjusted at will, the scale of just temperament can be used with any key-note. But in instruments with fixed key-boards such as pianos and organs, where each wire or pipe always has the same vibration frequency, the frequencies of several notes in the scale will not be in accordance with the requirements of just tem-

perament. This may be shown by a comparison of two scales as given below, the first being in the key of C and the second in the key of D. The numbers are the absolute frequencies in the middle octave for physicists' pitch.

	do	re	mi	fa	sol	la	si	do	re
	1	$\frac{9}{8}$	$\frac{5}{4}$	$\frac{4}{3}$	$\frac{3}{2}$	$\frac{5}{3}$	$\frac{15}{8}$	2	$\frac{9}{4}$
Key of C	256	288	320	341.3	384	426.6	480	512	576
		do	re	mi	fa	sol	la	si	do
		1	$\frac{9}{8}$	$\frac{5}{4}$	$\frac{4}{3}$	$\frac{3}{2}$	$\frac{5}{3}$	$\frac{15}{8}$	2
Key of D......		288	32	360	384	432	480	540	576

It is seen that instead of the 320 vibrations which the piano would give if it had only the major diatonic scale, there should be 324 if D is the key-note. Also, instead of 341.3 the number should be 360, instead of 426.6 it should be 432, and instead of 512 it should be 540. For the key of D, then, provision would have to be made for four extra notes. By comparing in this manner the frequencies for scales with other key-notes, it is found that there would need to be about 50 notes in each octave.

This difficulty could be partly remedied by introducing sharps and flats within the interval of the whole tones. The interval from C to $C\sharp$ is $\frac{25}{24}$ and from $D\flat$ to D is also $\frac{25}{24}$. If all the whole tones are broken up in this manner, we would have the so-called *harmonic* scale of 18 notes,

C	C♯	D♭	D	D♯	E♭	E
1	$\frac{25}{24}$	$\frac{27}{25}$	$\frac{9}{8}$	$\frac{75}{64}$	$\frac{6}{5}$	$\frac{5}{4}$, etc.

The intervals in this scale are much shorter, and the frequencies required for the extra notes when different keys are used will very nearly agree with one of these 18 notes. But the scale is long and the agreement not as close as it should be in many cases.

It has therefore been agreed that the scale shall consist of 12 notes with equal intervals throughout. This is called the *scale of even temperament.*

The frequency for any note in this scale multiplied by $\sqrt[12]{2}$ or 1.059 will give the frequency for the next note. A comparison of the frequencies as given below shows that the intervals do not seriously differ from those of the true scale.

	C		D		E	F		G		A		B	C
Just	256		288		320	341.3		384		426.7		480	512
Even	256	271.3	287.4	304.8	322.7	341.7	362.2	383.8	406.6	430.7	456.5	483.5	512

The numbers between the letters in the even scale are used for both the sharp of the letter below and the flat of the letter above. They correspond to the black keys of the piano.

Problems

1. Compare the velocity of sound in air at $-17°$ C. and at $16°$ C.

2. A bomb dropped from an air-ship explodes on striking the ground and the sound is heard in the ship 10 seconds after the bomb has started to fall. Temperature of air $= 10°$ C. How high was the ship?

3. What must be the tension of a string 1 m. long and weighing 4 mg. that it may make 256 complete vibrations per second? (Use grams, centimetres, and dynes.)

4. A closed organ pipe 1 m. in length gives the correct pitch at $15°$ C. Will the pitch be raised or lowered when the temperature rises to $35°$ C.?

5. If the frequency of c'' is 512 and an observer moves toward the source of sound at the rate of 41.37 m. per second in air at $0°$ C., what tone will he hear?

6. A metal rod 1 m. long vibrates longitudinally and produces nodes and antinodes in a Kundt's tube. If the distance between the nodes in air at $20°$ C. is 9 cm., what is the velocity of sound in the metal rod?

7. What is the ratio of the radii of the e and g strings on a violin, all other conditions being the same?

8. What is the interval between F and c on the major scale? Between C and $F\#$ on the even-tempered scale?

Ans. 1. 16 : 17.
2. 384+ m.
3. 1.7 kg.
4. Raised 3 vibrations per sec.
5. d''.
6. 3811.2.
7. 6 : 5.
8. (1) $\frac{3}{2}$. (2) $\sqrt{2}$.

APPENDIX

1. Prove that the absolute potential at a distance r_1 from a charged point is $\frac{Q}{r_1}$.

It is desired to find the work required to bring unit charge from infinity to the point r_1 (see Fig. 7). The intensity of the field at r_1 in air is $\frac{Q}{r_1^2}$, *i.e.*, intensity or strength varies inversely as the square of the distance. If F is the force at any point, the work done in moving unit positive charge a distance dr is Fdr.

Substituting the value of $F=\frac{Q}{r^2}$ and letting W stand for work,

$$W=\int_{r_1}^{\infty}\frac{Q}{r^2}dr=Q\int_{r_1}^{\infty}\frac{dr}{r^2}$$

$$\therefore W=Q\int_{r_1}^{\infty}r^{-2}dr$$

$$=Q\Big|_{r_1}^{\infty}-\frac{1}{r}$$

$$=Q\left(\frac{1}{r_1}-\frac{1}{\infty}\right)$$

$$=\frac{Q}{r_1}$$

2. Prove that the intensity of the magnetic field within a solenoid is

$$H=\frac{4\pi ni}{10}\text{ gausses}$$

Let a continuous wire be wrapped so as to form the hollow cylinder shown in Fig. 218. This constitutes a solenoid. A current flowing on the conductor must pass many times around the enclosed space.

Let o be a point within the solenoid and A an elementary ring having a width l and circumference $2\pi r'$.

If there are n turns of wire per centimetre in the solenoid, there are nl turns on the ring. Each short arc of the ring may be regarded as an arc of another circle having its centre at o. Hence a unit pole at o will be urged in a direction at right angles to the plane of the circle around o, *e.g.*, the small arc ab will urge the unit pole in the direction of. A similar arc on the opposite side of the ring will urge the unit pole along of'. The sum of all these elementary portions is $2\pi r'$ for each turn, and $2\pi r'nl$ for nl turns. The sum of all the forces on o, according to the definition of unit *e.m.* current, is, therefore,

$$\frac{2\pi r'lni}{r^2}$$

where i is strength of current and r is the radius of an imaginary circle having its centre at o.

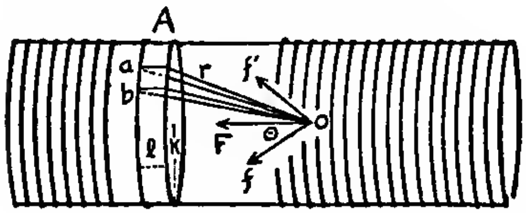

FIG. 218.

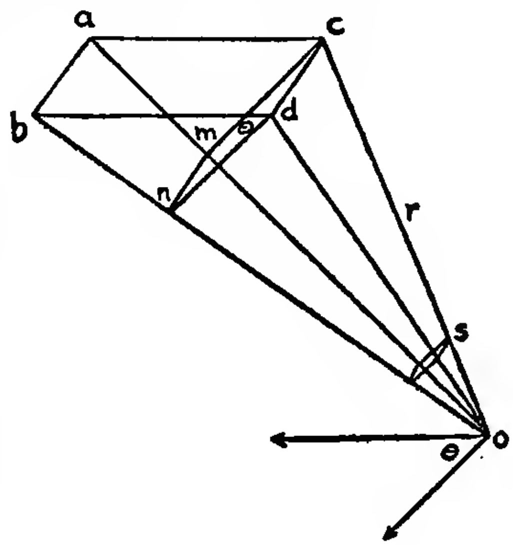

FIG. 219.

The resultant of all the forces due to each elementary part of the ring is oF, parallel to the axis of the solenoid and making an angle θ with of or of'. Hence the total force exerted by the elementary ring on unit pole at o is

$$F = in\frac{2\pi r'l}{r^2}\cos\theta$$

Now $\dfrac{2\pi r'l}{r^2}\cos\theta$ is the solid angle at o included between lines drawn from both edges of the elementary ring to o. This will be plain from Fig. 219, where $abcd$ is a small portion of the ring of Fig. 218. If o is the centre of a unit sphere, say 1 cm. in radius, there will be 4π square centimetres on the surface, each of which may be regarded as subtending unit solid angle at o. There would then be 4π solid angles about o. But at any distance r, the area of the

base of a pyramid varies directly as the square of r. Hence if the area *cdmn* is divided by r^2 we get the area s on the unit sphere.

The area $cdmn = abcd \cos\theta$, hence the solid angle at o, Fig. 219, is

$$\frac{abcd \cos\theta}{r^2}$$

The area of the ring, Fig. 218, is made up of such areas as *abcd* and the total number of them is $2\pi r'l$. Hence the solid angle subtended by the ring is

$$\frac{2\pi r'l}{r^2}\cos\theta$$

The point o is surrounded by portions of the solenoid in every direction except at the ends and, if the solenoid is long as compared with its diameter, o may be regarded as completely surrounded. Then by applying the same argument as above, the entire volume of the unit sphere at o will be filled with solid angles except where subtended by the small open space at the ends, and this is negligible under the conditions given.

Since there are 4π solid angles at o,

$$F = 4\pi ni \text{ gausses}$$

if i is in *e.m.* units, and

$$F = \frac{4\pi ni}{10} \text{gausses}$$

if i is in amperes.

3. The square root of the average of all the squares of the current or *e.m.f.* at each instant during a period gives the effective current or *e.m.f.*, called also the virtual current or virtual *e.m.f.*

Since the square root of the average of the squares of a series of numbers is larger than the average of those numbers, the virtual current is larger than the average current.

To calculate virtual *e.m.f.* of an alternating current we have, from the sine curve,

$$y = r\sin\theta$$

$$\text{Mean square of } y = \frac{\int_0^{2\pi} y^2 d\theta}{\int_0^{2\pi} d\theta}$$

Substituting the value of $y=r \sin\theta$, and integrating the denominator,

$$\text{Mean square of } y=\frac{r^2}{2\pi}\int_0^{2\pi} \sin^2\theta d\theta$$

But $\sin^2\theta=\frac{1}{2}-\frac{\cos 2\theta}{2}$. Substituting this value of $\sin^2\theta$,

$$\text{Mean square of } y=\frac{r^2}{2\pi}\int_0^{2\pi} \frac{d\theta}{2}-\frac{\cos 2\theta}{4}2d\theta$$

$$=\frac{r^2}{2\pi}\Big|_0^{2\pi}\frac{\theta}{2}-\frac{\sin 2\theta}{4}$$

$$=\frac{r^2}{2\pi}\pi-o$$

$$=\frac{r^2}{2}$$

$$\sqrt{\text{mean square of } y}=\frac{r}{\sqrt{2}}=.707r$$

Let i_v be the virtual strength of current. The maximum ordinate in a sine curve is $r=$maximum current i_m.

Hence

$$i_v=\frac{i_m}{\sqrt{2}}=.707i_m$$

Likewise for virtual *e.m.f.*

$$E_v=\frac{E_m}{\sqrt{2}}=.707E_m$$

4. Prove that if a current i with a frequency n is flowing through a circuit having inductance L, the counter *e.m.f.* is $2\pi nLi$.

If the current is expressed in virtual amperes and the inductance in henrys, the *e.m.f.* will be in virtual volts.

Inductance, L, is for unit current, hence the self-induction for current i is Li. Since the self-induced *e.m.f.* is proportional to the rate of change of Li,

$$e.m.f.=L\frac{di}{dt} \tag{1}$$

But $$i = i_m \sin 2\pi nt \tag{2*}$$

and $$\frac{di}{dt} = 2\pi n i_m \cos 2\pi nt \tag{3}$$

Substituting this value of $\frac{di}{dt}$ in (1) and keeping in mind that virtual values of sine and cosine are equal,

$$e.m.f. = 2\pi n L i_m \sin 2\pi nt \tag{4}$$

Then substituting in (4) the value of i as found in (2),

$$e.m.f. = 2\pi n L i$$

5. Prove that the velocity of a wave in an elastic medium is

$$V = \frac{\sqrt{E}}{\sqrt{\rho}}$$

It may be assumed, to begin with, that velocity varies directly as the elasticity E which determines the rapidity of transmission from particle to particle and inversely as the density which has a retarding influence.

Elasticity is a force per unit area, *i.e.*, $\frac{F}{A}$. This, expressed in fundamental units, is $\frac{[MLT^{-2}]}{[L^2]}$. Density is mass per unit volume or $[ML^{-3}]$.

But velocity $V = \frac{L}{T}$, hence

$$V = \left[\frac{L}{T}\right] = \frac{\left(\frac{[MLT^{-2}]}{[L^2]}\right)^a}{\left(\frac{[ML^{-3}]}{1}\right)^b}$$

Now if the exponents a and b are each made to equal $\frac{1}{2}$, the last term of this dimensional equation becomes equal to $\left[\frac{L}{T}\right]$. Hence, by choosing such units that no constant need be introduced,

$$V = \sqrt{\frac{E}{\rho}}$$

---

* See p. 32, "Mechanics and Heat."

6. Magnifying Power

1. *Simple Microscope.*—The principle of the simple microscope has already been illustrated in Fig. 177, *B*. The magnifying power of such an instrument is the ratio of the linear dimensions of an image to that of an object. Thus, in Fig. 220, *oo′* is an object placed less than the focal distance from the lens. An eye at *E* will then see a virtual image at *ii′*. The magnifying power is then the ratio of *ii′* to *oo′*. These lines bear the same ratio as their distances from the lens, so if O is the object distance and I the image distance,

$$\frac{I}{O} = \frac{ii'}{oo'}$$

The general equation for lenses is

$$\frac{1}{O} + \frac{1}{I} = \frac{1}{F}$$

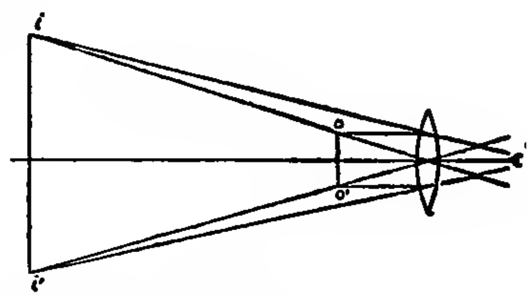

FIG. 220.

but since the image in a simple microscope is always virtual,

$$\frac{1}{O} - \frac{1}{I} = \frac{1}{F}$$

or

$$\frac{I}{O} - 1 = \frac{I}{F}$$

$$\therefore \frac{I}{O} = \frac{I}{F} + 1$$

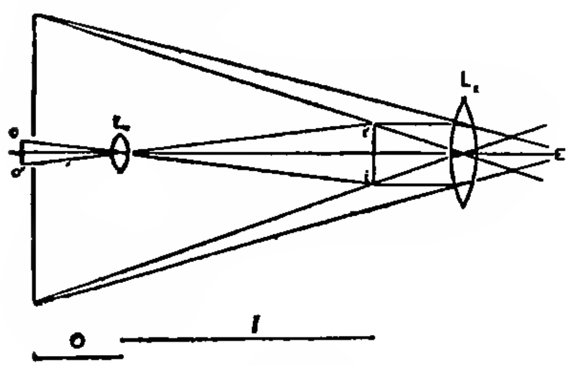

FIG. 221.

Since the distance I for distinct vision is 25 cm., or 10 inches, we may write

$$\frac{I}{O} = \frac{25}{F} + 1 = \text{magnifying power.}$$

2. *Compound Microscope.*—In the compound microscope the objective L_1, Fig. 221, forms a real image of *oo′* at *ii′*. The length of the image is as many times greater than the object as the distance I_1 is times O_1. Hence the magnification of *oo′* by the lens L_1 is

$$\frac{I_1}{O_1}$$

The image ii' falls within the focal length of L_2, hence the magnification of L_2 is the same as that given above for the simple microscope. The magnifying power of both lenses is therefore

$$\frac{I_1}{O_1}\left(\frac{25}{F}+1\right)$$

3. *Astronomical Telescope.*—In an astronomical refracting telescope the object glass is of large diameter and long focus.

Let the diameter of the object oo', Fig. 222, be denoted by D_1 and its distance by O. Let the angle which rays from o and o' make at lens L_1 be θ. The object is far distant and so the wave-front from it may be regarded as plane. The distance from c_1 to the image ii' is therefore the focal length, F_1, of L_1.

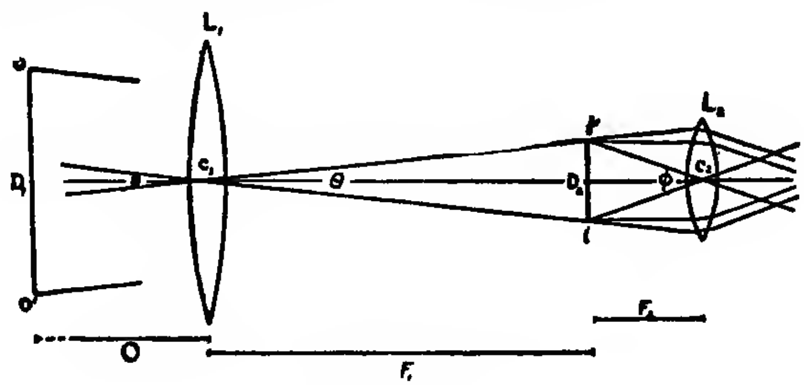

FIG. 222.

If the object is viewed directly by the eye without any lens, its diameter, θ, measured in radians, is

$$\frac{D_1}{O}$$

The ratio of D_2 to D_1 is equal to the ratio of F_1 to O. Hence

$$\frac{D_2}{D_1}=\frac{F_1}{O}$$

and $$D_2=\frac{D_1F_1}{O}$$

Now if the image D_2 is viewed through an eye-piece L_2 placed at its focal distance, F_2, from ii', the apparent angular dimensions, ϕ, of the image will be

$$\frac{D_2}{F_2}$$

Substituting the value of D_2 found above, we have for the apparent size of the image

$$\frac{D_1F_1}{OF_2}$$

But $\frac{D_1}{O}$ is the diameter of the object as seen without the telescope, hence this has been multiplied by $\frac{F_1}{F_2}$ by means of the telescope. The magnifying power is therefore $\frac{F_1}{F_2}$, or the ratio of the focal length of objective to eye-piece.

7. Diopters

Opticians and spectacle makers usually express the power of a lens in *diopters*. A diopter is the reciprocal of focal length in metres. If the focal length is 1 m., the power is 1 diopter. If the focal length is 50 cm., 2 diopters; if 4 m., .25 diopter, etc.

One metre is 39.37 inches, but 40 inches is usually taken as equal to 1 m. for this purpose. A lens having a 5-inch focus would then have a power of 8 diopters.

If an eye cannot clearly see objects at less than 2 m. distant, the focal length in the eye is too long and an image, if it could be formed, would be back of the retina. To remedy this defect a convex lens must be used. If distinct vision at 25 cm. is desired the curvature of the refracted waves must be increased from $\frac{1}{200}$ to $\frac{1}{25}$. The increase is therefore $\frac{7}{200}$ and so the necessary power of the glasses is 3.5 diopters.

8. The Interferometer

An interferometer is any device by which waves in two beams of light are made to interfere and produce fringes or bands which are alternately dark and light if monochromatic light is used, or colored bands in case of white light.

A very simple arrangement for showing interference bands is shown in Fig. 223. Two pieces of plate glass, A and B, about 8 cm. long and 1 cm. wide, are placed in contact at a and separated very slightly at b. Thus a very thin wedge of air is included between the two plates. When light falls on the top of A it is partly

transmitted through both plates and partly reflected from each of the four surfaces, but that reflected from the bottom of A and the top of B will produce interference. That portion of a beam reflected from the top of B must, after reflection, lag behind the portion reflected from the bottom of A by a distance $2d$ where d is the thickness of the wedge at that point. These two trains of waves will therefore differ in phase, and if this difference amounts to half a wave-length there is destructive interference or darkness.

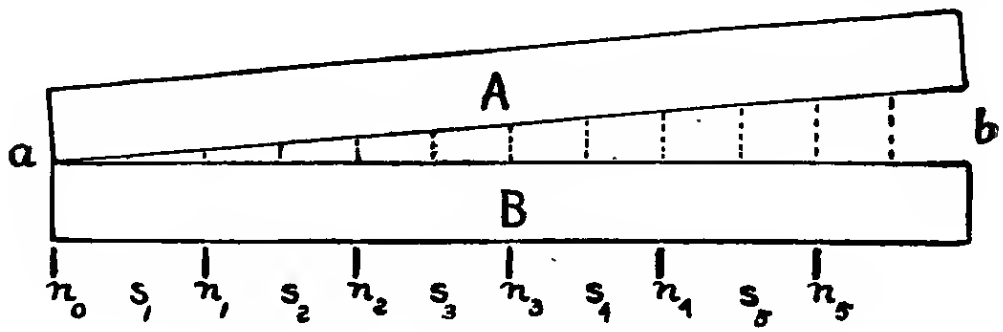

FIG. 223.

But the conditions of reflection are different in these two cases, for in A the ray is reflected in glass against air while on B the reflection is in air against glass. The effect of a change of medium is just the same as in case of sound waves (§ 190), and the difference of phase is therefore increased by $\frac{\lambda}{2}$ on this account. Hence the total retardation or difference of phase is

$$2d+\frac{\lambda}{2}$$

When this retardation amounts to an even number of half wave-lengths, the band seen at that part of the wedge is bright. When it equals an odd number of half wave-lengths, the bands are dark.

At a the two plates are in contact and $d=0$. Hence $2d+\frac{\lambda}{2}=\frac{\lambda}{2}$ and there is a black band at this point. At s_1, $d=\frac{\lambda}{4}$. Hence $2d+\frac{\lambda}{2}=\lambda$. Here, then, is a bright band, for the two sets of waves are in step. At n_1, $d=\frac{\lambda}{2}$. Hence $2d+\frac{\lambda}{2}=\frac{3\lambda}{2}$, and again there is

a dark line. In like manner it can be shown that there is a dark band when $d=\frac{2\lambda}{2}, \frac{3\lambda}{2}, \frac{4\lambda}{2}$, etc., for the corresponding retardation is $\frac{5\lambda}{2}, \frac{7\lambda}{2}, \frac{9\lambda}{2}$, etc.

If the dark lines are numbered in order from a to b, 0, 1, 2, 3, 4, etc., it is seen that twice any one of these numbers, plus 1, gives the number of half wave-lengths of retardation. Hence

$$(2n+1)\frac{\lambda}{2} = 2d + \frac{\lambda}{2}$$

$$\therefore \frac{2n\lambda}{2} = 2d$$

and
$$\lambda = \frac{2d}{n}$$

9. The Michelson Interferometer

A form of interferometer invented by Professor Michelson can be conveniently used in making exact physical measurements in a variety of experiments.

The apparatus in a simple and useful form consists of four glass plates whose faces are optically plane. M_1 and M_2, Fig. 224, are silvered on the front surface. M_1 is fixed in position except that its plane may be finely adjusted by means of thumb-screws. M_2 may be moved back and forth by turning a small crank or a worm gear. A and C are polished glass plates with parallel sides. Rays of light from some source x are made parallel by a lens L, pass through plate A, and are in part reflected from the surface ab to M_2, the other part passing on to M_1. The rays in both cases are reflected back to the surface ab and pass thence to the eye at E. The purpose of the plate C is to make the two paths optically equal. Plates A and C are first made in one piece and then cut in two and made exactly equal in thickness. The rays reflected to M_2 pass three times through A, and those reflected from M_1 pass once through A and twice through C before reaching the eye at E. Plate C is therefore called the compensator.

By careful adjustment the mirror M_2 and the virtual image of M_1 may be made exactly parallel, and when the difference in their optical paths is one-half wave-length, there will be destructive

interference and a series of dark and bright bands will be observed. These will move across the field of view when M_2 is moved forward or backward. These bands or fringes are most readily found when the light is monochromatic, as when a sodium flame is used. A mercury vapor lamp is also excellent for this purpose.

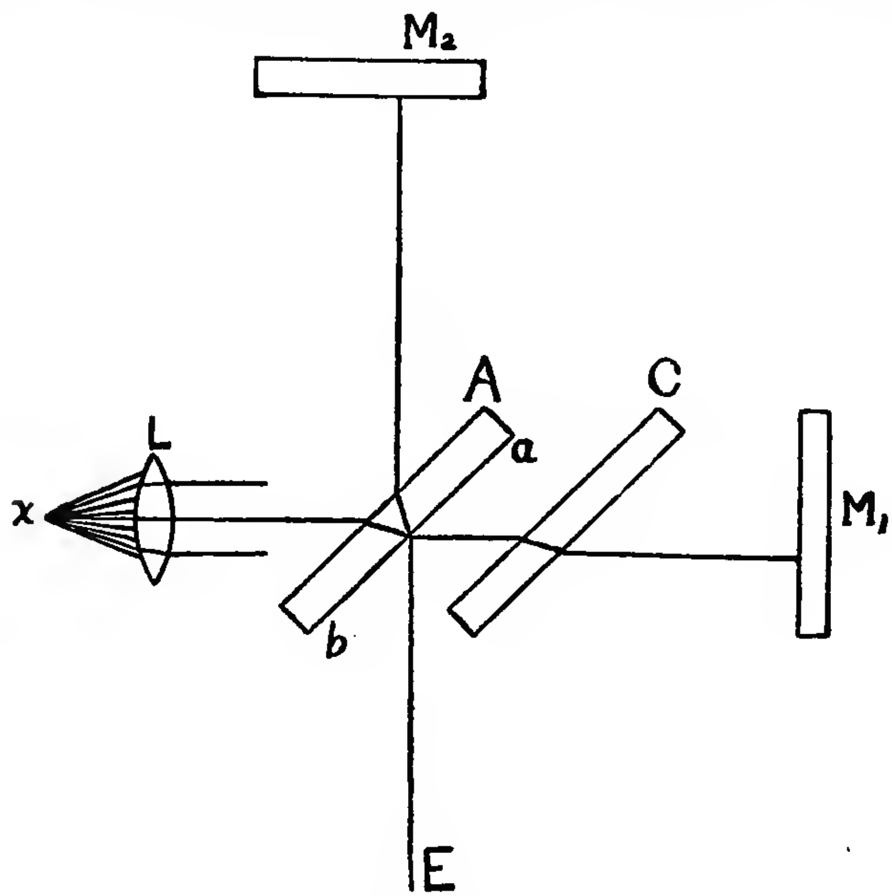

FIG. 224.

The number of waves of light in a given length may be counted by focusing a micrometer microscope on a given mark of a scale placed on the carrier of mirror M_2, and then counting the number of dark bands that cross the field while M_2 is moved, say, 2 or 3 tenths of a millimetre. The number for 1 mm. is about 3000, but will be more or less for different wave-lengths.

When white light is used the mirrors M_1 and M_2 must be exactly the same distance from ab. The fringes are then colored except the central band, which is dark.

A great deal of patience and care is necessary at first in attempting to operate an instrument of this kind, for wave-lengths are very short and a very slight change in the adjustment will produce a marked change in results. This is particularly true for white light.

In a book entitled " Light Waves and Their Uses," by Professor Michelson, the student will find a description of many uses to which an interferometer may be put, and also a full account of the methods employed in measuring the standard metre in terms of light waves.

10. Specific Inductive Capacity, K

Air, standard pressure	1	Mica	4–8
Beeswax	1.86	Paraffin, solid	2–2.3
Flint glass, light	6.85	Petroleum	2–3
Flint glass, heavy	10	Porcelain	4.38
Glass, common	3	Resin	2.5
Hard rubber	2–3	Shellac	2.7–3.6
Hydrogen	.9997	Sulphur	2.5–4
India rubber, pure	2.2–2.5	Turpentine	2.2

11. Electrochemical Equivalents

Substance.	Atomic weight.	Valence.	Grams per coulomb.
Aluminum	27.1	3	.000936
Antimony	120.2	3	.000415
Arsenic	75	3	.000259
Bismuth	208.5	3	.0007199
Bromine	79.96	1	.0008283
Chlorine	35.45	1	.0003672
Copper	63.6	1	.0006588
		2	.0003294
Gold	197.2	3	.0006809
Helium	4	..	......
Hydrogen	1.008	1	.00001036
Iron	55.9	2	.0002895
		3	.000193
Lead	206.9	2	.0010716
Lithium	7.03	1	.0000728
Magnesium	24.36	2	.0001262
Manganese	55	2	.0002849
		4	.0001424
Mercury	200	1	.0020717
Nickel	58.7	2	.0003040
Nitrogen	14.04	3	.0000485
Oxygen	16	2	.0000829
Platinum	194.8	2	.0010098
Potassium	39.15	1	.0004055
Radium	225	..	......
Silver	107.93	1	.0011180
Sodium	23.05	1	.0002387
Sulphur	32.06	2	.000166
Tin	119	2	.0006163
Uranium	238.5	2	.0012353
Zinc	65.4	2	.0003387

12. Specific Resistance and Temperature Coefficient

Metal.	Resistance per c.c.	Resistance per mil-foot at 20° C.	Temperature coefficient.
Aluminum	$3(10)^{-6}$	17.4	.00435
Copper, annealed	$1.584(10)^{-6}$	10.4	.0042
Copper, hard	$1.619(10)^{-6}$	10.65	......
German silver	$20(10)^{-6}$	114–270	.00025
Iron, annealed	$10.5(10)^{-6}$	90	.005
Manganin	$42(10)^{-6}$	250–450	.00001
Mercury	$94(10)^{-6}$	....	.00075
Platinum	$8.9(10)^{-6}$	58	.00366
Silver	$1.5(10)^{-6}$	9.53	.00377
Tungsten	....	....	.005

13. Thermoelectric Power in Microvolts with Respect to Lead

Temp.=20° C.

Metal.	Microvolts.	Metal.	Microvolts.
Bismuth	−89	Silver	3
Cobalt	−22	Zinc	3.7
German silver	−12	Copper, pure	3.8
Mercury	− 4.2	Iron	17.5
Lead	0	Antimony	24
Tin	1.0	Tellurium	502
Gold	1.2	Selenium	807

When a junction of any two of these metals is heated a current will flow from the higher to the lower metal in the series.

The thermoelectric power of any two metals is the difference of their numbers as given in the table.

14. Wave-lengths of Fraunhofer Lines

Letter.	Line due to	Wave-length in microns.	Letter.	Line due to	Wave-length in microns.
A	Sun	.7604	E_1	Fe	.5270
a	Sun	.718478	b_1	Mg	.518379
B	O	.6870	F	H	.48615
C	H	.6563	G	Fe	.4308
D_1	Na	.5896	H	Ca	.39686
D_2	Na	.5890	K	Ca	.3934
D_3	He	.587598			

15. Indices of Refraction

Substance.	Description.	Indices.			
		Red (C).	Yellow (D).	Blue (F).	Violet (H).
Air		1.000293	1.000294	1.000296	1.0003
Water		1.3318	1.3336	1.3377	1.3442
Carbon disulphide		1.6336	1.6433	1.6688	1.7175
Quartz	Ordinary ray	1.5419	1.5442	1.5496	1.5581
Quartz	Extraordinary ray	1.5509	1.5534	1.5589	1.5677
Iceland spar	Ordinary ray	1.65446	1.65846	1.6679	1.6833
Iceland spar	Extraordinary ray	1.48474	1.48654	1.4908	1.4978
Crown glass	Light	1.5254	1.5280	1.5343	1.5443
Crown glass	Dense	1.5568	1.5604	1.5690	1.5836
Flint glass	Light	1.5783	1.5822	1.5929	1.6098
Flint glass	Dense	1.6795	1.6858	1.7019	1.7306

16. Velocity of Sound

Substance.	Meters per second.	Substance.	Meters per second.
Air (0° C.)	331	Hydrogen	1269
Alcohol	1260	Iron	5000
Aluminum	5100	Marble	3810
Brass	3500	Oak wood (along the fibre)	3850
Brick	3652	Oxygen	317
Carbon dioxide	262	Pine wood (along the fibre)	3320
Copper	3560	Platinum	2690
Glass	5000 to 6000	Silver	2610
Granite	3950	Slate	4510

17. Natural Sines and Cosines

	SINE							
Deg.	0′	10′	20′	30′	40′	50′	60′	Deg.
0	0.00000	0.00291	0.00582	0.00873	0.01164	0.01454	0.01745	89
1	0.01745	0.02036	0.02327	0.02618	0.02908	0.03199	0.03490	88
2	0.03490	0.03781	0.04071	0.04362	0.04653	0.04943	0.05234	87
3	0.05234	0.05524	0.05814	0.06105	0.06395	0.06685	0.06976	86
4	0.06976	0.07266	0.07556	0.07846	0.08136	0.08426	0.08716	85
5	0.08716	0.09005	0.09295	0.09585	0.09874	0.10164	0.10453	84
6	0.10453	0.10742	0.11031	0.11320	0.11609	0.11898	0.12187	83
7	0.12187	0.12476	0.12764	0.13053	0.13341	0.13629	0.13917	82
8	0.13917	0.14205	0.14493	0.14781	0.15069	0.15356	0.15643	81
9	0.15643	0.15931	0.16218	0.16505	0.16792	0.17078	0.17365	80
10	0.17365	0.17651	0.17937	0.18224	0.18509	0.18795	0.19081	79
11	0.19081	0.19366	0.19652	0.19937	0.20222	0.20507	0.20791	78
12	0.20791	0.21076	0.21360	0.21644	0.21928	0.22212	0.22495	77
13	0.22495	0.22778	0.23062	0.23345	0.23627	0.23910	0.24192	76
14	0.24192	0.24474	0.24756	0.25038	0.25320	0.25601	0.25882	75
15	0.25882	0.26163	0.26443	0.26724	0.27004	0.27284	0.27564	74
16	0.27564	0.27843	0.28123	0.28402	0.28680	0.28959	0.29237	73
17	0.29237	0.29515	0.29793	0.30071	0.30348	0.30625	0.30902	72
18	0.30902	0.31178	0.31454	0.31730	0.32006	0.32282	0.32557	71
19	0.32557	0.32832	0.33106	0.33381	0.33655	0.33929	0.34202	70
20	0.34202	0.34475	0.34748	0.35021	0.35293	0.35565	0.35837	69
21	0.35837	0.36108	0.36379	0.36650	0.36921	0.37191	0.37461	68
22	0.37461	0.37730	0.37999	0.38268	0.38537	0.38805	0.39073	67
23	0.39073	0.39341	0.39608	0.39875	0.40142	0.40408	0.40674	66
24	0.40674	0.40939	0.41204	0.41469	0.41734	0.41998	0.42262	65
25	0.42262	0.42525	0.42788	0.43051	0.43313	0.43575	0.43837	64
26	0.43837	0.44098	0.44359	0.44620	0.44880	0.45140	0.45399	63
27	0.45399	0.45658	0.45917	0.46175	0.46433	0.46690	0.46947	62
28	0.46947	0.47204	0.47460	0.47716	0.47971	0.48226	0.48481	61
29	0.48481	0.48735	0.48989	0.49242	0.49495	0.49748	0.50000	60
30	0.50000	0.50252	0.50503	0.50754	0.51004	0.51254	0.51504	59
31	0.51504	0.51753	0.52002	0.52250	0.52498	0.52745	0.52992	58
32	0.52992	0.53238	0.53484	0.53730	0.53975	0.54220	0.54464	57
33	0.54464	0.54708	0.54951	0.55194	0.55436	0.55678	0.55919	56
34	0.55919	0.56160	0.56401	0.56641	0.56880	0.57119	0.57358	55
35	0.57358	0.57596	0.57833	0.58070	0.58307	0.58543	0.58779	54
36	0.58779	0.59014	0.59248	0.59482	0.59716	0.59949	0.60182	53
37	0.60182	0.60414	0.60645	0.60876	0.61107	0.61337	0.61566	52
38	0.61566	0.61795	0.62024	0.62251	0.62479	0.62706	0.62932	51
39	0.62932	0.63158	0.63383	0.63608	0.63832	0.64056	0.64279	50
40	0.64279	0.64501	0.64723	0.64945	0.65166	0.65386	0.65606	49
41	0.65606	0.65825	0.66044	0.66262	0.66480	0.66697	0.66913	48
42	0.66913	0.67129	0.67344	0.67559	0.67773	0.67987	0.68200	47
43	0.68200	0.68412	0.68624	0.68835	0.69046	0.69256	0.69466	46
44	0.69466	0.69675	0.69883	0.70091	0.70298	0.70505	0.70711	45
	60′	50′	40′	30′	20′	10′	0′	
	COSINE							

Natural Sines and Cosines—*Continued*

	COSINE							
Deg.	0′	10′	20′	30′	40′	50′	60′	Deg.
0	1.00000	1.00000	0.99998	0.99996	0.99993	0.99989	0.99985	89
1	0.99985	0.99979	0.99973	0.99966	0.99958	0.99949	0.99939	88
2	0.99939	0.99929	0.99917	0.99905	0.99892	0.99878	0.99863	87
3	0.99863	0.99847	0.99831	0.99813	0.99795	0.99776	0.99756	86
4	0.99756	0.99736	0.99714	0.99692	0.99668	0.99644	0.99619	85
5	0.99619	0.99594	0.99567	0.99540	0.99511	0.99482	0.99452	84
6	0.99452	0.99421	0.99390	0.99357	0.99324	0.99290	0.99255	83
7	0.99255	0.99219	0.99182	0.99144	0.99106	0.99067	0.99027	82
8	0.99027	0.98986	0.98944	0.98902	0.98858	0.98814	0.98769	81
9	0.98769	0.98723	0.98676	0.98629	0.98580	0.98531	0.98481	80
10	0.98481	0.98430	0.98378	0.98325	0.98272	0.98218	0.98163	79
11	0.98163	0.98107	0.98050	0.97992	0.97934	0.97875	0.97815	78
12	0.97815	0.97754	0.97692	0.97630	0.97566	0.97502	0.97437	77
13	0.97437	0.97371	0.97304	0.97237	0.97169	0.97100	0.97030	76
14	0.97030	0.96959	0.96887	0.96815	0.96742	0.96667	0.96593	75
15	0.96593	0.96517	0.96440	0.96363	0.96285	0.96206	0.96126	74
16	0.96126	0.96046	0.95964	0.95882	0.95799	0.95715	0.95630	73
17	0.95630	0.95545	0.95459	0.95372	0.95284	0.95195	0.95106	72
18	0.95106	0.95015	0.94924	0.94832	0.94740	0.94646	0.94552	71
19	0.94552	0.94457	0.94361	0.94264	0.94167	0.94068	0.93969	70
20	0.93969	0.93869	0.93769	0.93667	0.93565	0.93462	0.93358	69
21	0.93358	0.93253	0.93148	0.93042	0.92935	0.92827	0.92718	68
22	0.92718	0.92609	0.92499	0.92388	0.92276	0.92164	0.92050	67
23	0.92050	0.91936	0.91822	0.91706	0.91590	0.91472	0.91355	66
24	0.91355	0.91236	0.91116	0.90996	0.90875	0.90753	0.90631	65
25	0.90631	0.90507	0.90383	0.90259	0.90133	0.90007	0.89879	64
26	0.89879	0.89752	0.89623	0.89493	0.89363	0.89232	0.89101	63
27	0.89101	0.88968	0.88835	0.88701	0.88566	0.88431	0.88295	62
28	0.88295	0.88158	0.88020	0.87882	0.87743	0.87603	0.87462	61
29	0.87462	0.87321	0.87178	0.87036	0.86892	0.86748	0.86603	60
30	0.86603	0.86457	0.86310	0.86163	0.86015	0.85866	0.85717	59
31	0.85717	0.85567	0.85416	0.85264	0.85112	0.84959	0.84805	58
32	0.84805	0.84650	0.84495	0.84339	0.84182	0.84025	0.83867	57
33	0.83867	0.83708	0.83549	0.83389	0.83228	0.83066	0.82904	56
34	0.82904	0.82741	0.82577	0.82413	0.82248	0.82082	0.81915	55
35	0.81915	0.81748	0.81580	0.81412	0.81242	0.81072	0.80902	54
36	0.80902	0.80730	0.80558	0.80386	0.80212	0.80038	0.79864	53
37	0.79864	0.79688	0.79512	0.79335	0.79158	0.78980	0.78801	52
38	0.78801	0.78622	0.78442	0.78261	0.78079	0.77897	0.77715	51
39	0.77715	0.77531	0.77347	0.77162	0.76977	0.76791	0.76604	50
40	0.76604	0.76417	0.76229	0.76041	0.75851	0.75661	0.75471	49
41	0.75471	0.75280	0.75088	0.74896	0.74703	0.74509	0.74314	48
42	0.74314	0.74120	0.73924	0.73728	0.73531	0.73333	0.73135	47
43	0.73135	0.72937	0.72737	0.72537	0.72337	0.72136	0.71934	46
44	0.71934	0.71732	0.71529	0.71325	0.71121	0.70916	0.70711	45
	60′	50′	40′	30′	20′	10′	0′	
	SINE							

18. Natural Tangents and Cotangents

	TANGENT							
Deg.	0′	10′	20′	30′	40′	50′	60′	Deg.
0	0.00000	0.00291	0.00582	0.00873	0.01164	0.01455	0.01746	89
1	0.01746	0.02036	0.02328	0.02619	0.02910	0.03201	0.03492	88
2	0.03492	0.03783	0.04075	0.04366	0.04658	0.04949	0.05241	87
3	0.05241	0.05533	0.05824	0.06116	0.06408	0.06700	0.06993	86
4	0.06993	0.07285	0.07578	0.07870	0.08163	0.08456	0.08749	85
5	0.08749	0.09042	0.09335	0.09629	0.09923	0.10216	0.10510	84
6	0.10510	0.10805	0.11099	0.11394	0.11688	0.11983	0.12278	83
7	0.12278	0.12574	0.12869	0.13165	0.13461	0.13758	0.14054	82
8	0.14054	0.14351	0.14648	0.14945	0.15243	0.15540	0.15838	81
9	0.15838	0.16137	0.16435	0.16734	0.17033	0.17333	0.17633	80
10	0.17633	0.17933	0.18233	0.18534	0.18835	0.19136	0.19438	79
11	0.19438	0.19740	0.20042	0.20345	0.20648	0.20952	0.21256	78
12	0.21256	0.21560	0.21864	0.22169	0.22475	0.22781	0.23087	77
13	0.23087	0.23393	0.23700	0.24008	0.24316	0.24624	0.24933	76
14	0.24933	0.25242	0.25552	0.25862	0.26172	0.26483	0.26795	75
15	0.26795	0.27107	0.27419	0.27732	0.28046	0.28360	0.28675	74
16	0.28675	0.28990	0.29305	0.29621	0.29938	0.30255	0.30573	73
17	0.30573	0.30891	0.31210	0.31530	0.31850	0.32171	0.32492	72
18	0.32492	0.32814	0.33136	0.33460	0.33783	0.34108	0.34433	71
19	0.34433	0.34758	0.35085	0.35412	0.35740	0.36068	0.36397	70
20	0.36397	0.36727	0.37057	0.37388	0.37720	0.38053	0.38386	69
21	0.38386	0.38721	0.39055	0.39391	0.39727	0.40065	0.40403	68
22	0.40403	0.40741	0.41081	0.41421	0.41763	0.42105	0.42447	67
23	0.42447	0.42791	0.43136	0.43481	0.43828	0.44175	0.44523	66
24	0.44523	0.44872	0.45222	0.45573	0.45924	0.46277	0.46631	65
25	0.46631	0.46985	0.47341	0.47698	0.48055	0.48414	0.48773	64
26	0.48773	0.49134	0.49495	0.49858	0.50222	0.50587	0.50953	63
27	0.50953	0.51320	0.51688	0.52057	0.52427	0.52798	0.53171	62
28	0.53171	0.53545	0.53920	0.54296	0.54673	0.55051	0.55431	61
29	0.55431	0.55812	0.56194	0.56577	0.56962	0.57348	0.57735	60
30	0.57735	0.58124	0.58513	0.58905	0.59297	0.59691	0.60086	59
31	0.60086	0.60483	0.60881	0.61280	0.61681	0.62083	0.62487	58
32	0.62487	0.62892	0.63299	0.63707	0.64117	0.64528	0.64941	57
33	0.64941	0.65355	0.65771	0.66189	0.66608	0.67028	0.67451	56
34	0.67451	0.67875	0.68301	0.68728	0.69157	0.69588	0.70021	55
35	0.70021	0.70455	0.70891	0.71329	0.71769	0.72211	0.72654	54
36	0.72654	0.73100	0.73547	0.73996	0.74447	0.74900	0.75355	53
37	0.75355	0.75812	0.76272	0.76733	0.77196	0.77661	0.78129	52
38	0.78129	0.78598	0.79070	0.79544	0.80020	0.80498	0.80978	51
39	0.80978	0.81461	0.81946	0.82434	0.82923	0.83415	0.83910	50
40	0.83910	0.84407	0.84906	0.85408	0.85912	0.86419	0.86929	49
41	0.86929	0.87441	0.87955	0.88473	0.88992	0.89515	0.90040	48
42	0.90040	0.90569	0.91099	0.91633	0.92170	0.92709	0.93252	47
43	0.93252	0.93797	0.94345	0.94896	0.95451	0.96008	0.96569	46
44	0.96569	0.97133	0.97700	0.98270	0.98843	0.99420	1.00000	45
	60′	50′	40′	30′	20′	10′	0′	
	COTANGENT							

Natural Tangents and Cotangents—*Continued*

	COTANGENT							
Deg.	0′	10′	20′	30′	40′	50′	60′	Deg.
0	Infini	343.77371	171.88540	114.58865	85.93979	68.75009	57.28996	89
1	57.28996	49.10388	42.96408	38.18846	34.36777	31.24158	28.63625	88
2	28.63625	26.43160	24.54176	22.90377	21.47040	20.20555	19.08114	87
3	19.08114	18.07498	17.16934	16.34986	15.60478	14.92442	14.30067	86
4	14.30067	13.72674	13.19688	12.70621	12.25051	11.82617	11.43005	85
5	11.43005	11.05943	10.71191	10.38540	10.07803	9.78817	9.51436	84
6	9.51436	9.25530	9.00983	8.77689	8.55555	8.34496	8.14435	83
7	8.14435	7.95302	7.77035	7.59575	7.42871	7.26873	7.11537	82
8	7.11537	6.96823	6.82694	6.69116	6.56055	6.43484	6.31375	81
9	6.31375	6.19703	6.08444	5.97576	5.87080	5.76937	5.67128	80
10	5.67128	5.57638	5.48451	5.39552	5.30928	5.22566	5.14455	79
11	5.14455	5.06584	4.98940	4.91516	4.84300	4.77286	4.70463	78
12	4.70463	4.63825	4.57363	4.51071	4.44942	4.38969	4.33148	77
13	4.33148	4.27471	4.21933	4.16530	4.11256	4.06107	4.01078	76
14	4.01078	3.96165	3.91364	3.86671	3.82083	3.77595	3.73205	75
15	3.73205	3.68909	3.64705	3.60588	3.56577	3.52609	3.48741	74
16	3.48741	3.44951	3.41236	3.37594	3.34023	3.30521	3.27085	73
17	3.27085	3.23714	3.20406	3.17159	3.13972	3.10842	3.07768	72
18	3.07768	3.04749	3.01783	2.98869	2.96004	2.93189	2.90421	71
19	2.90421	2.87700	2.85023	2.82391	2.79802	2.77254	2.74748	70
20	2.74748	2.72281	2.69853	2.67462	2.65109	2.62791	2.60509	69
21	2.60509	2.58261	2.56046	2.53865	2.51715	2.49597	2.47509	68
22	2.47509	2.45451	2.43422	2.41421	2.39449	2.37504	2.35585	67
23	2.35585	2.33693	2.31826	2.29984	2.28167	2.26374	2.24604	66
24	2.24604	2.22857	2.21132	2.19430	2.17749	2.16090	2.14451	65
25	2.14451	2.12832	2.11233	2.09654	2.08094	2.06553	2.05030	64
26	2.05030	2.03526	2.02039	2.00569	1.99116	1.97680	1.96261	63
27	1.96261	1.94858	1.93470	1.92098	1.90741	1.89400	1.88073	62
28	1.88073	1.86760	1.85462	1.84177	1.82906	1.81649	1.80405	61
29	1.80405	1.79174	1.77955	1.76749	1.75556	1.74375	1.73205	60
30	1.73205	1.72047	1.70901	1.69766	1.68643	1.67530	1.66428	59
31	1.66428	1.65337	1.64256	1.63185	1.62125	1.61074	1.60033	58
32	1.60033	1.59002	1.57981	1.56969	1.55966	1.54972	1.53987	57
33	1.53987	1.53010	1.52043	1.50184	1.50133	1.49190	1.48256	56
34	1.48256	1.47330	1.46411	1.45501	1.44598	1.43703	1.42815	55
35	1.42815	1.41934	1.41061	1.40195	1.39336	1.38484	1.37638	54
36	1.37638	1.36800	1.35968	1.35142	1.34323	1.33511	1.32704	53
37	1.32704	1.31904	1.31110	1.30323	1.29541	1.28764	1.27994	52
38	1.27994	1.27230	1.26471	1.25717	1.24969	1.24227	1.23490	51
39	1.23490	1.22758	1.22031	1.21310	1.20593	1.19882	1.19175	50
40	1.19175	1.18474	1.17777	1.17085	1.16398	1.15715	1.15037	49
41	1.15037	1.14363	1.13694	1.13029	1.12369	1.11713	1.11061	48
42	1.11061	1.10414	1.09770	1.09131	1.08496	1.07864	1.07237	47
43	1.07237	1.06613	1.05994	1.05378	1.04766	1.04158	1.03553	46
44	1.03553	1.02952	1.02355	1.01761	1.01170	1.00583	1.00000	45
	60′	50′	40′	30′	20′	10′	0′	
	TANGENT							

INDEX

E

F

L

M